

Gravitational-Wave Asteroseismology Illuminating Dense Nuclear Matter through Dynamical Tides

Fabian Gittins | CoCoNuT Meeting, Strasbourg | 28 Oct 2025



Utrecht
University

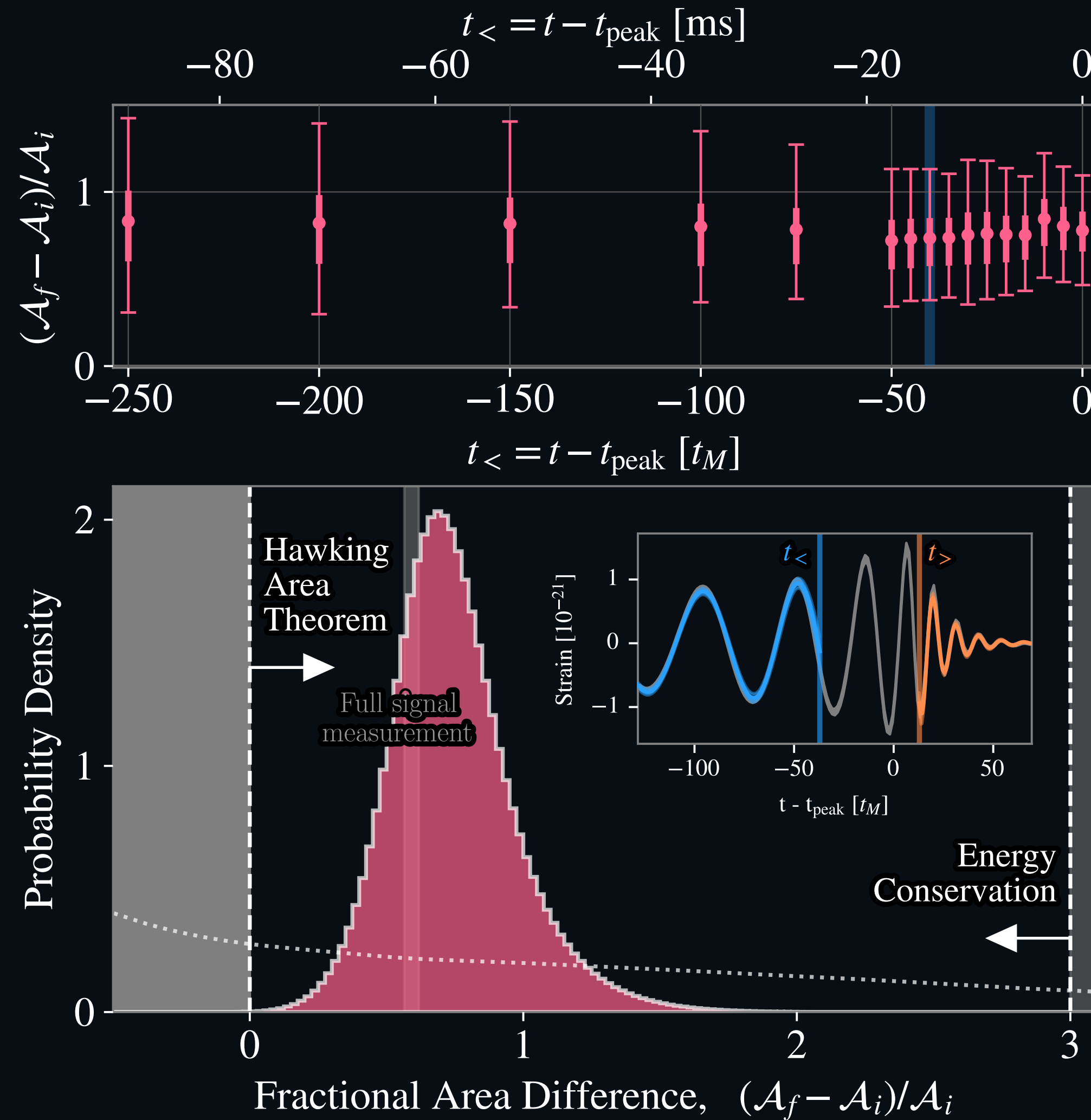


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Overview

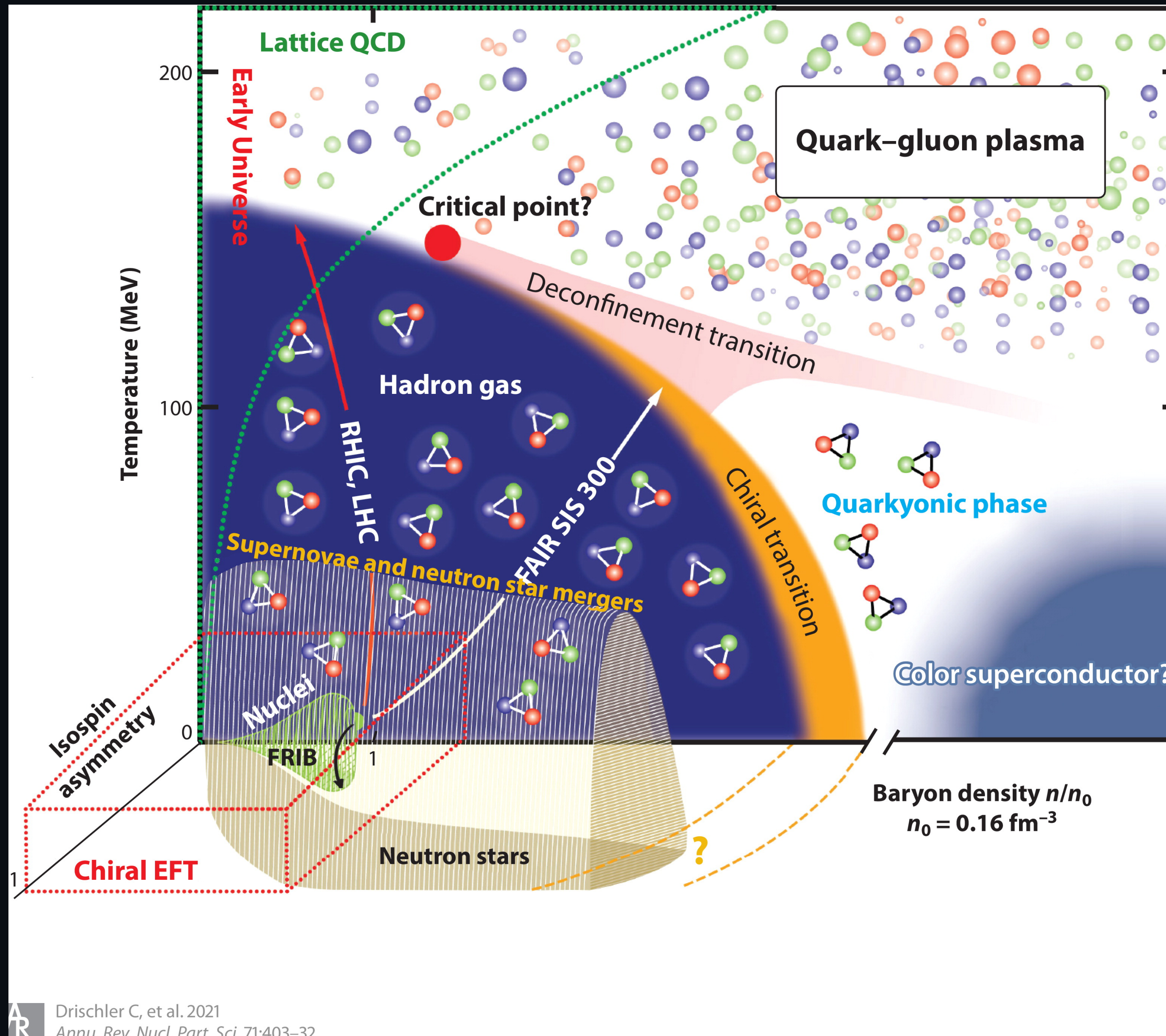
- Gravitational waves probe *fundamental physics* through observations of neutron stars
- Tidal dynamics present opportunity to conduct asteroseismology
- Neutron-star oscillation modes are notably rich
- Opportunities and challenges ahead

Black-Hole Spectroscopy: GW250114



$$\mathcal{A}(M, \chi) = 8\pi \left(\frac{GM}{c^2} \right)^2 \left(1 + \sqrt{1 - \chi^2} \right)$$

Quantum Chromodynamics (QCD)



