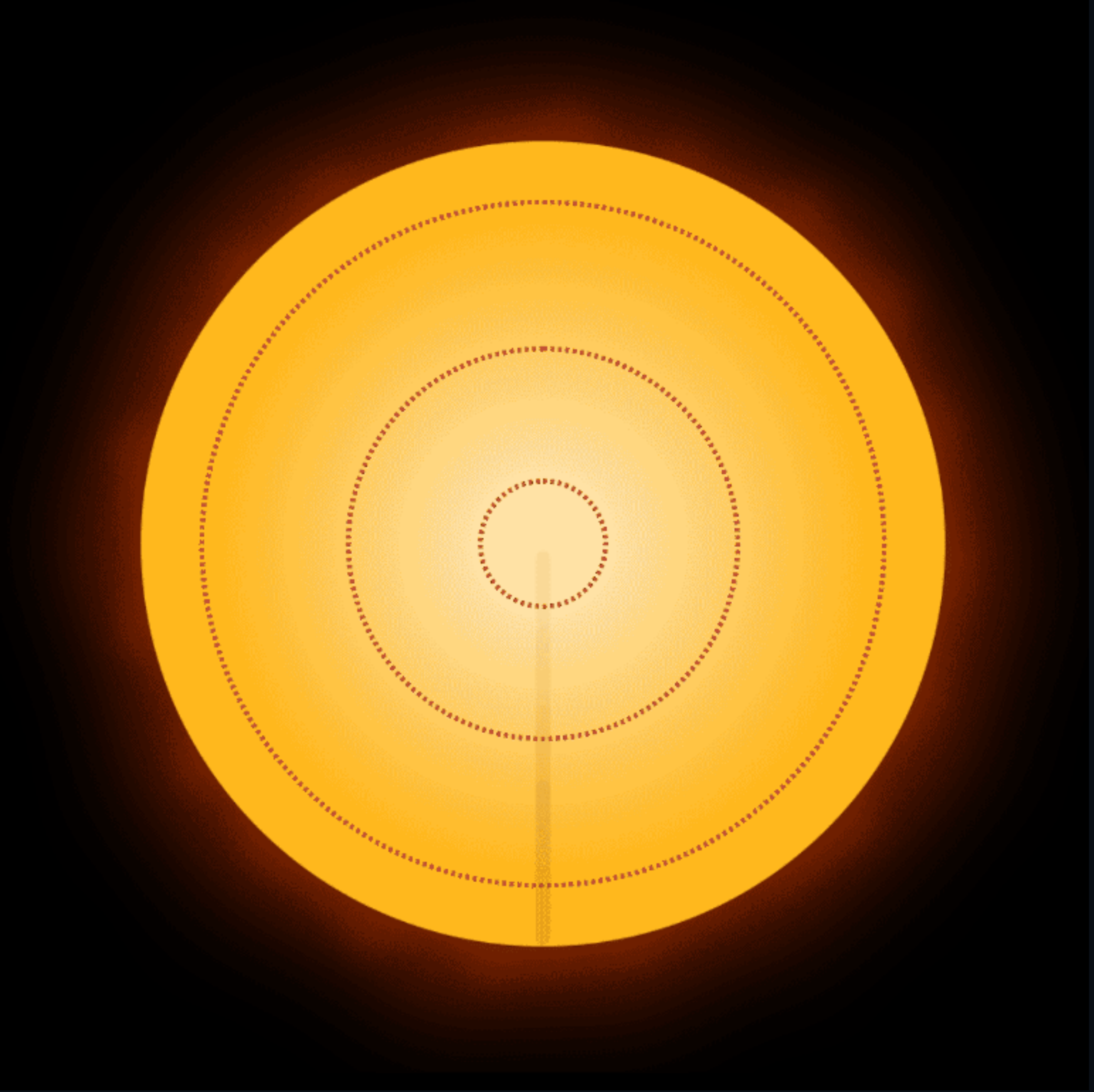


Gravitational-Wave Asteroseismology

Illuminating Dense Nuclear Matter through Dynamical Tides

Fabian Gittins | IReNA Online Seminar | 16 Jan 2026

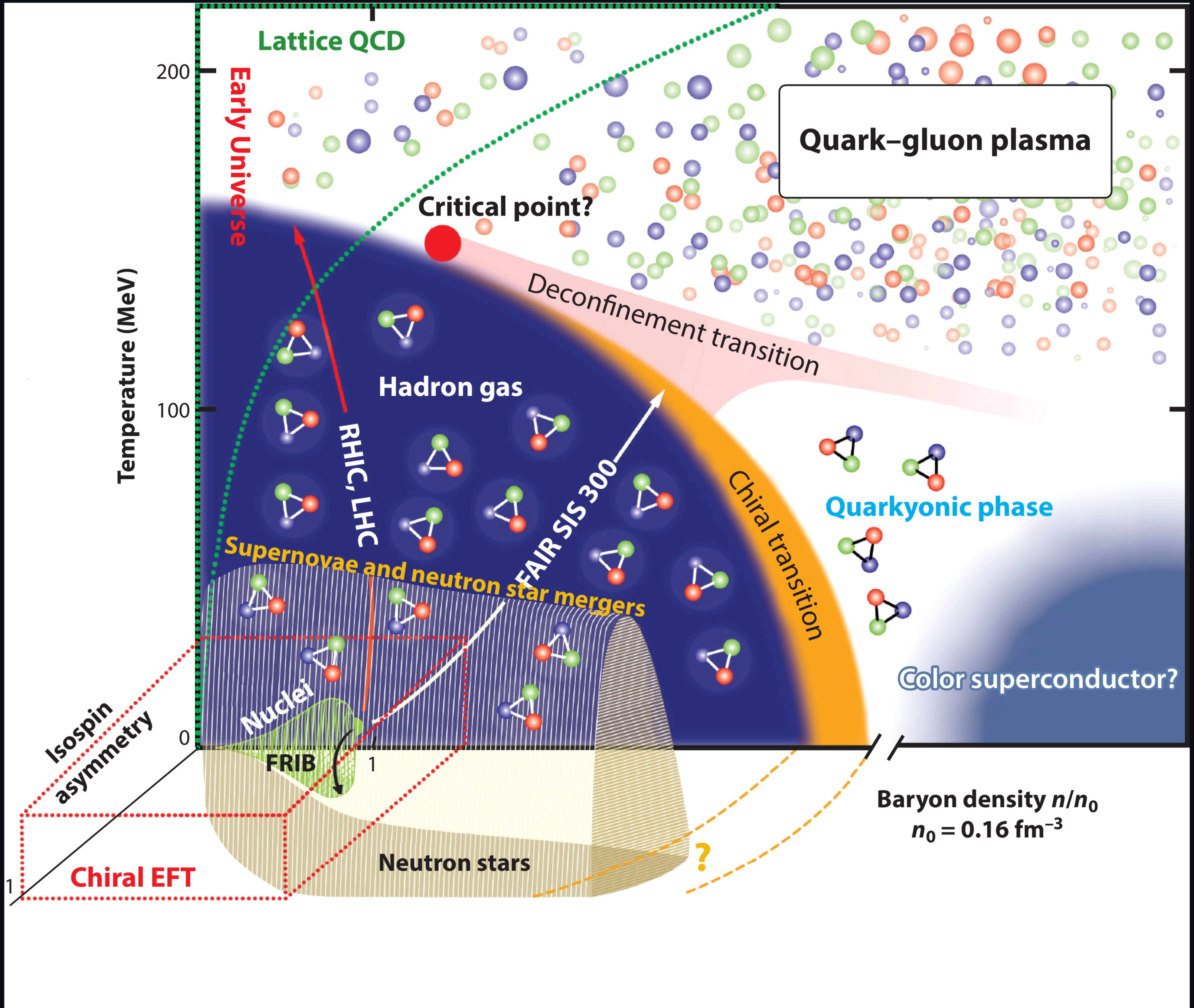
Utrecht University

Funded by the European Union

Overview

- Gravitational waves probe *nuclear physics* through observations of neutron stars
- Tidal dynamics present opportunity to conduct asteroseismology
- Neutron-star oscillation modes are notably rich
- Opportunities and challenges ahead

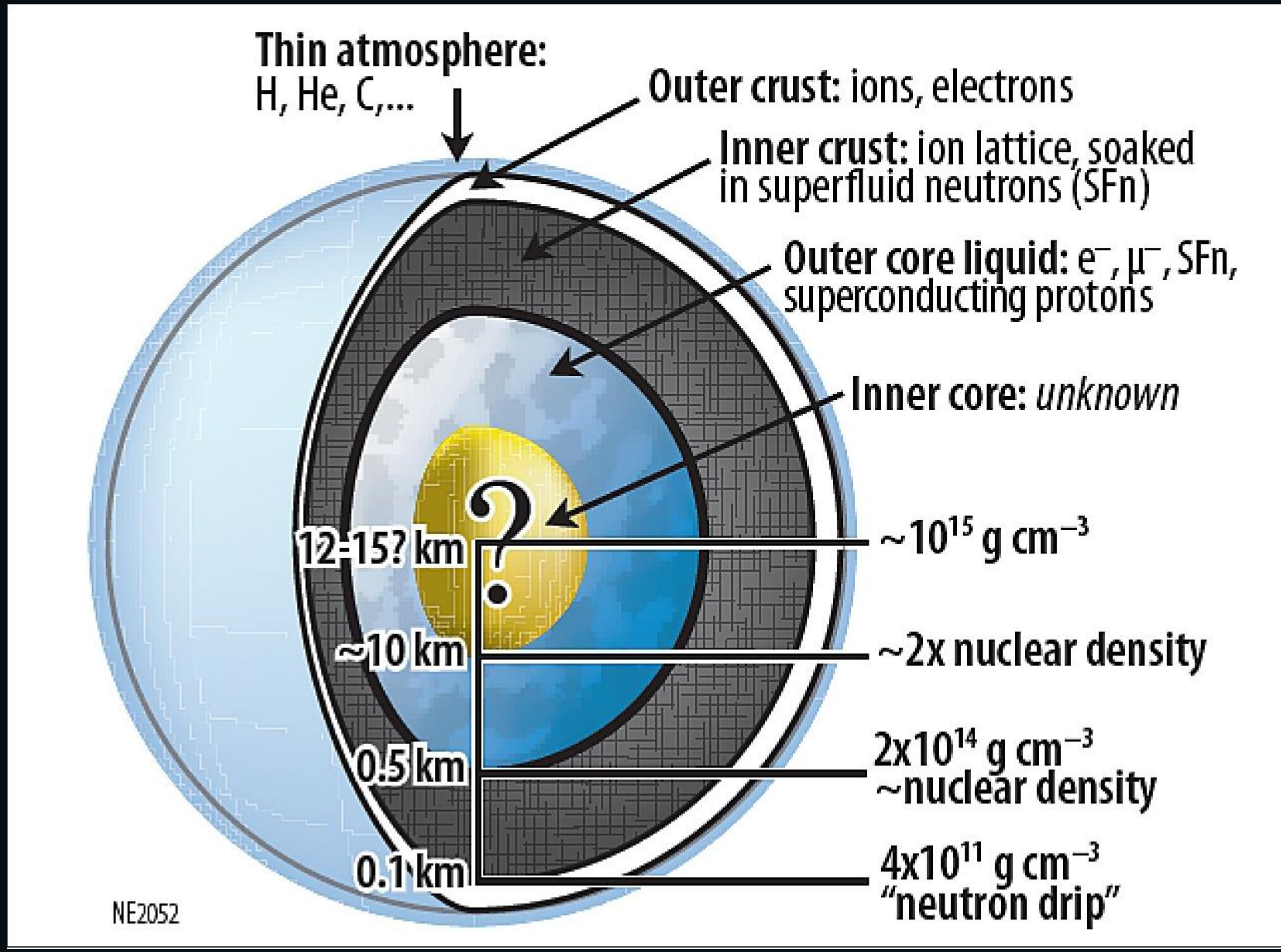
Quantum Chromodynamics (QCD)



Drischler C, et al. 2021
Annu. Rev. Nucl. Part. Sci. 71:403–32

Physics of Neutron Stars

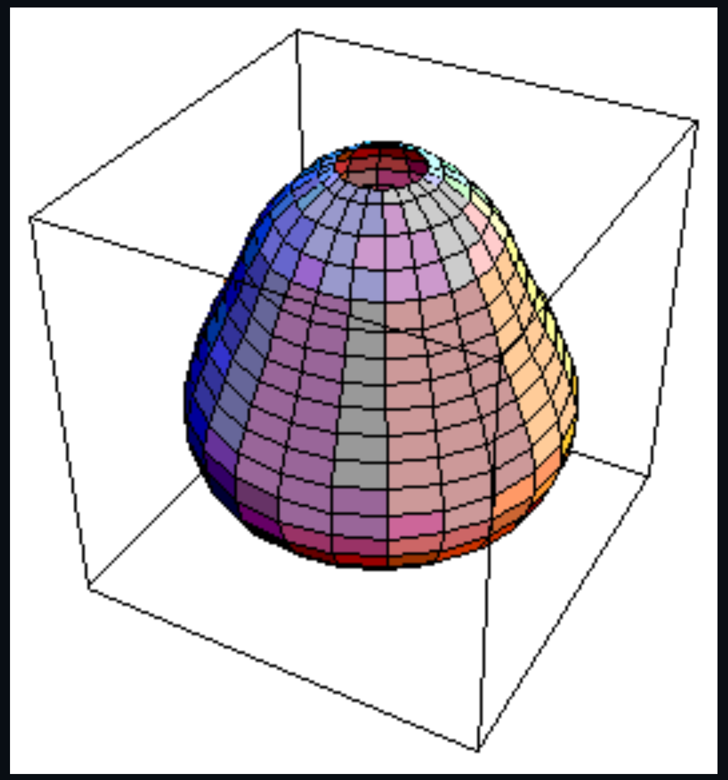
- Neutron stars are extreme laboratories
  - Strong-field gravity
  - Dense nuclear matter
  - Rapid rotation
  - Strong magnetic fields
  - Superfluidity
  - Solid crusts