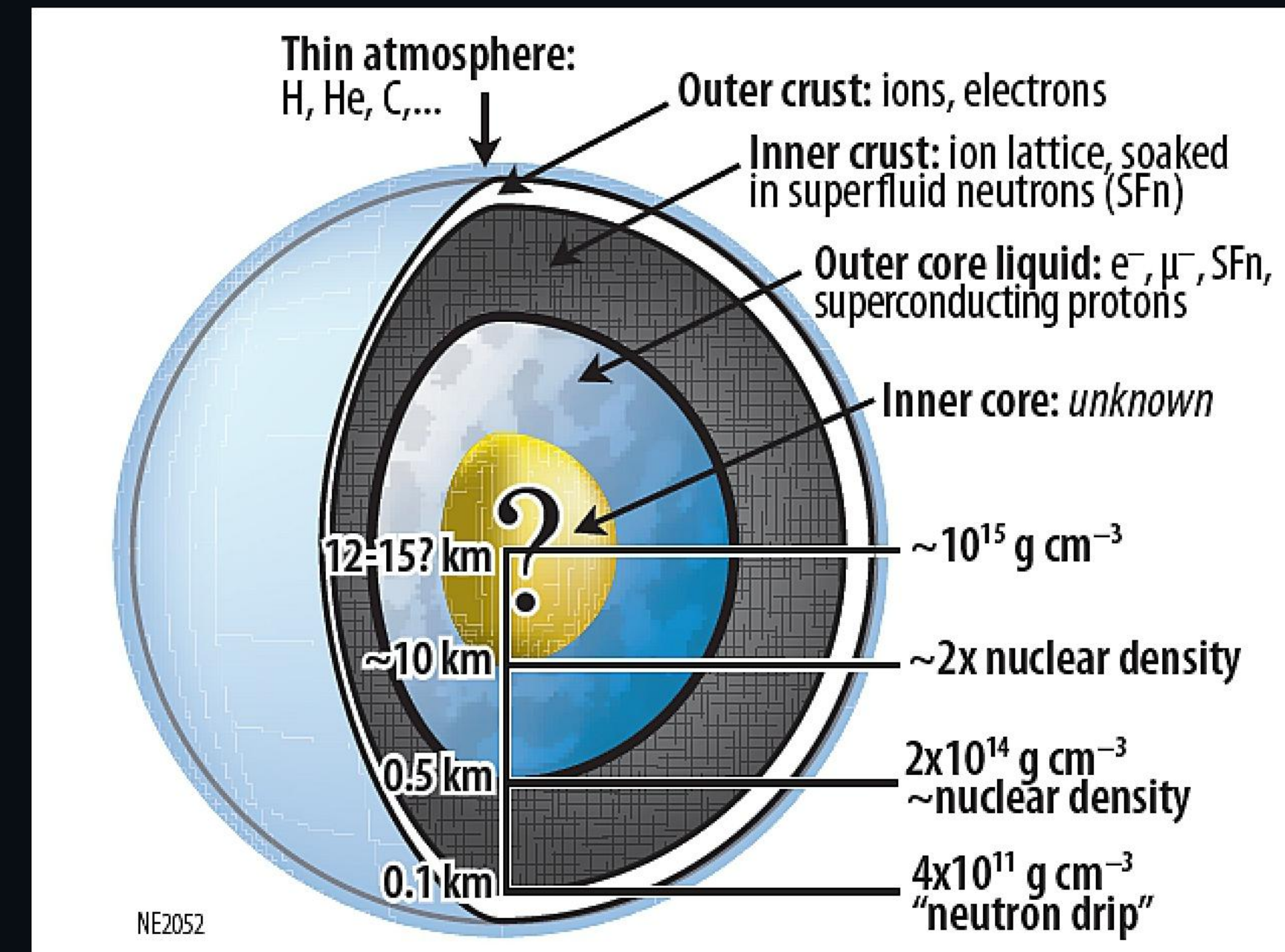
Physics of Neutron Stars

[see talk by *D. Chatterjee* (Mon)]

- Neutron stars are extreme laboratories.

- Strong-field gravity
- Dense nuclear matter
- Rapid rotation
- Strong magnetic fields
- Superfluidity
- Solid crusts

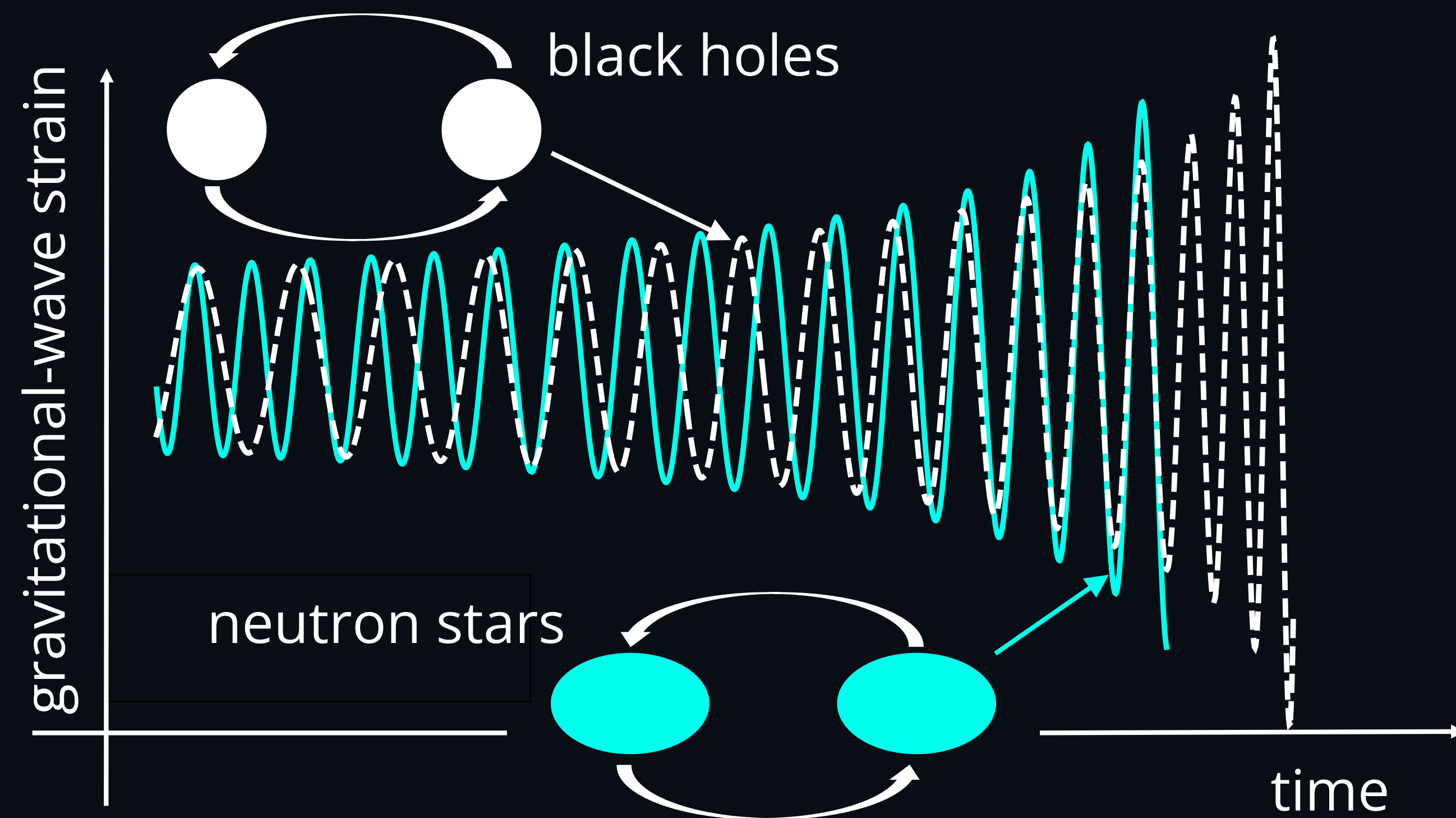
[see talks by *A. R. Raduta, M. Shahrbafe, P. Davis, G. Riviello, M. Bigdeli* (Mon); *S. Tsiopelas, G. Montefusco* (Tue)]



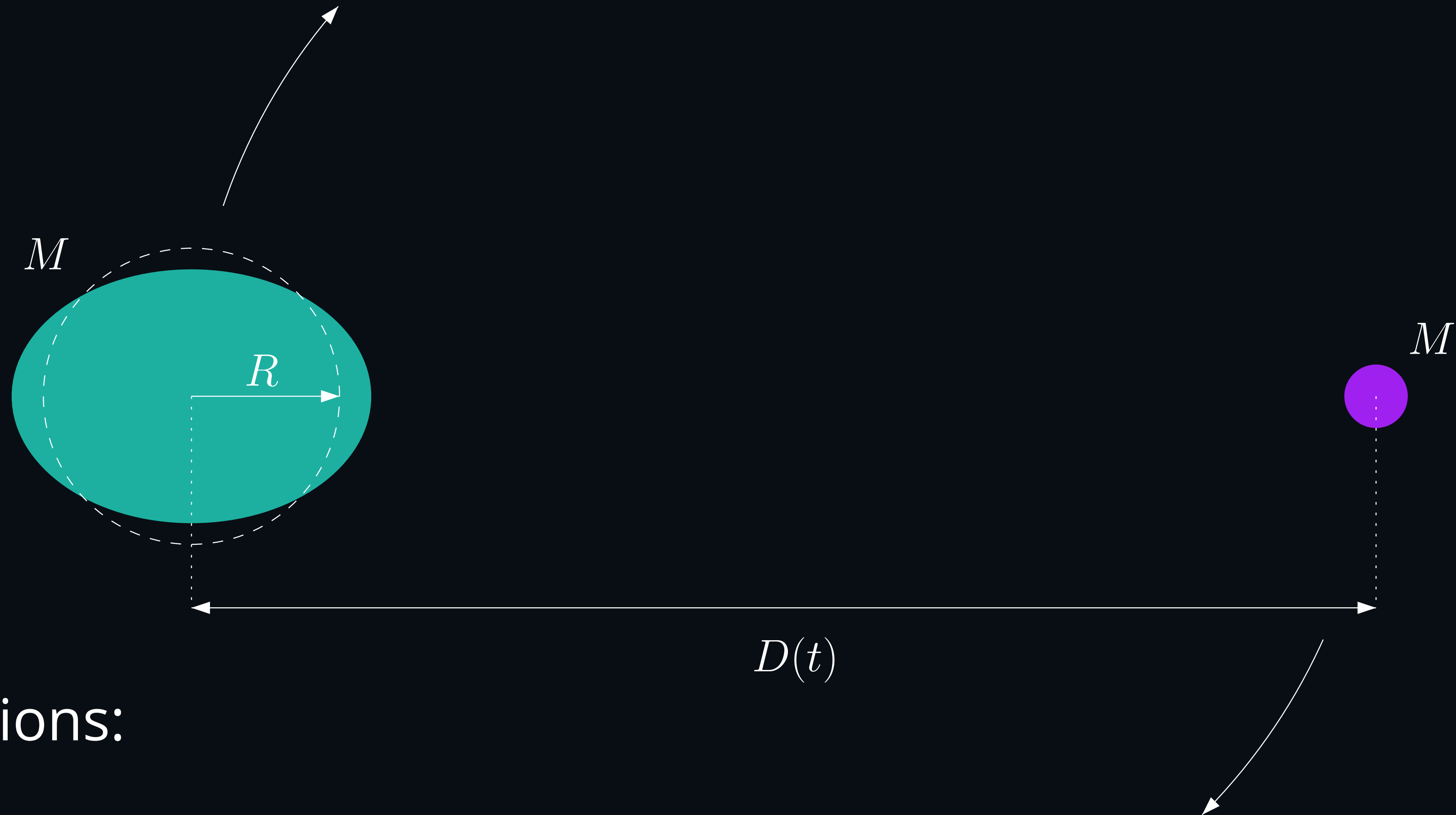
- Each of these aspects give rise to their own family of **oscillation modes**.