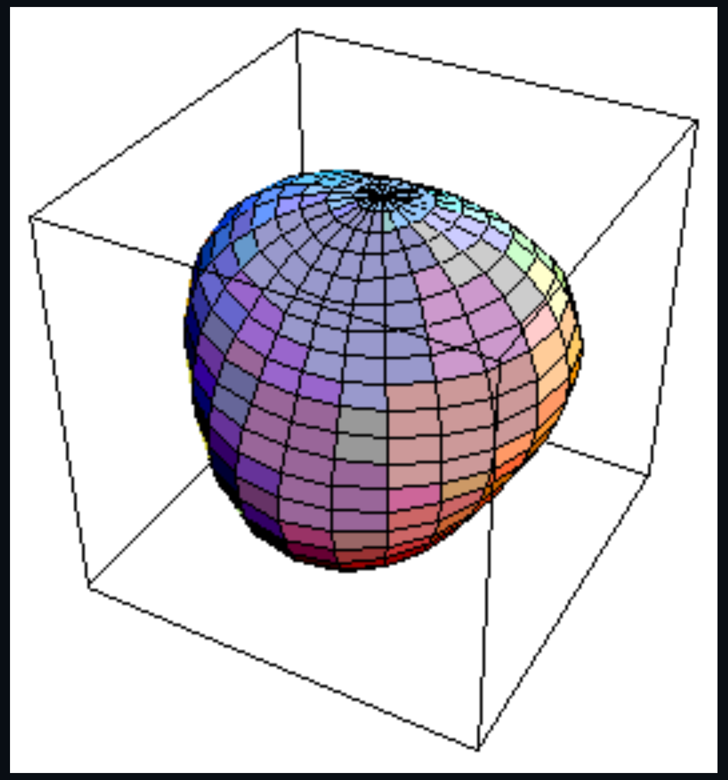
- Each of these aspects give rise to their own family of oscillation modes

Neutron–Star Mode Compendium

- *f*-mode: scales with average density
- *p*-modes: sound waves in the star (overtones of the *f*-mode)
- *g*-modes: buoyancy waves from thermal/composition gradients
- inertial modes (including *r*-modes): associated with rotation
- *i*-modes: arise from phase transitions
- Also:
    - *w*-modes, *s*-modes, Alfvèn modes, …



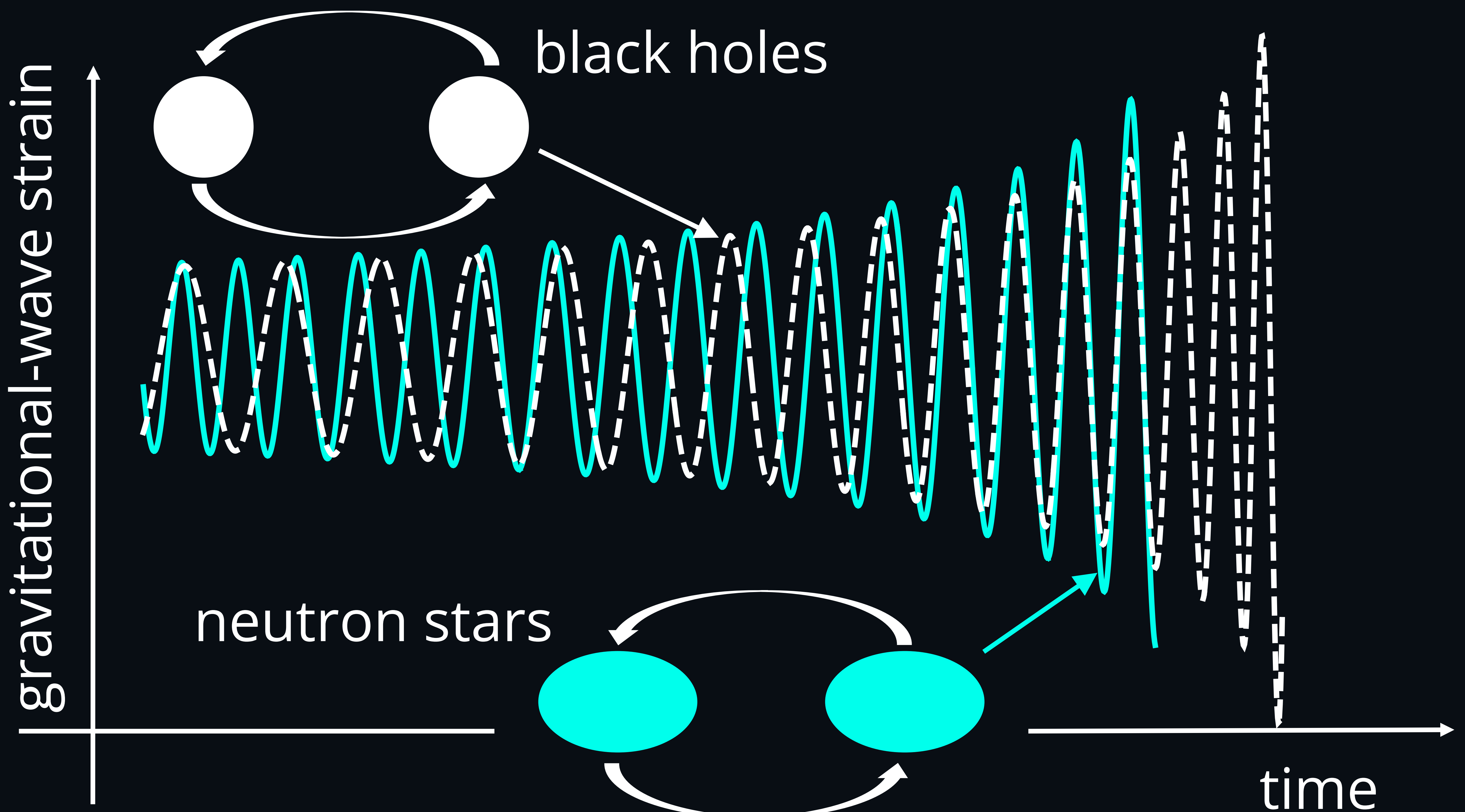
$(l, m) = (3,0)$



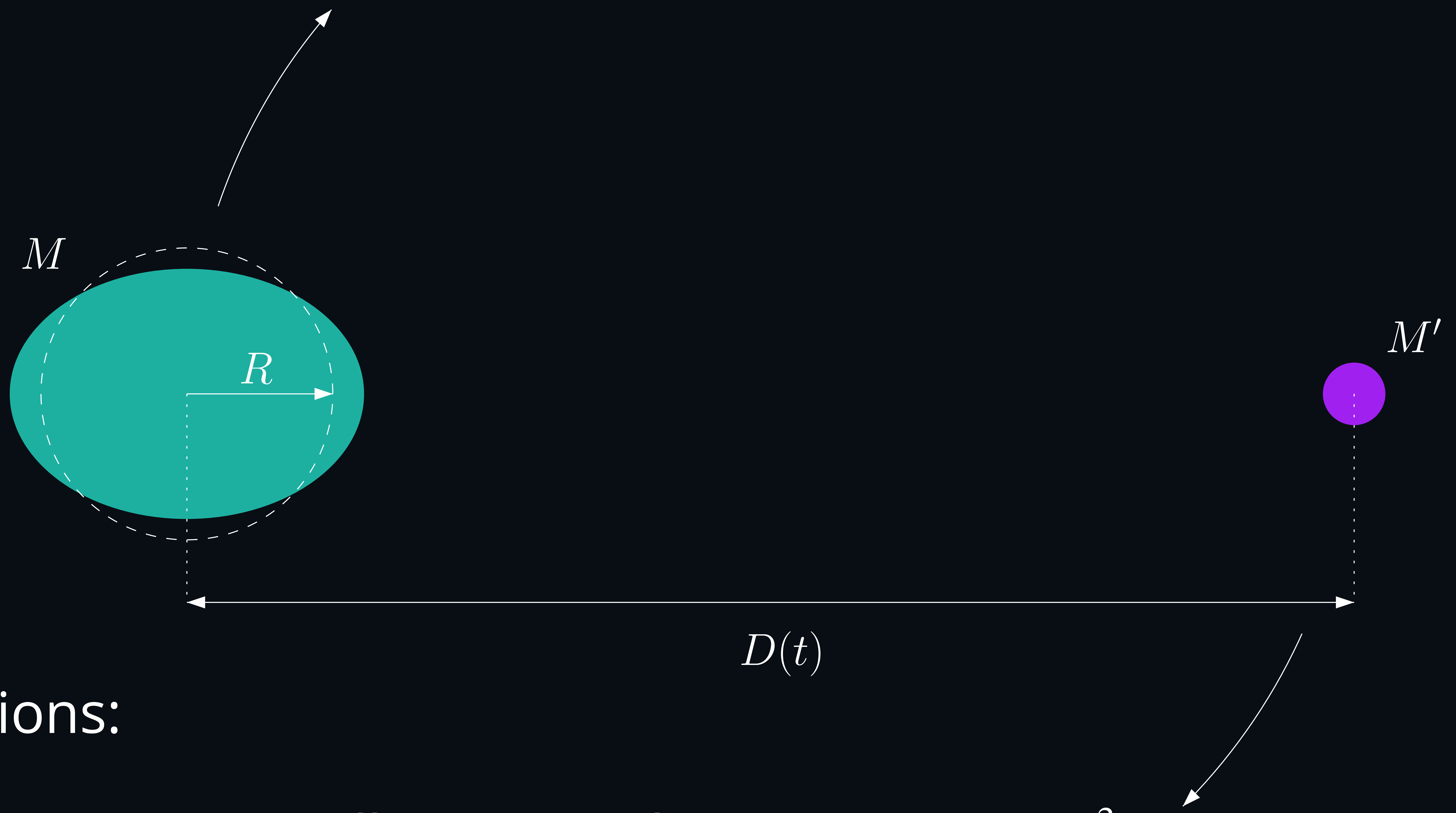
$(l, m) = (3,2)$

Matter Effects

- Consider two compact binaries: one with black holes, while the other comprises neutron stars
- The binaries are otherwise identical; same component *masses*, *spins*, *binary orientation* and *position* with respect to the detectors

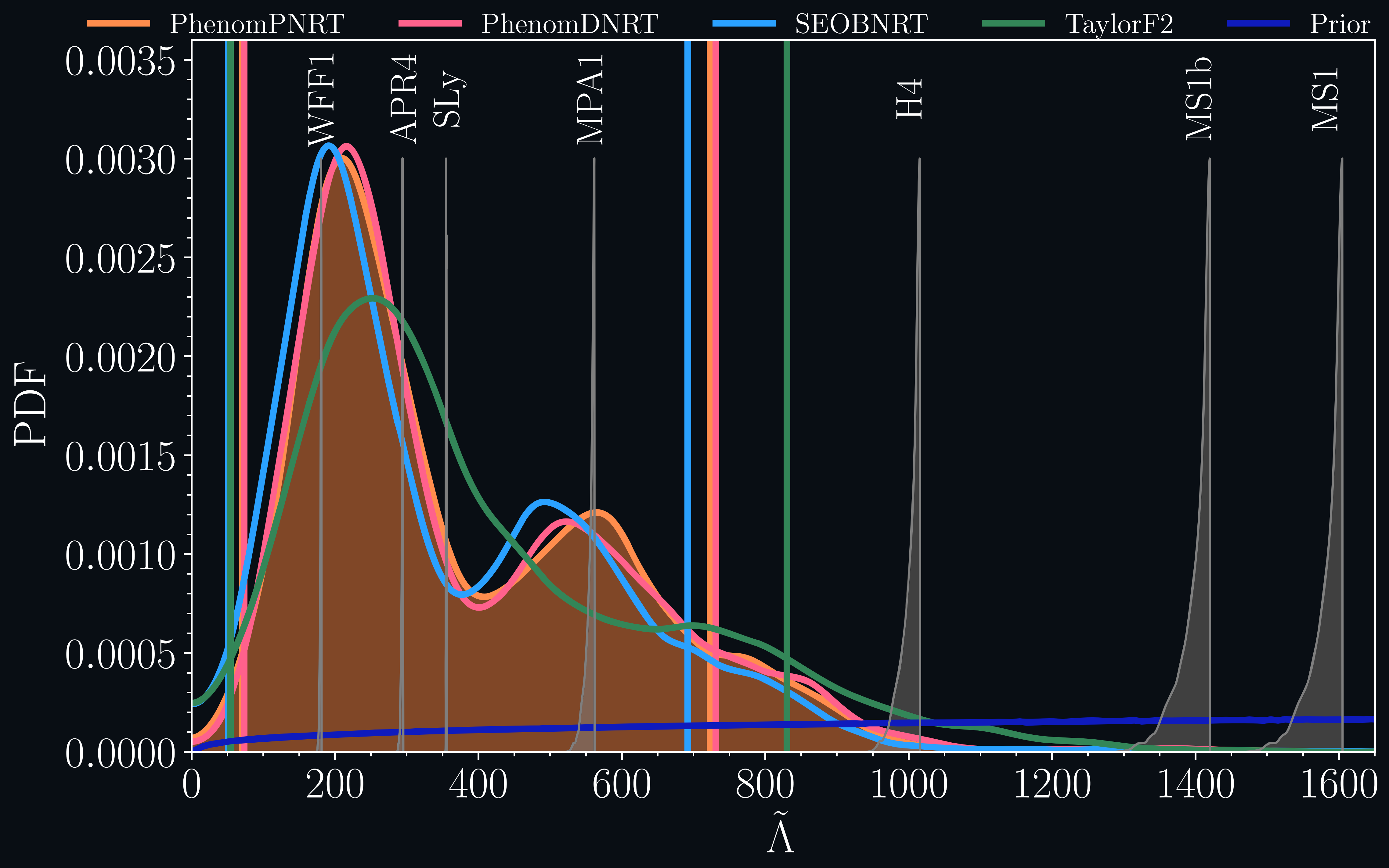


Static Tide

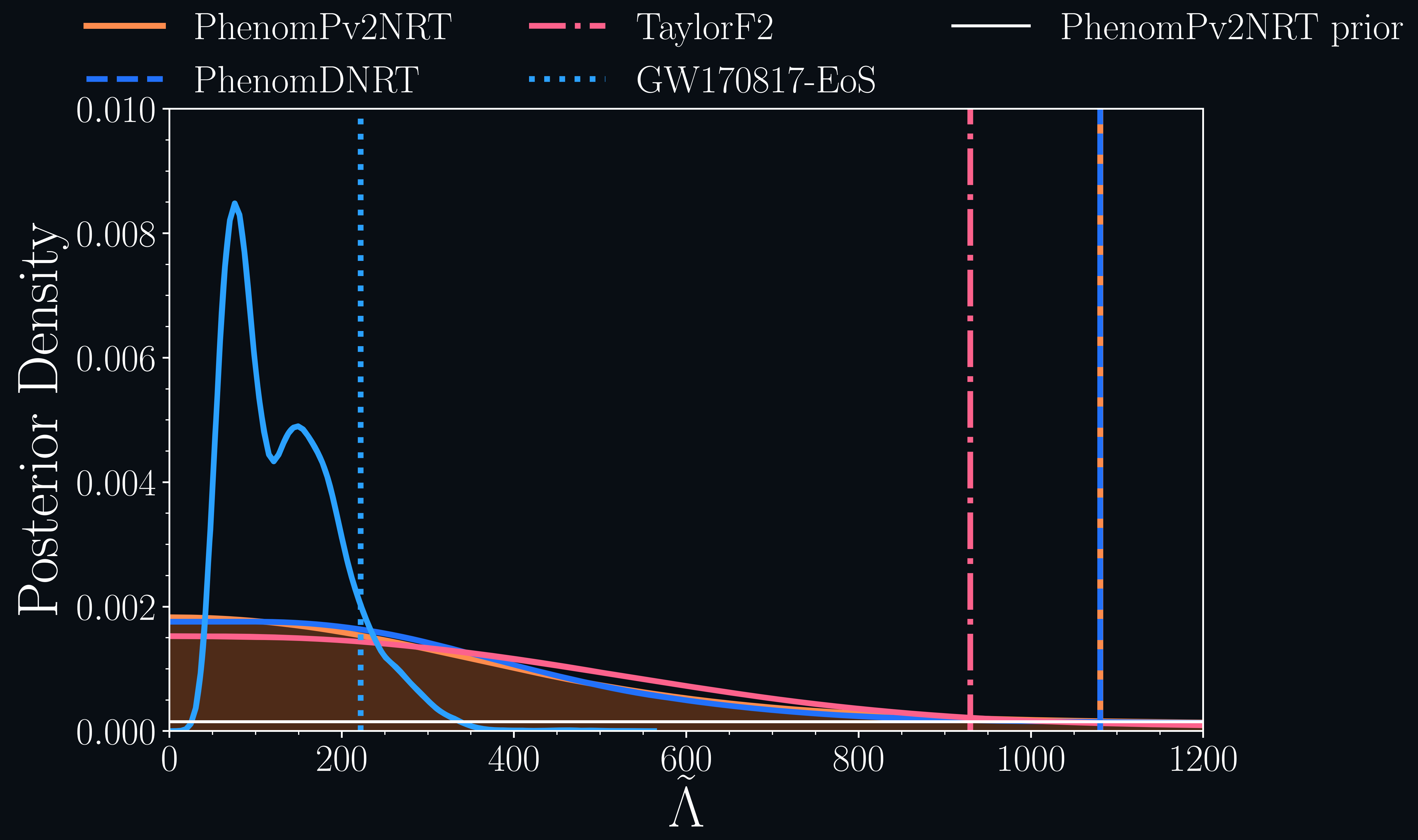


- Assumptions:
  - Components are well separated, $\epsilon = (M'/M)(R/D)^3 \ll 1$
  - The orbital frequency is slow, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Matter Constraints: GW170817 & GW190425



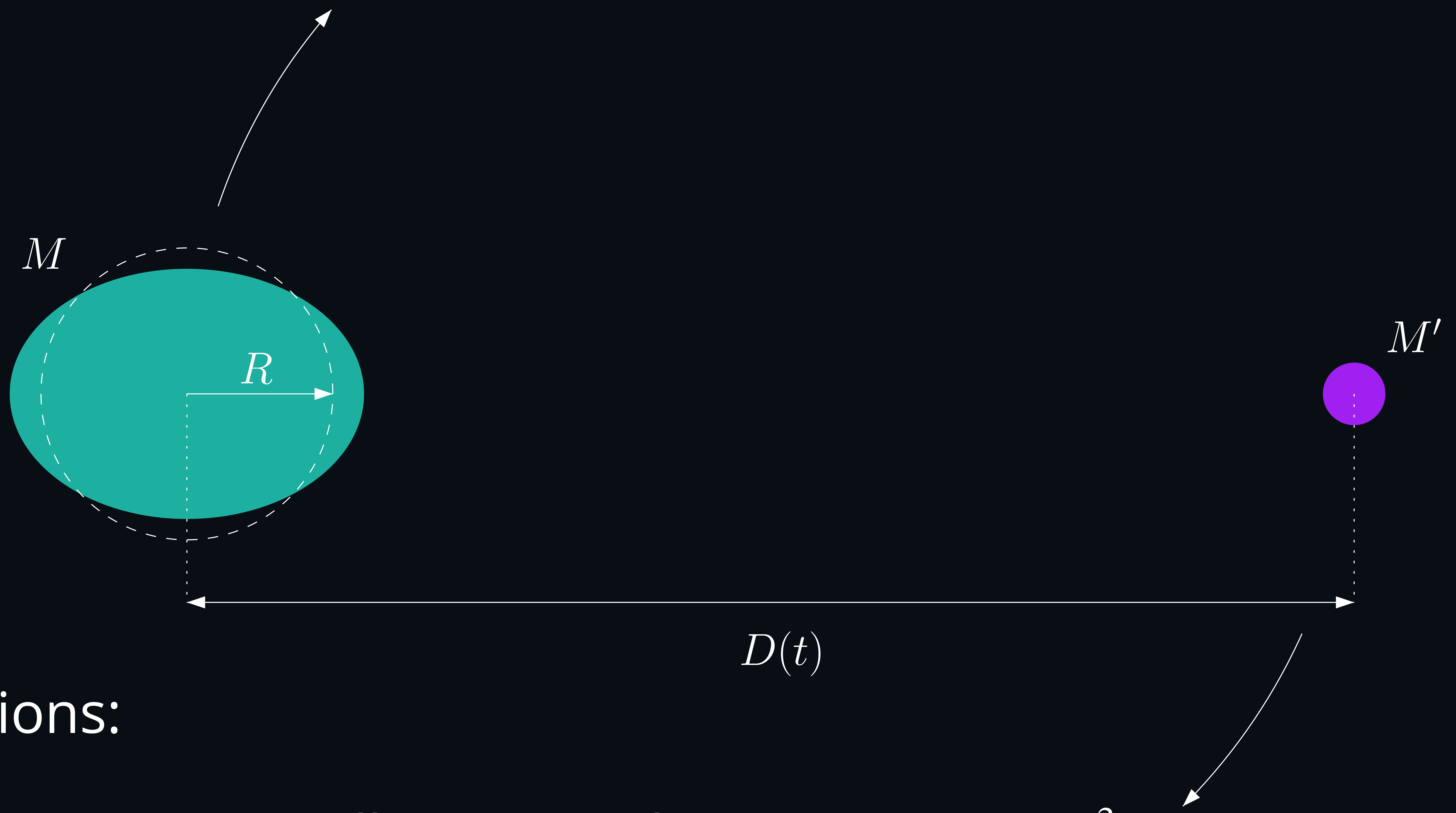
[LIGO-Virgo Collaboration, Phys. Rev. X **9**, 011001 (2019)]



[LIGO-Virgo Collaboration, Astrophys. J. **892**, L3 (2020)]

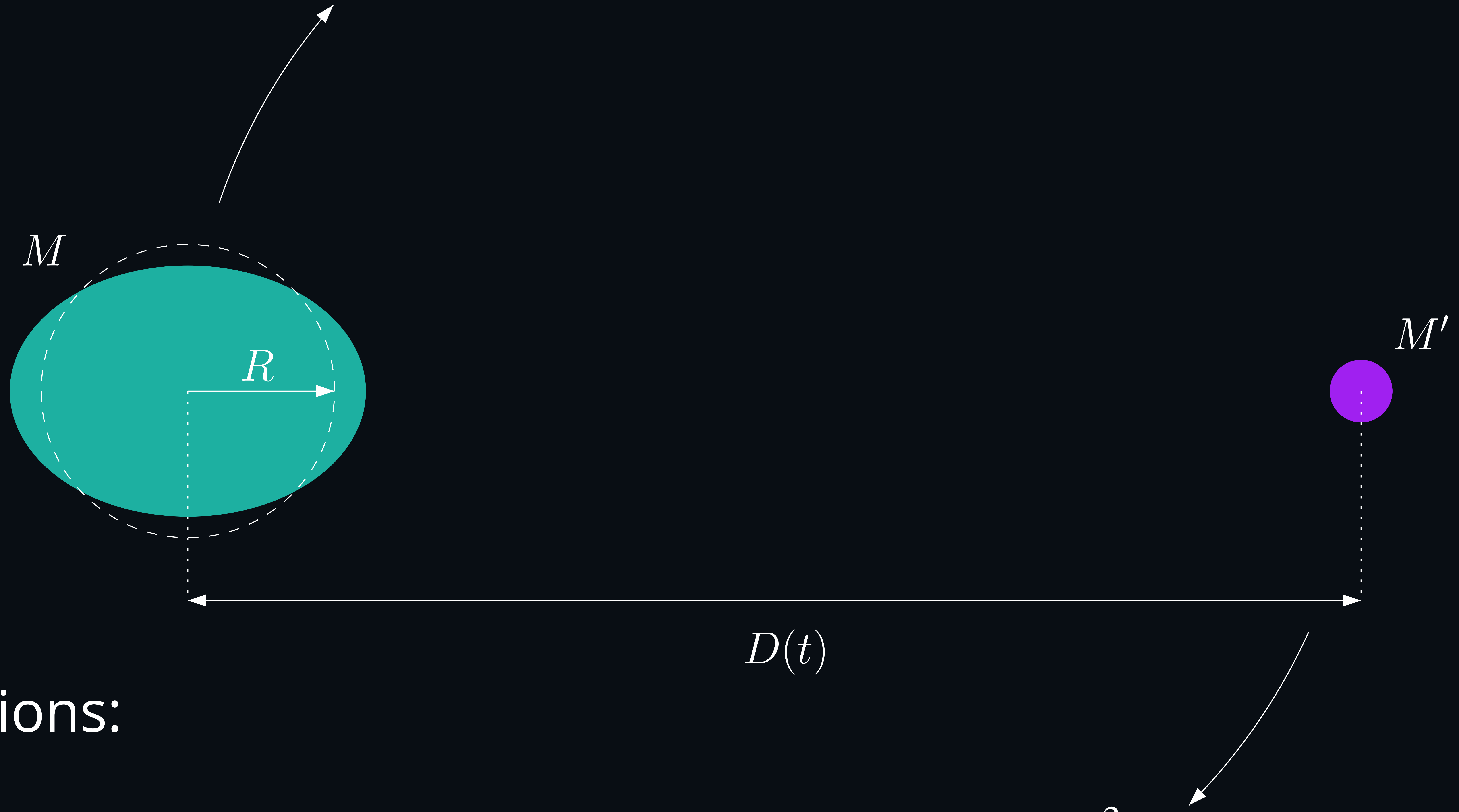
➡Provided the function $\rho = \rho(p)$, one can solve for M and Λ

Static Tide



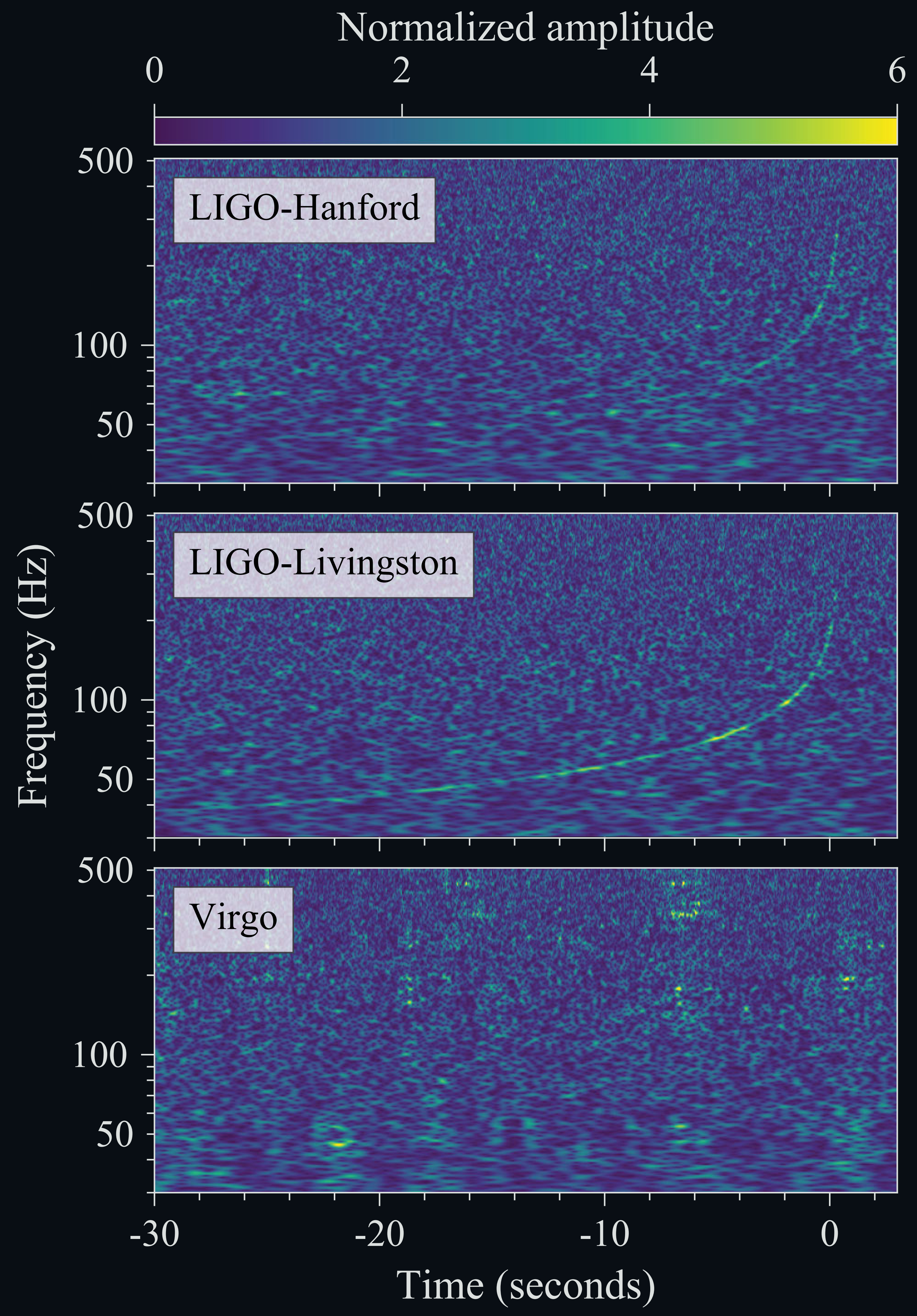
- Assumptions:
  - Components are well separated, $\epsilon = (M'/M)(R/D)^3 \ll 1$
  - The orbital frequency is slow, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Dynamical Tide