Matter Effects

- Consider two compact binaries: one with **black holes**, while the other comprises **neutron stars**
- The binaries are otherwise identical; same component *masses*, *spins*, *binary orientation* and *position* with respect to the detectors

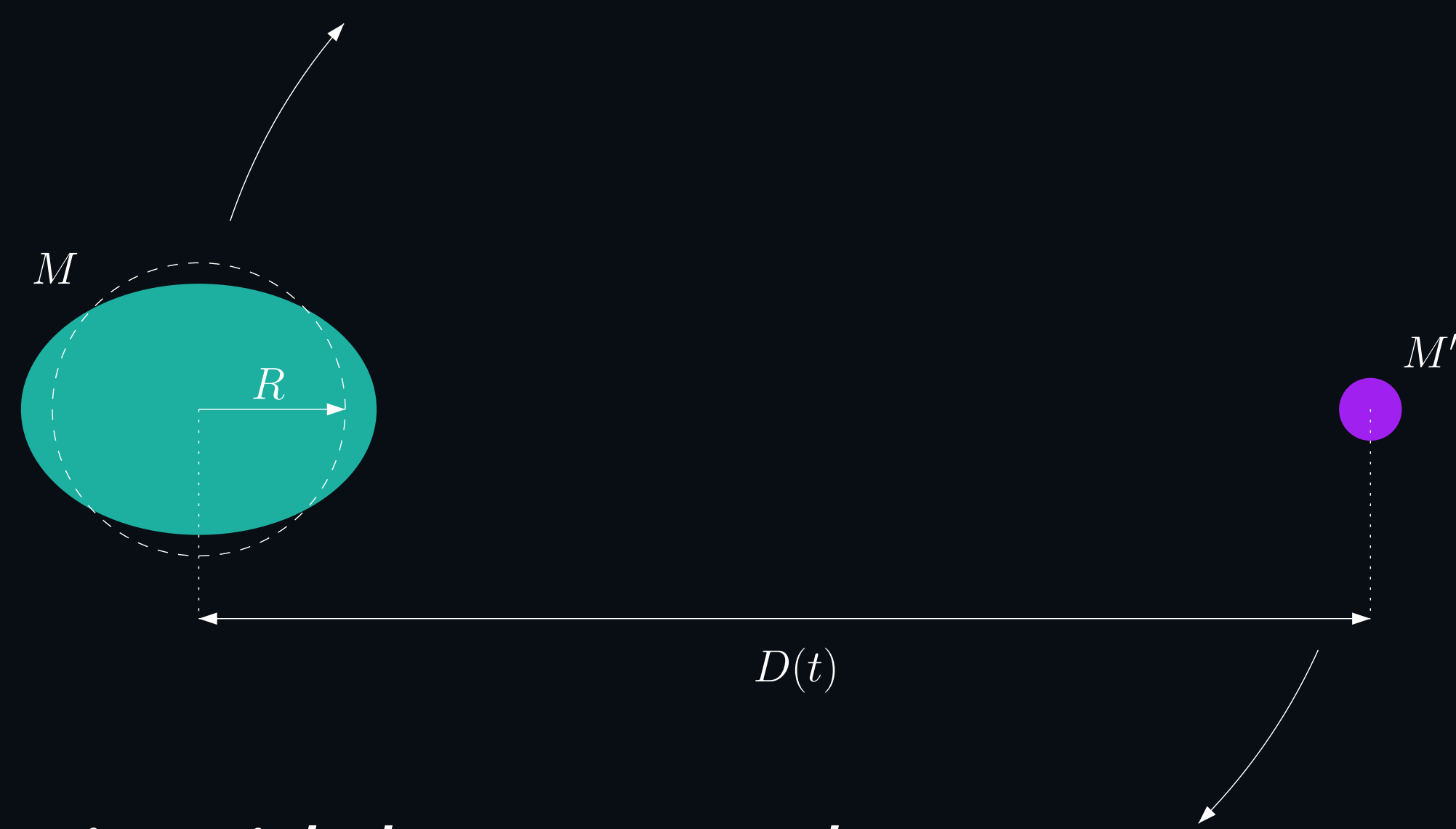


Static Tide



- Assumptions:
 - Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$
 - Slowly varying external field, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Static Tide