- Assumptions:
  - Components are well separated, $\epsilon = (M'/M)(R/D)^3 \ll 1$
  - ~~The orbital frequency is slow, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$~~

Inspiral



- The static approximation inevitably breaks down,

$$\lambda = \dot{\Phi}/\omega_\alpha \sim O(1)$$

- The frequency ω_α represents a characteristic mode frequency,

$$\omega_f \sim \sqrt{\frac{GM}{R^3}} \approx 2\pi \cdot 2.2\,\text{kHz} \left(\frac{M}{1.4 M_\odot}\right)^{1/2} \left(\frac{10\,\text{km}}{R}\right)^{3/2}$$

[LIGO-Virgo Collaboration+, Phys. Rev. Lett. **119**, 161101 (2017)]

Mode-Sum Representation

- Normal modes form a complete basis [Chandrasekhar, Astrophys. J. **139**, 664 (1964)],

$$\boldsymbol{\xi}(t, \mathbf{x}) = \sum_\alpha q_\alpha(t)\, \boldsymbol{\xi}_\alpha(\mathbf{x}), \quad \mathbf{C}(\mathbf{x}) \cdot \boldsymbol{\xi}_\alpha(\mathbf{x}) = \omega_\alpha^2\, \boldsymbol{\xi}_\alpha(\mathbf{x})$$

- The tidal equation of motion simplifies to

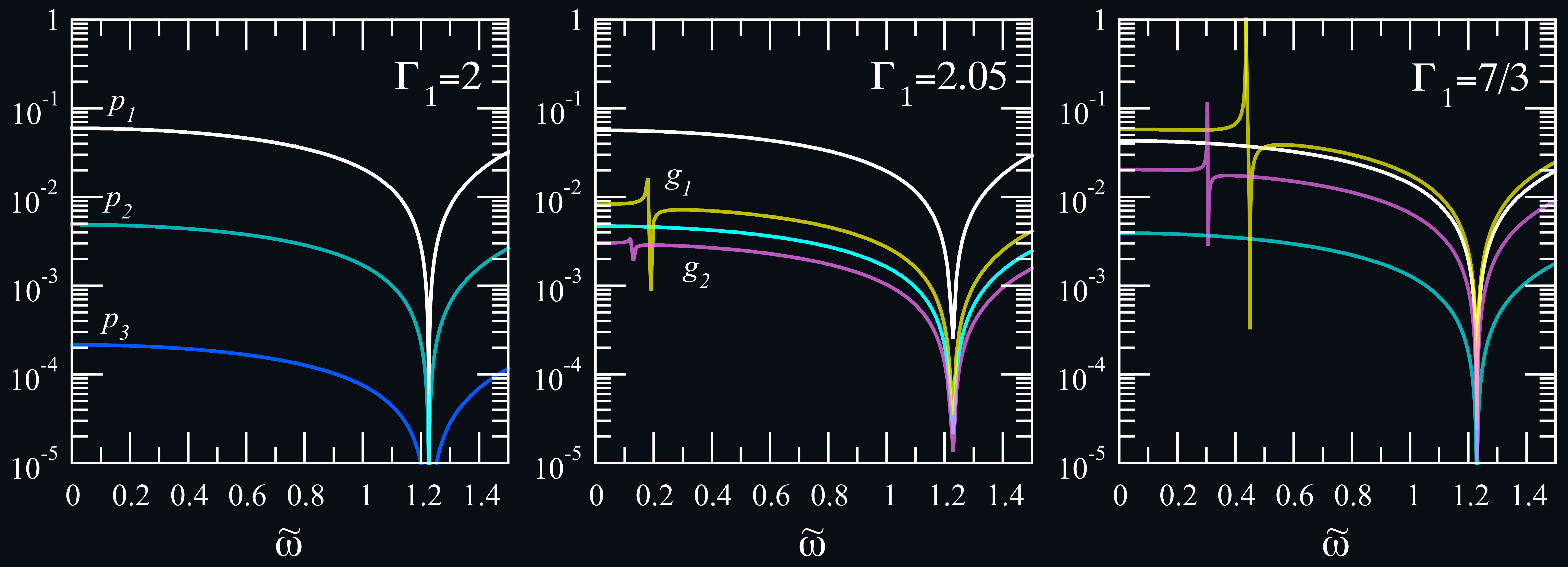
$$\ddot{q}_\alpha(t) + \omega_\alpha^2\, q_\alpha(t) = \frac{Q_\alpha(t)}{\mathcal{E}_\alpha} \propto e^{-im\Phi(t)}$$

- Challenge: Can this be formulated in general relativity?

Equilibrium Tide

- For an equilibrium orbit, $\dot{\Phi} = \text{const}$,

$$q_\alpha(t) = \frac{Q_\alpha(t)}{\mathcal{E}_\alpha} \frac{1}{\omega_\alpha^2 - (m\dot{\Phi})^2}$$



[Andersson+Pnigouras, Phys. Rev. D **101**, 083001 (2020)]

Static Limit

- In the static limit, $\dot{\Phi} = 0$,

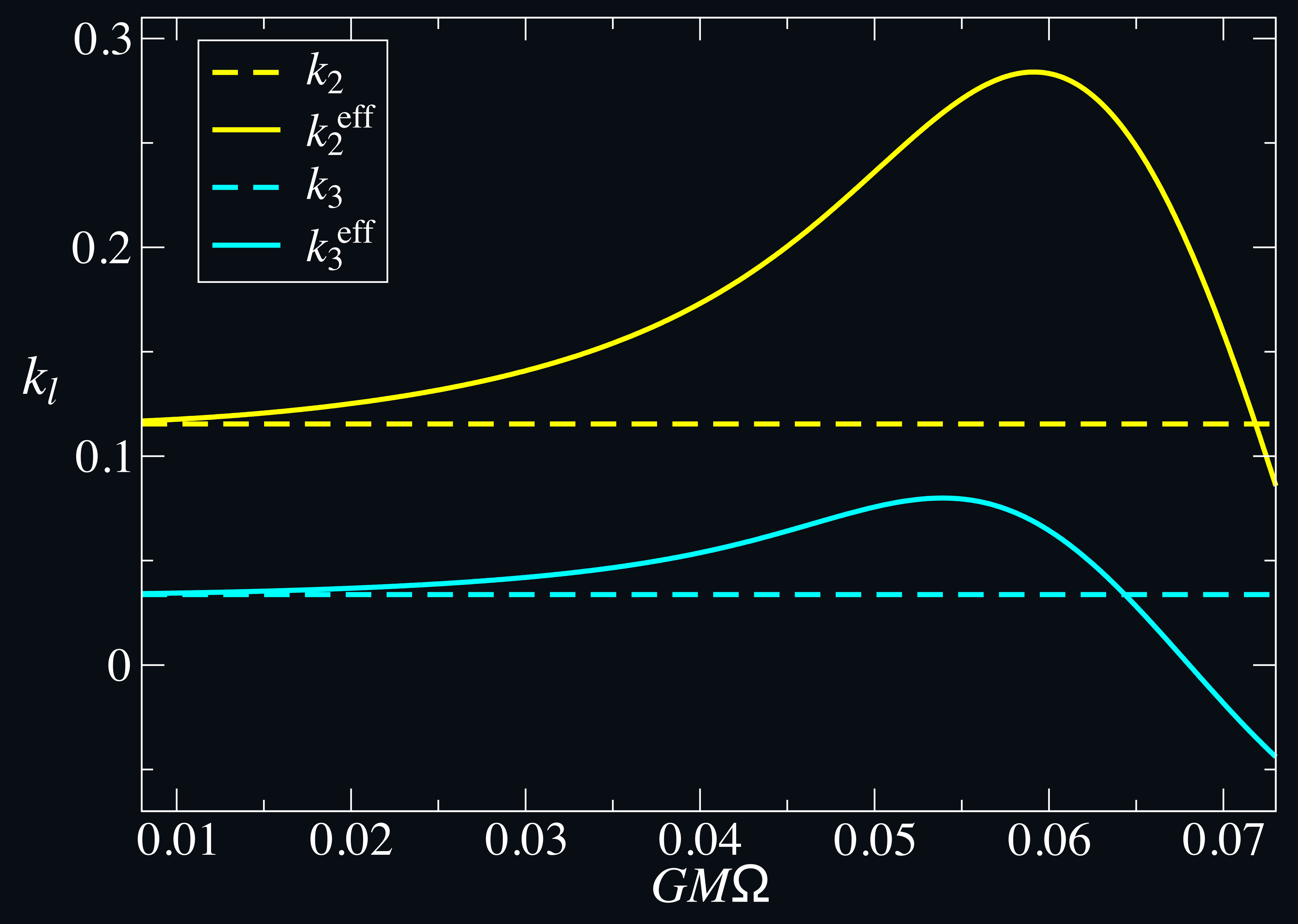
$$q_\alpha = \frac{Q_\alpha}{\mathcal{E}_\alpha} \frac{1}{\omega_\alpha^2}$$

$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
Mode	k_l	Mode	k_l	Mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
	9×10^{-4}		7×10^{-4}		3×10^{-4}

[Andersson+Pnigouras, Phys. Rev. D **101**, 083001 (2020)]

f–mode Approximation

- The dynamical tide is dominated by the f-mode
- There have been models developed for the f-mode dynamical tide that use
  - *effective-one-body* [Steinhoff+, Phys. Rev. D **94**, 104028 (2016)],
  - *Newtonian* [Schmidt+Hinderer, Phys. Rev. D **100**, 021501 (2019)] and
  - *phenomenological* approaches [Abac+, Phys. Rev. D **109**, 024062 (2024)]



Sub-Dominant Modes

- Low-frequency modes (including *g*-modes, *r*-modes and *i*-modes) will become resonant during inspiral,

$$q_\alpha(t) = \frac{Q_\alpha(t)}{\mathcal{E}_\alpha} \frac{1}{\omega_\alpha^2 - (m\dot{\Phi})^2} \qquad \Longrightarrow \qquad |m|\dot{\Phi} \approx \omega_\alpha$$

- Energy is extracted from the orbit,

$$\Delta E_\alpha \sim |q_\alpha|^2,$$

which results in a phase shift $\Delta\Phi$

Composition

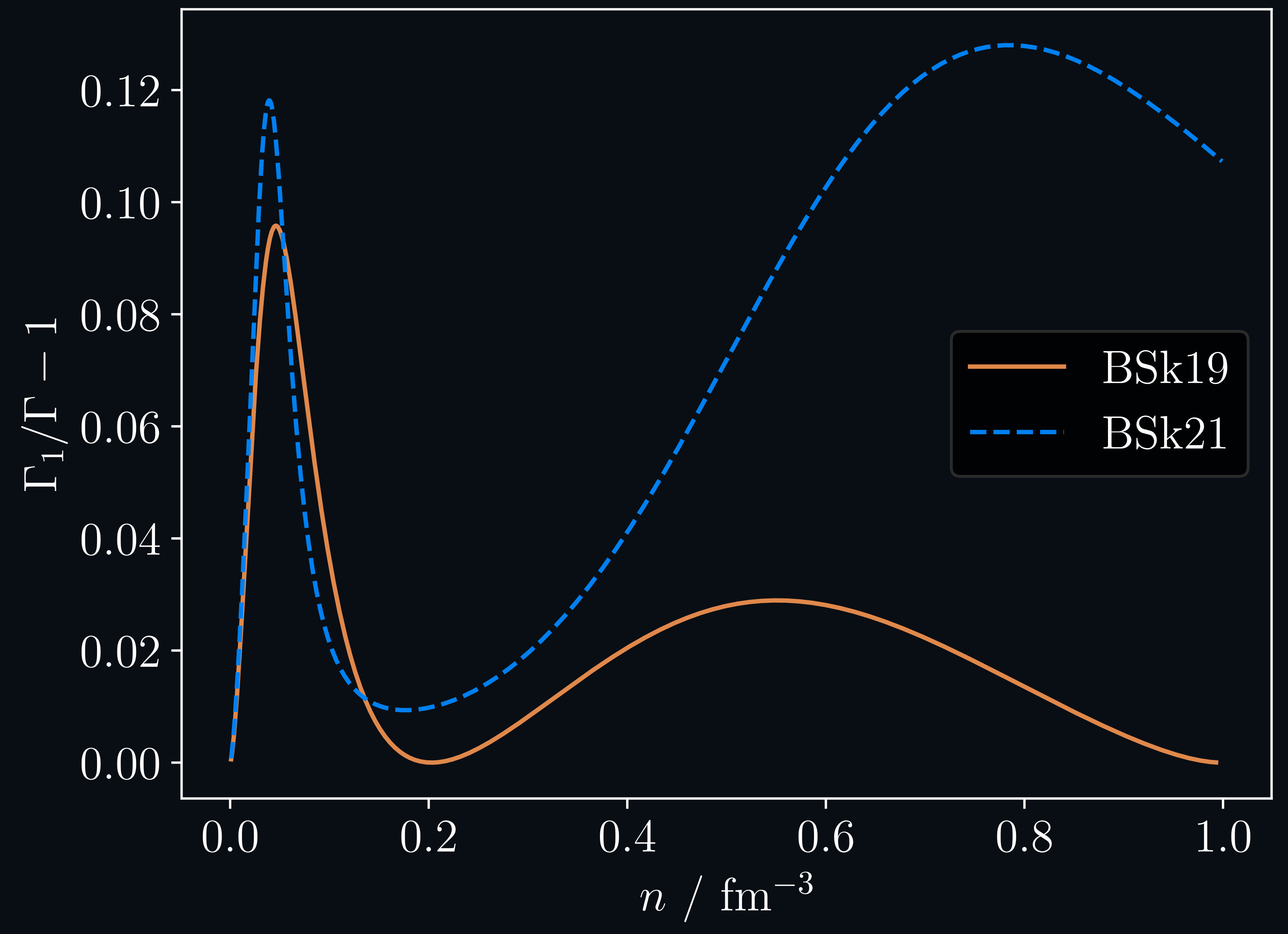
- Instead of

$$d\varepsilon = \frac{\varepsilon + p}{n_\mathrm{b}} dn_\mathrm{b} \implies \varepsilon = \varepsilon(n_\mathrm{b}),$$

the first law is

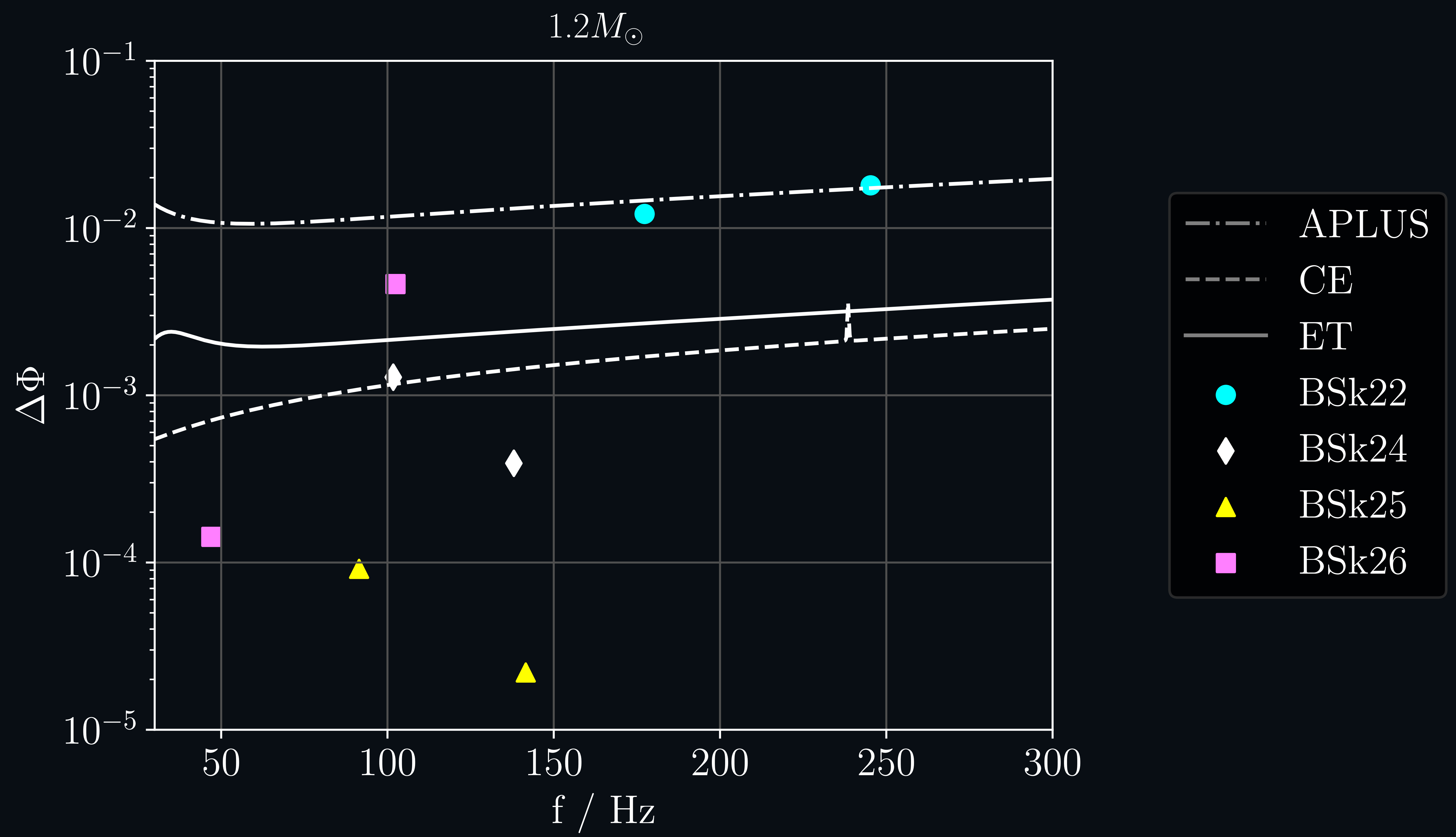
$$d\varepsilon = \frac{\varepsilon + p}{n_\mathrm{b}} dn_\mathrm{b} + n_\mathrm{b}\mu_\Delta dY_\mathrm{e} \implies \varepsilon = \varepsilon(n_\mathrm{b}, Y_\mathrm{e})$$

- When there are slow weak nuclear reactions, $\mu_\Delta \neq 0$



[FG + Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

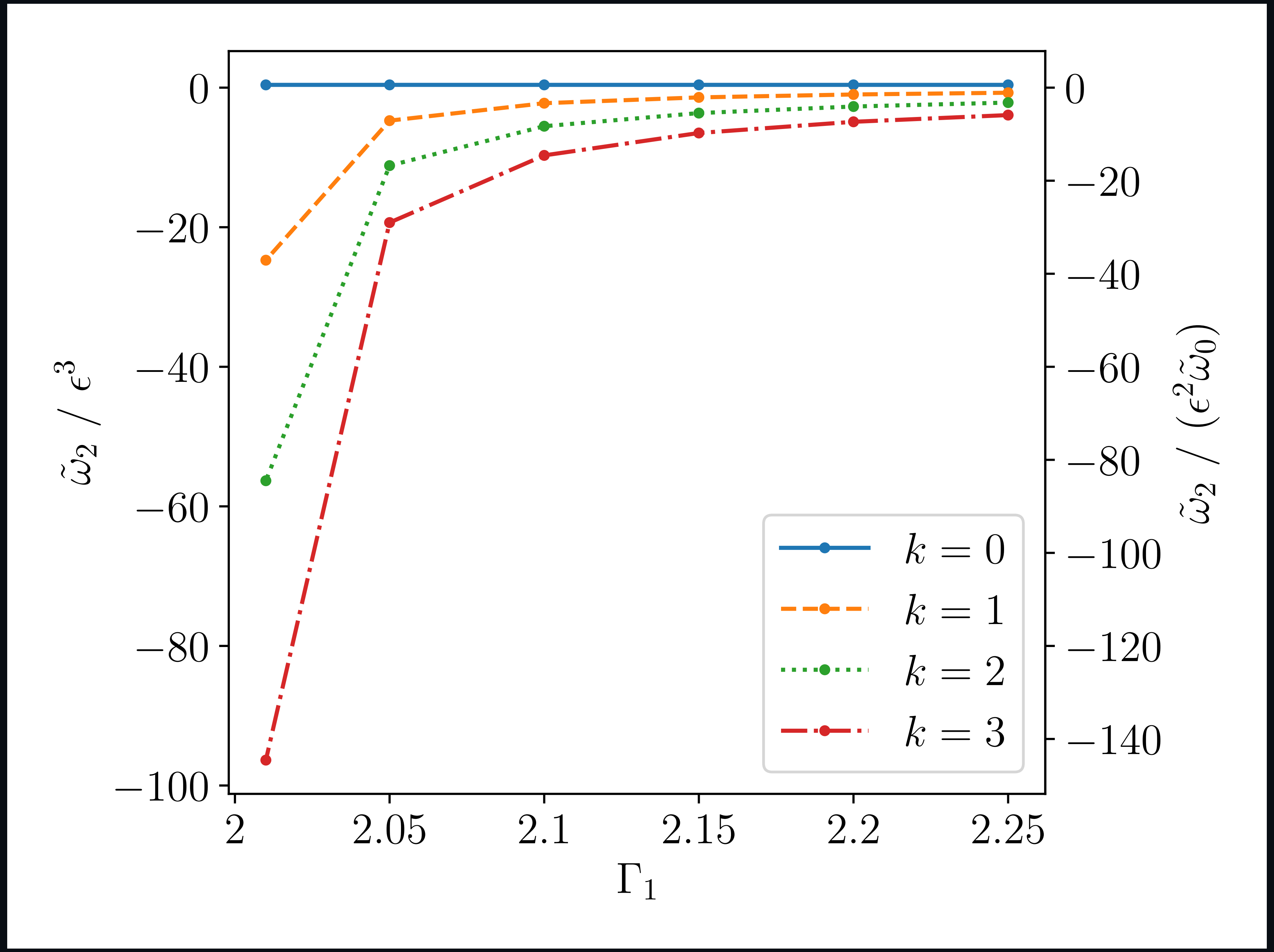
g-modes



[Counsell, FG + Andersson, Mon. Not. R. Astron. Soc. **536**, 1967 (2025)]

Rotation

- A special class of inertial modes have *axial* parity: the *r*-modes
- The *r*-modes are famous for their gravitational-wave-driven instability
- They also probe composition gradients



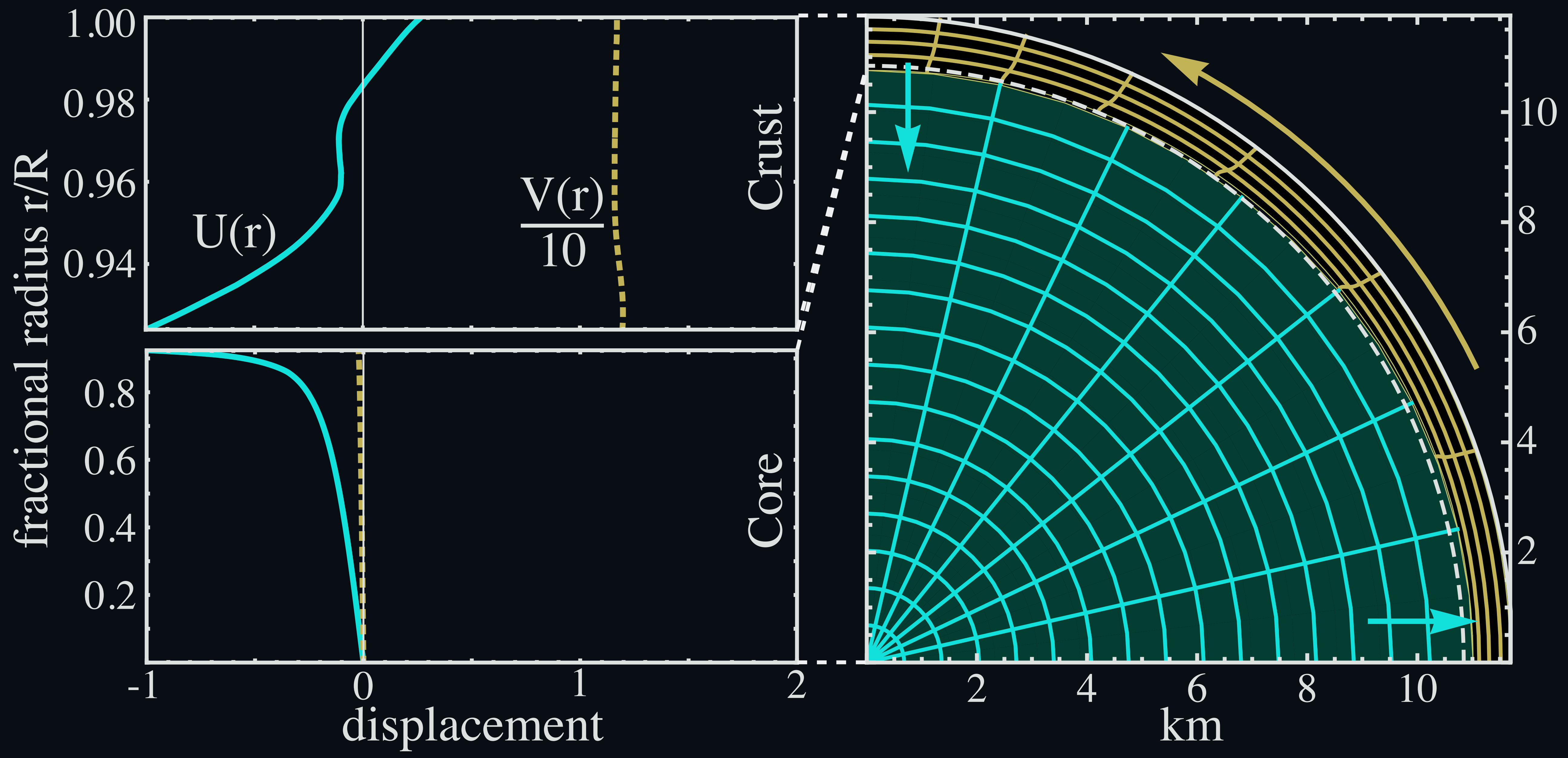
[FG + Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

r–modes

- The r-mode couples strongly to the gravito-magnetic tide
- Estimates give [Flanagan+Racine, Phys. Rev. D **75**, 044001 (2007)]

$$\Delta\Phi \approx -0.03\left(\frac{R}{10\,\mathrm{km}}\right)^4\left(\frac{f_{\mathrm{spin}}}{100\,\mathrm{Hz}}\right)^{2/3}\left(\frac{1.4M_\odot}{M}\right)^{10/3}$$