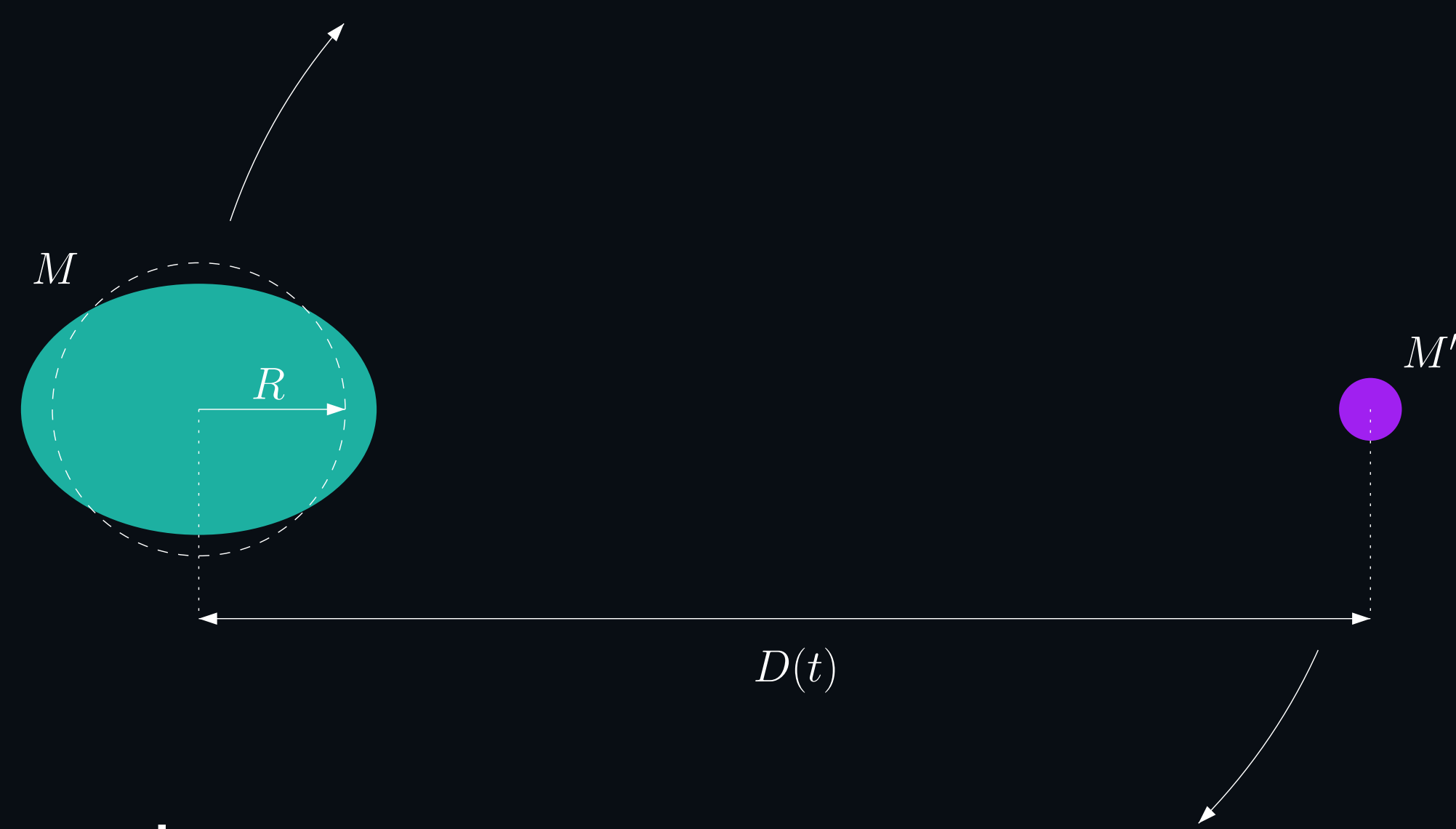
- Star's (static) shape is quantified by its *tidal Love numbers* k_l ,

$$U_l(r) = \left[2k_l \left(\frac{R}{r} \right)^{l+1} + \left(\frac{r}{R} \right)^l \right] \chi_l(R),$$

where

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dU_l}{dr} \right) - \frac{l(l+1)}{r^2} U_l = - \frac{4\pi G \rho}{dp/d\rho} U_l$$

Static Tide



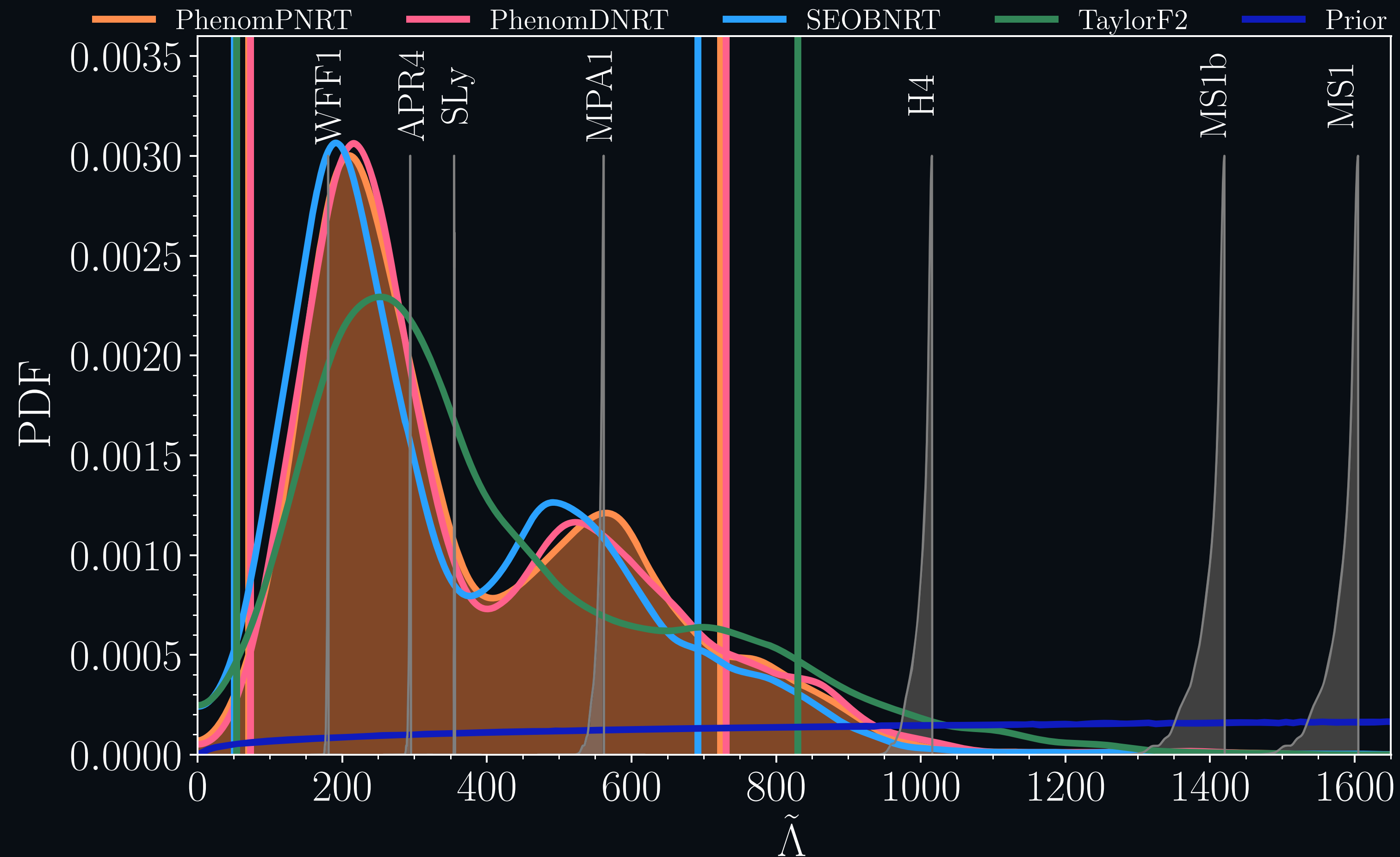
- Tide enters the gravitational-wave phase as [\[Flanagan+Hinderer, Phys. Rev. D **77**, 021502 \(2008\)\]](#)