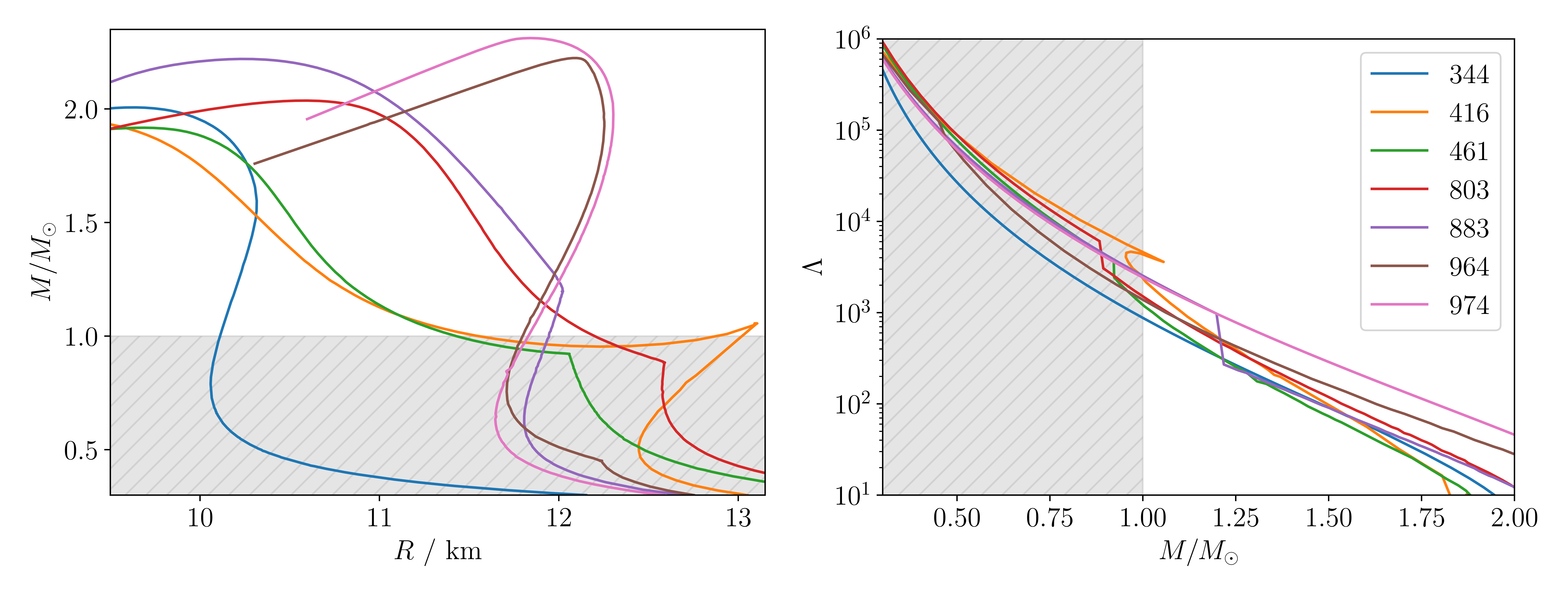
Phase Transitions



[Tsang+, Phys. Rev. Lett. **108**, 011102 (2012)]

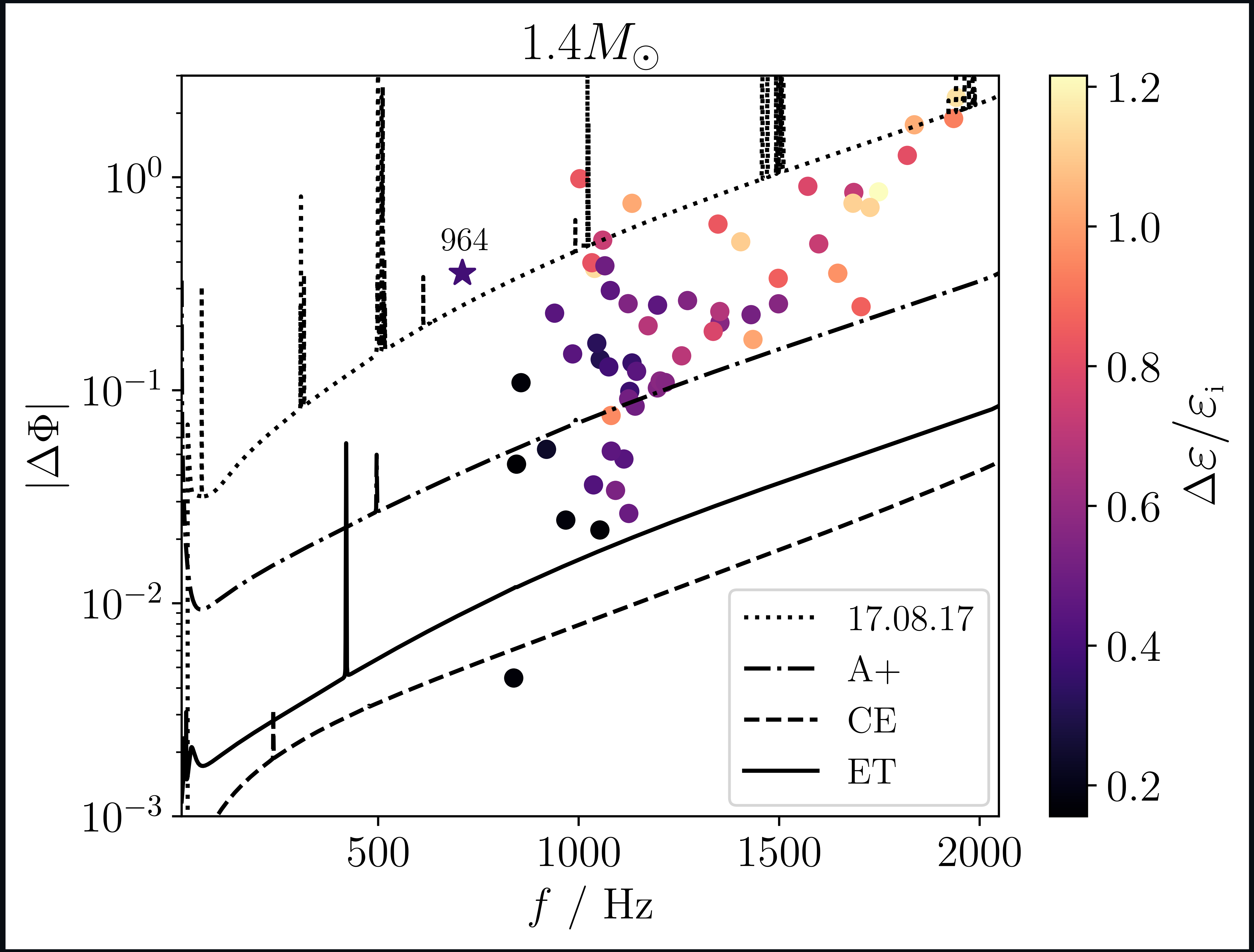
- The interfacial *i*-mode arises when there is a first-order phase transition in the star
- This may occur at the core-crust interface or (possibly) at a transition to deconfined quark matter in the core

Masquerade Problem



[Counsell, FG+, Phys. Rev. Lett. **135**, 081402 (2025)]

i-modes



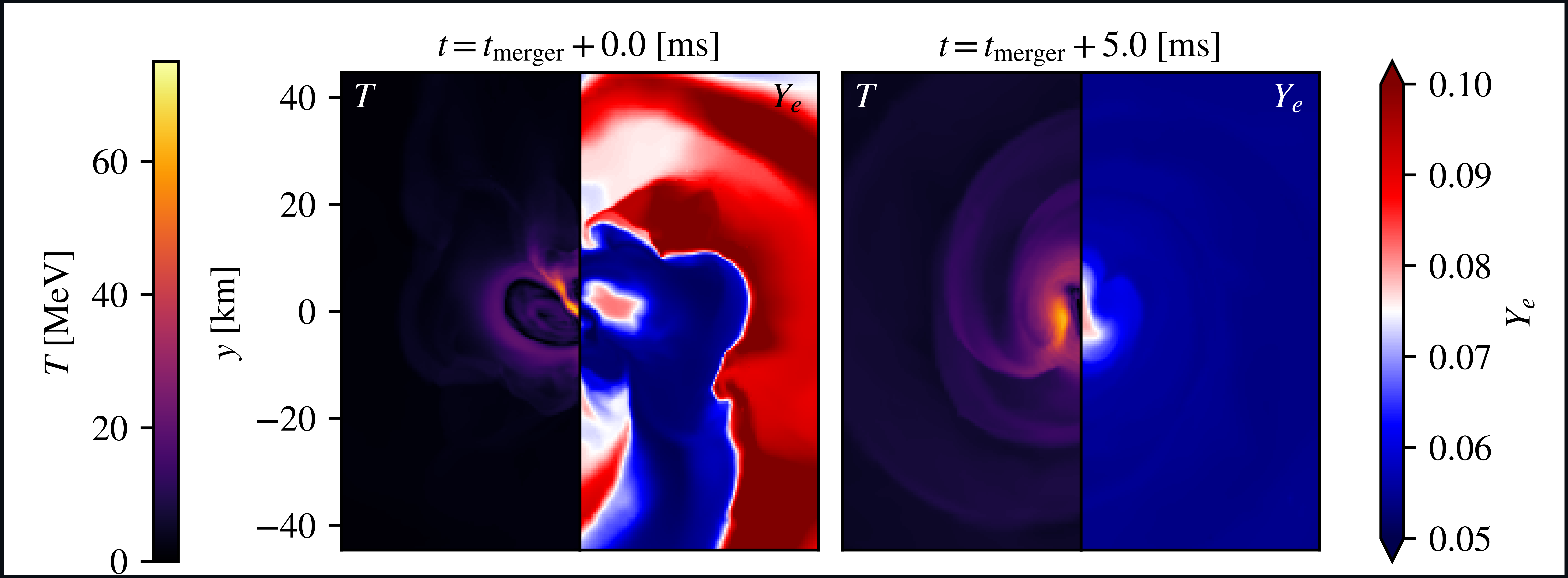
[Counsell, FG+, Phys. Rev. Lett. **135**, 081402 (2025)]

- Challenge: Develop models of this resonant behaviour

Role of Simulations

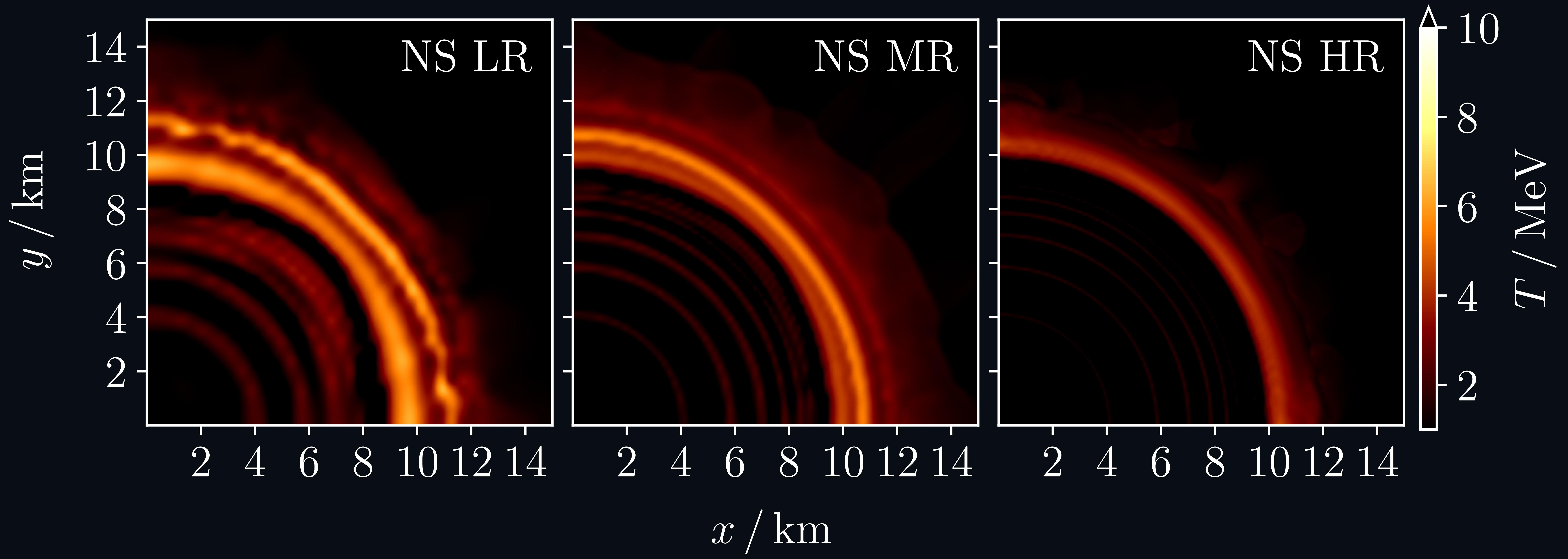
[Dietrich+, Gen. Relativ. Gravit. **53**, 27 (2021)]

Way too hot!



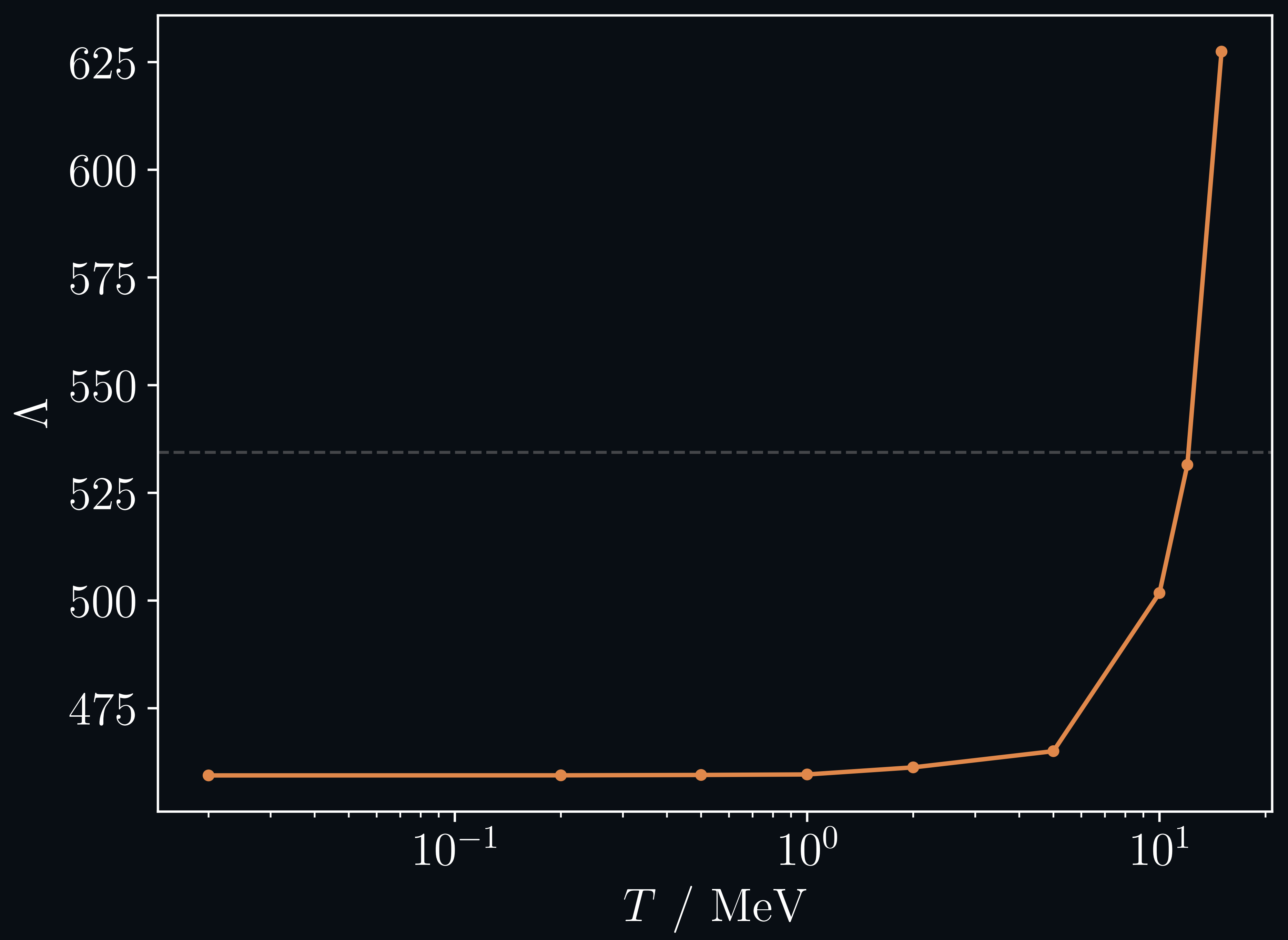
[Hammond+, Phys. Rev. D. **104**, 103006 (2021)]

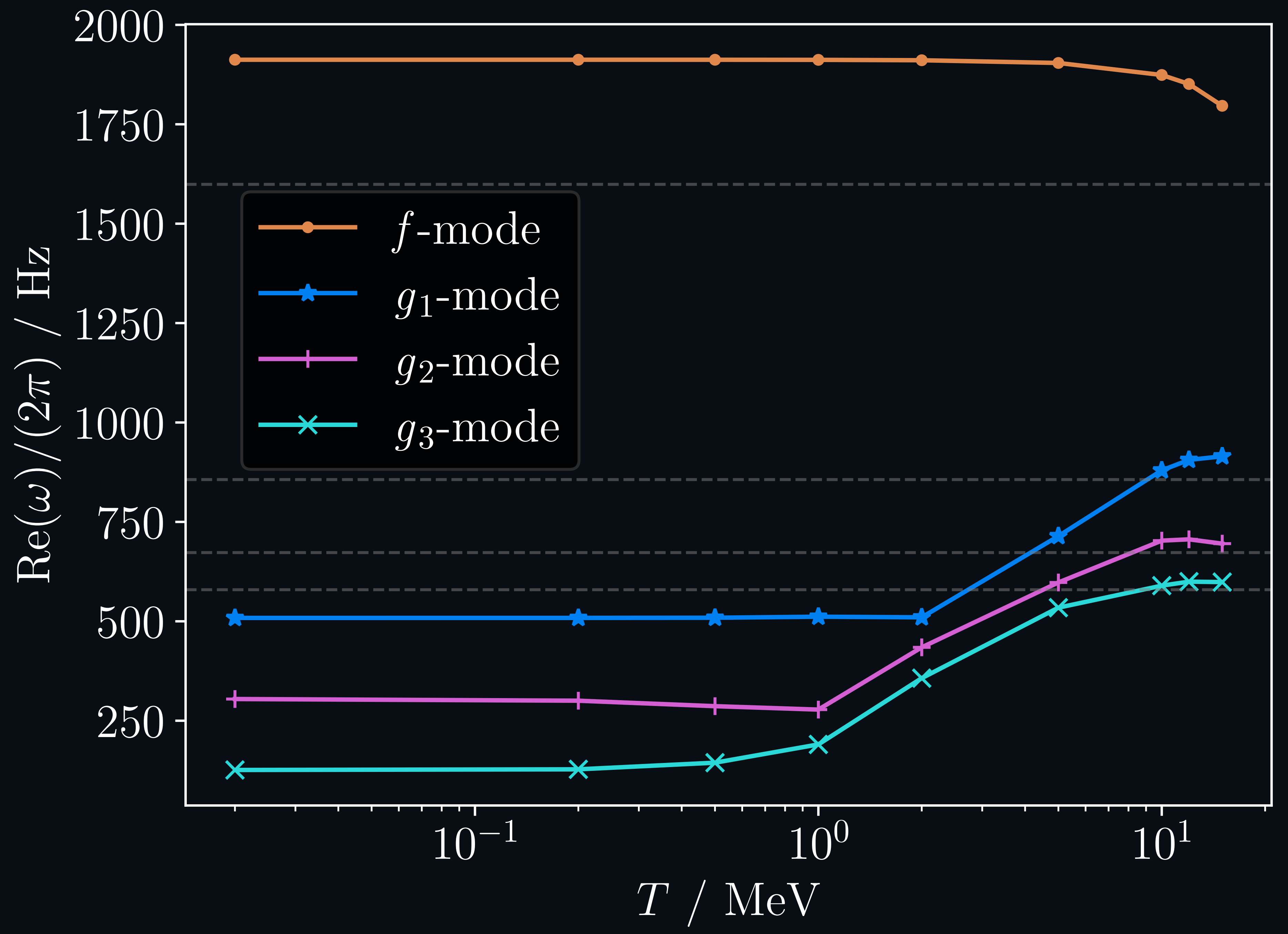
Way too hot!



[R. Matur]

Tidal Systematics





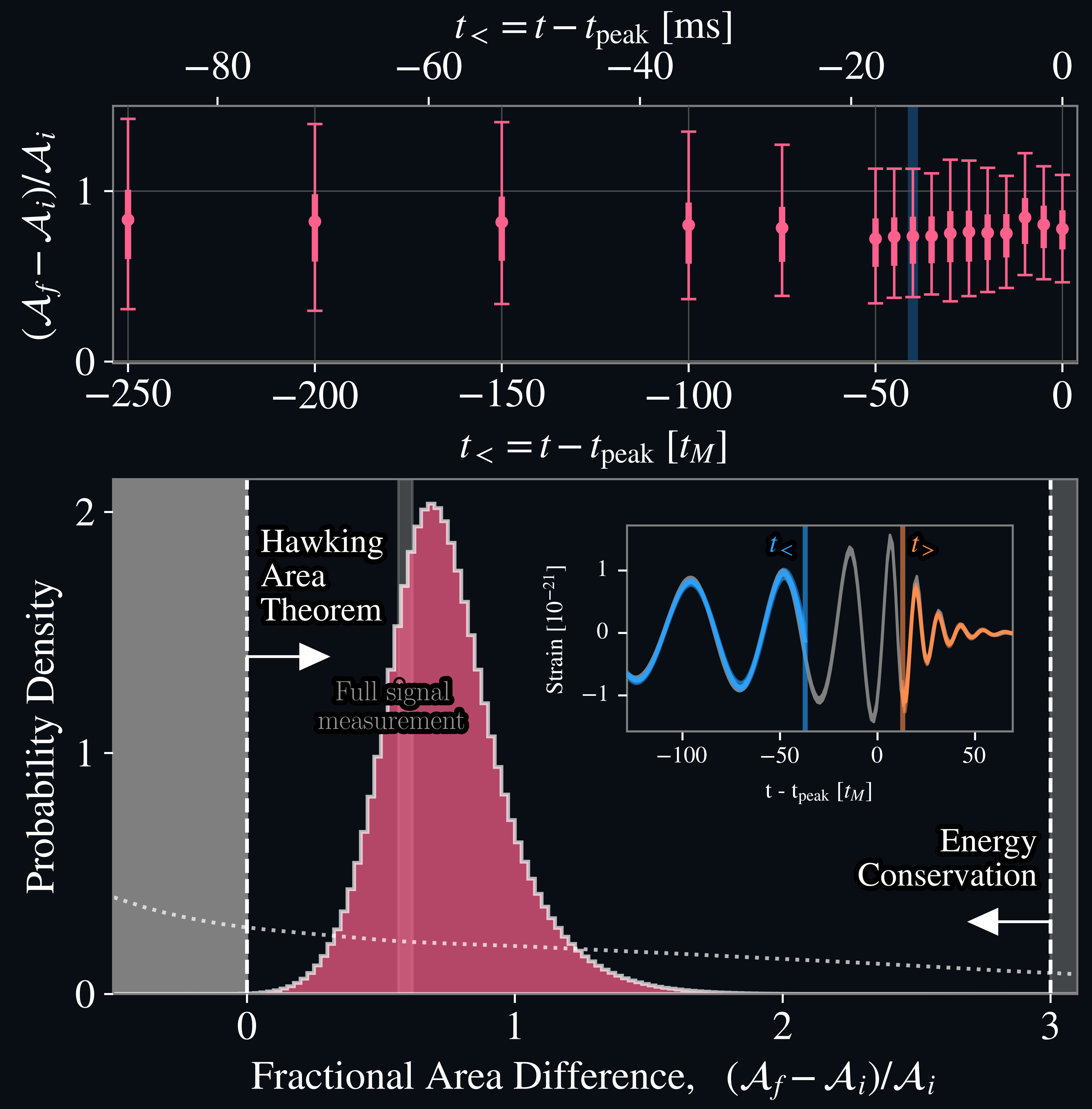
[FG+, Phys. Rev. D. **104**, 103006 (2021)]

Conclusions

Opportunities	Challenges
Gravitational waves probe dense nuclear matter by encoding fine tidal deformations	Can the mode-sum be formulated in general relativity?
The tide presents the opportunity to conduct neutron-star seismology	Go beyond universal relations in inference
Oscillation modes grant access to rich physics: composition and phase transitions	Develop gravitational-waveform models of resonant oscillation modes

Extra slides

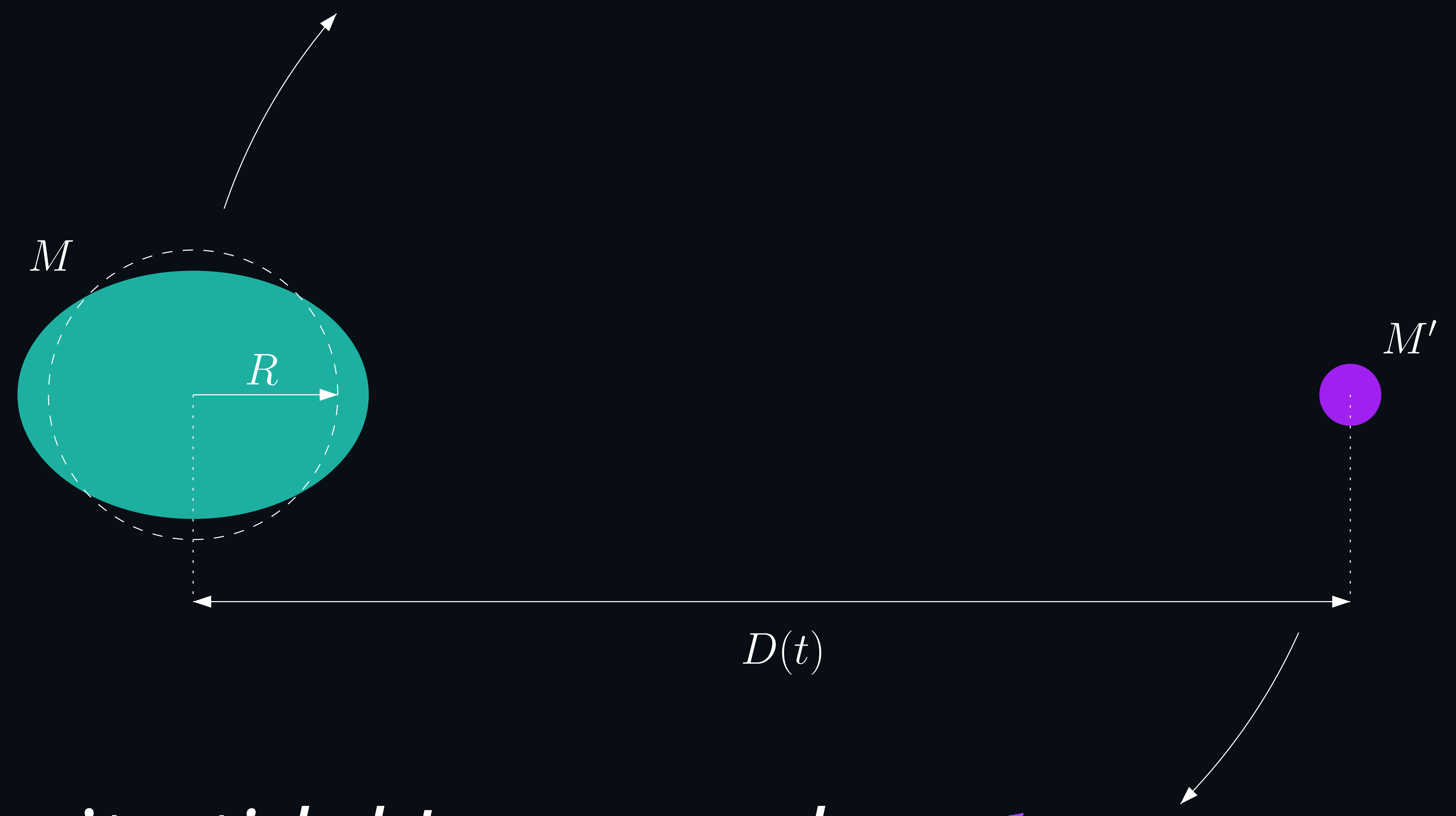
Black-Hole Spectroscopy: GW250114



$$\mathscr{A}(M,\chi) = 8\pi\left(\frac{GM}{c^2}\right)^2\left(1+\sqrt{1-\chi^2}\right)$$

[LIGO-Virgo-KAGRA Collaboration, Phys. Rev. Lett. **135**, 111403 (2025)]

Static Tide



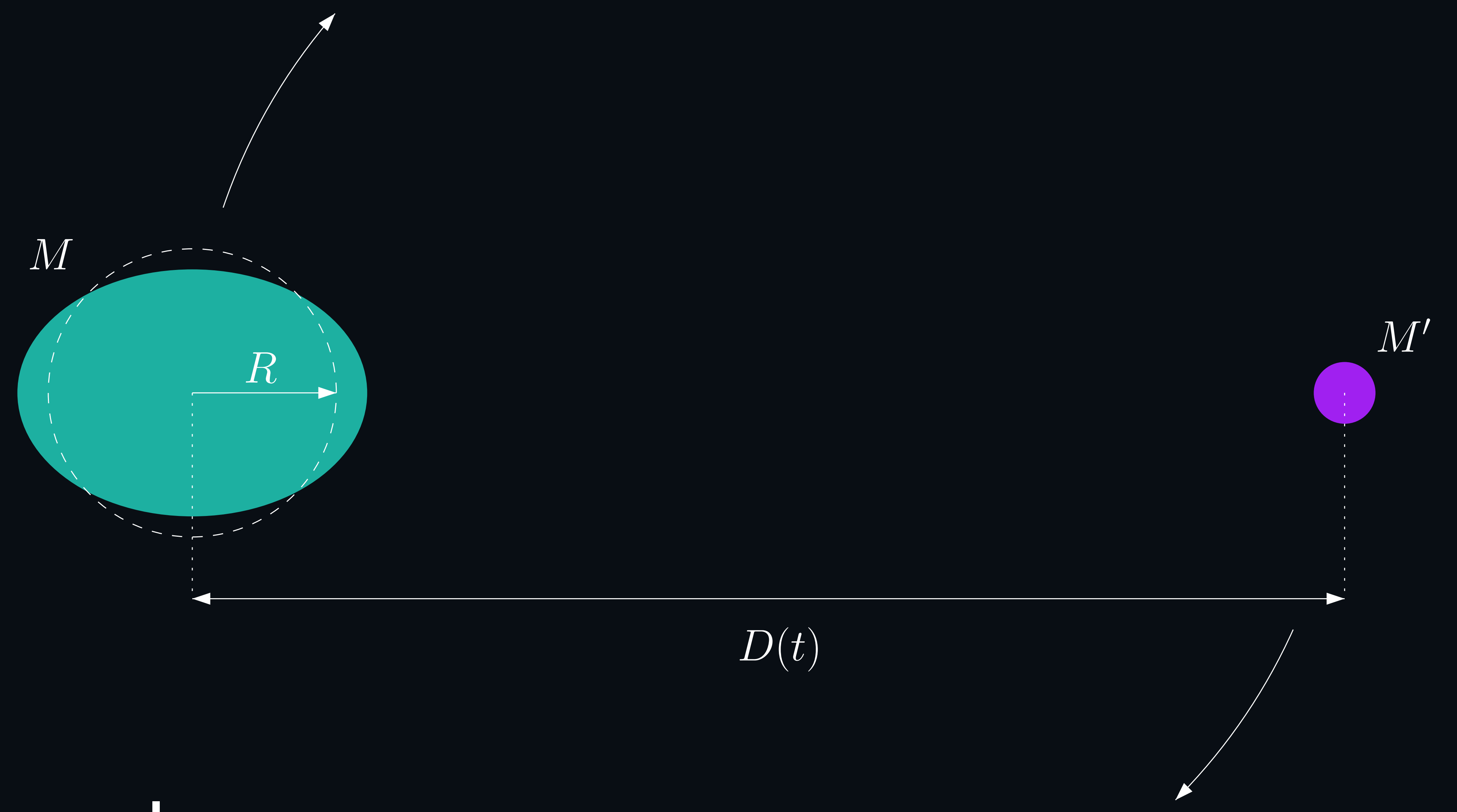
- Star's (static) shape is quantified by its *tidal Love numbers* k_l,

$$U_l(r) = \left[2k_l\left(\frac{R}{r}\right)^{l+1} + \left(\frac{r}{R}\right)^{l}\right]\chi_l(R),$$

where

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_l}{dr}\right) - \frac{l(l+1)}{r^2}U_l = -\frac{4\pi G\rho}{dp/d\rho}U_l$$

Static Tide



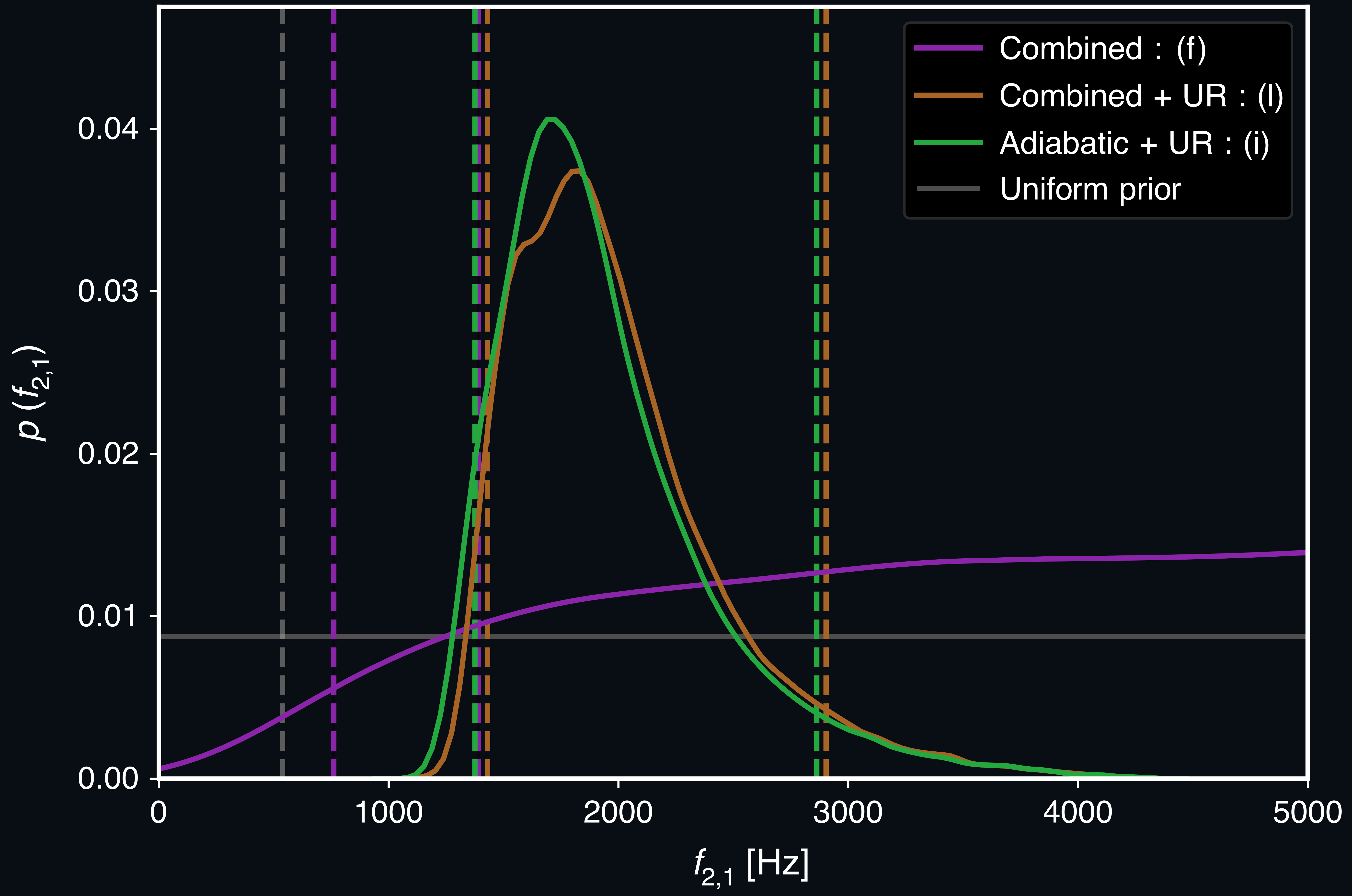
- Tide enters the gravitational-wave phase as [Flanagan+Hinderer, Phys. Rev. D **77**, 021502 (2008)]

$$\Psi_{\rm tide}(v) = -\frac{3}{128}\frac{M_{\rm total}}{\mu}\frac{1}{v^5}\cdot\left[\frac{39}{2}\tilde{\Lambda}v^{10} + O(v^2)\right],$$

where

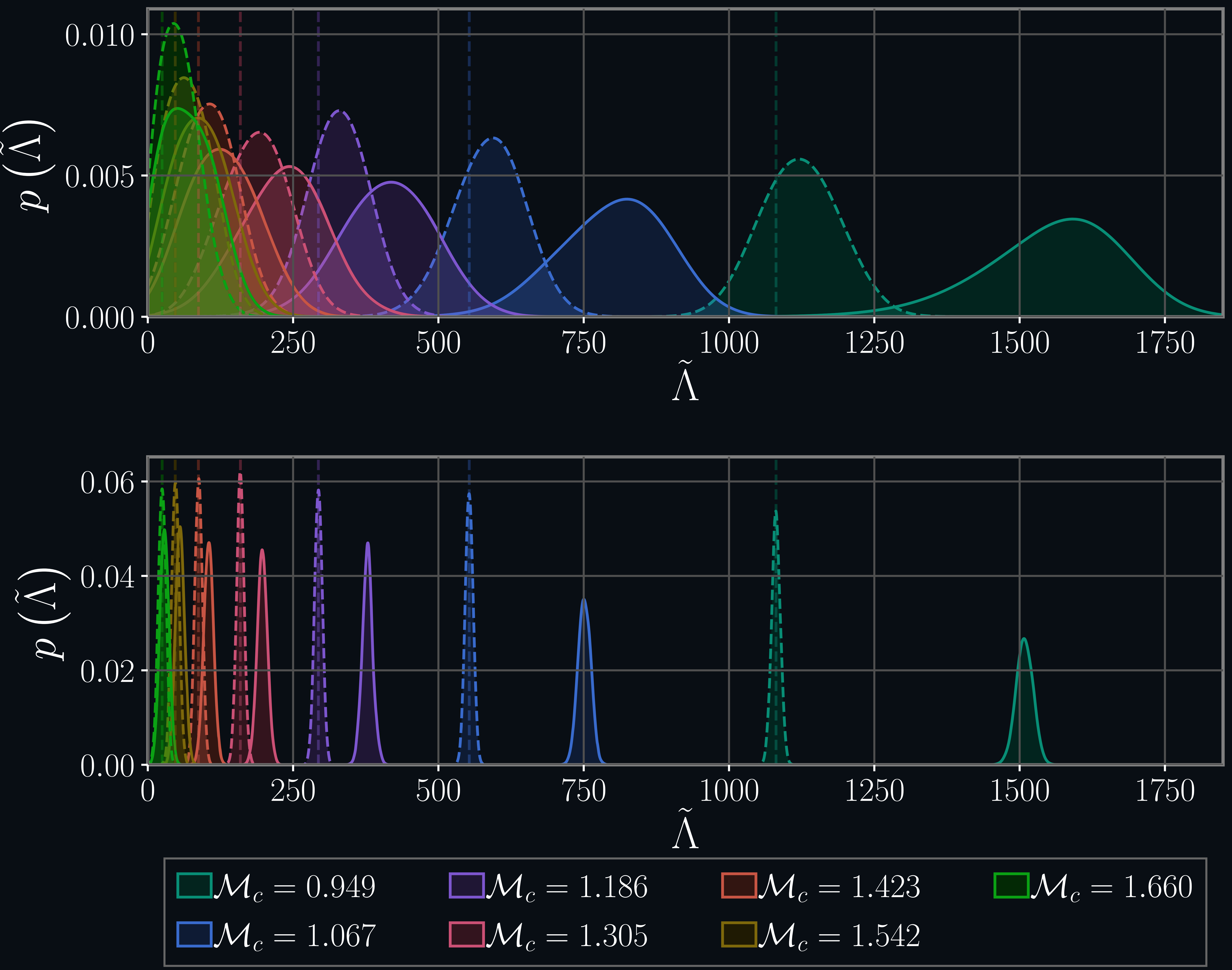
$$\tilde{\Lambda} = \frac{16}{13}\frac{1}{M_{\rm total}}\left[(M+12M')M^4\Lambda + (M'+12M)M'^4\Lambda'\right], \quad \Lambda = \frac{2}{3}\left(\frac{c^2R}{GM}\right)^5 k_2$$

Towards Asteroseismology



[Pratten+, Nat. Commun. **11**, 2553 (2020)]

Biases



[Pratten+, Phys. Rev. Lett. **129**, 081102 (2022)]