$$\Psi_{\text{tide}}(\nu) = -\frac{3}{128} \frac{M_{\text{total}}}{\mu} \frac{1}{\nu^5} \cdot \left[\frac{39}{2} \tilde{\Lambda} \nu^{10} + O(\nu^2) \right],$$

where

$$\tilde{\Lambda} = \frac{16}{13} \frac{1}{M_{\text{total}}} \left[(M + 12M') M^4 \Lambda + (M' + 12M) M'^4 \Lambda' \right], \quad \Lambda = \frac{2}{3} \left(\frac{c^2 R}{GM} \right)^5 k_2$$

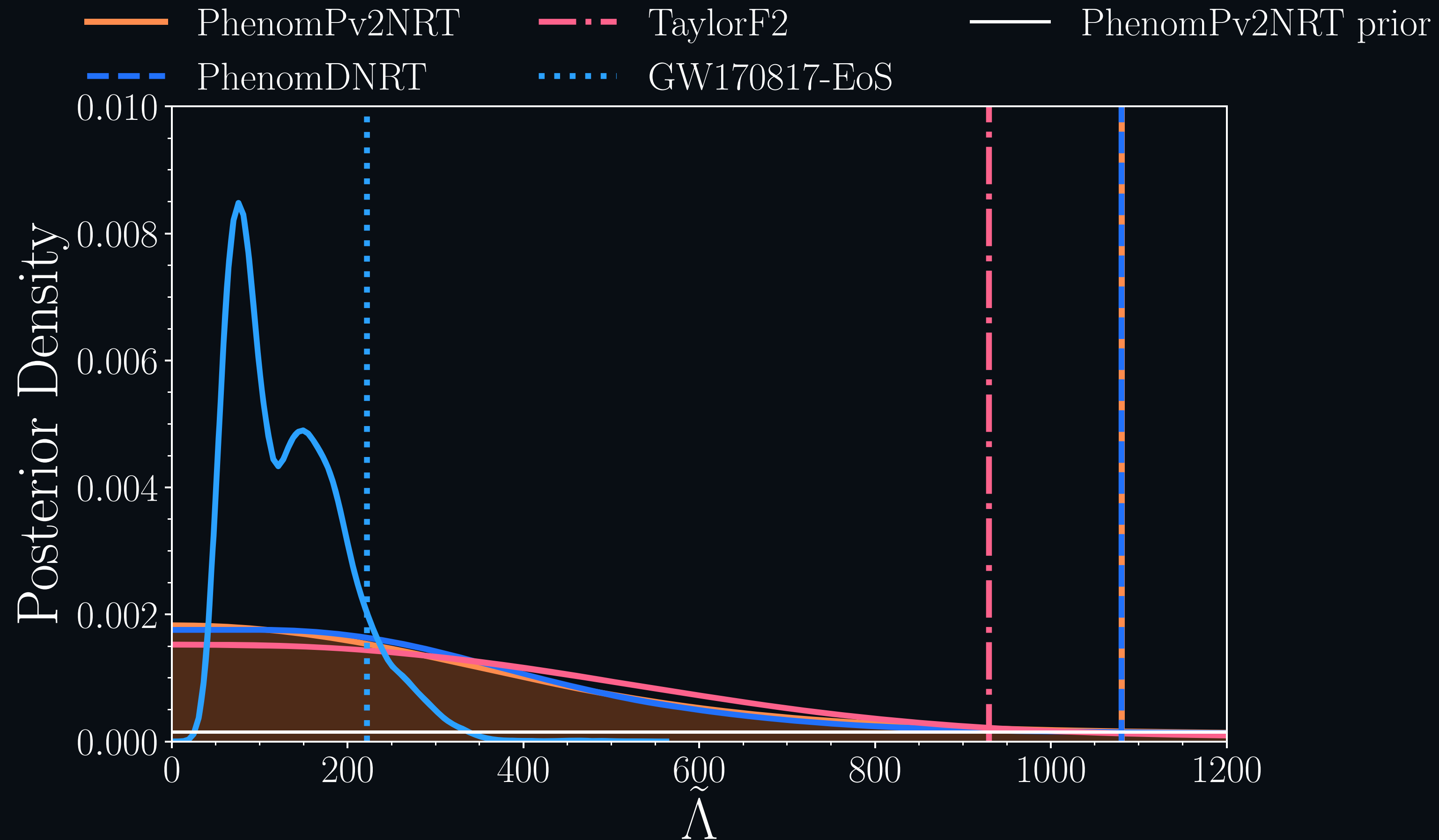
Matter Constraints: GW170817



[LIGO-Virgo Collaboration, Phys. Rev. X **9**, 011001 (2019)]

➡ Provided the function $\rho = \rho(p)$, one can solve for M and Λ

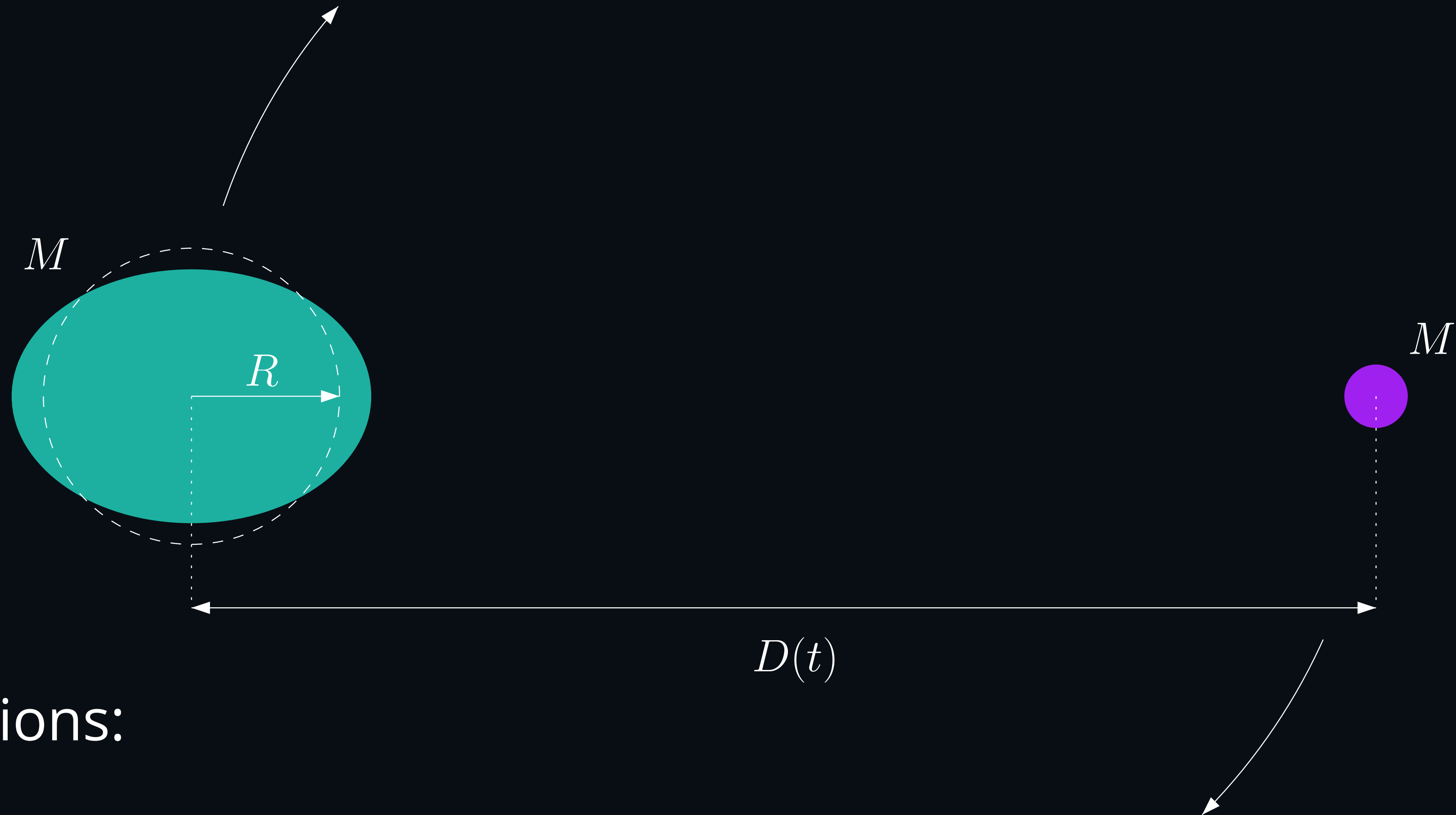
Matter Constraints: GW190425



[LIGO-Virgo Collaboration, *Astrophys. J.* **892**, L3 (2020)]

- However, there is more physics in the tide...

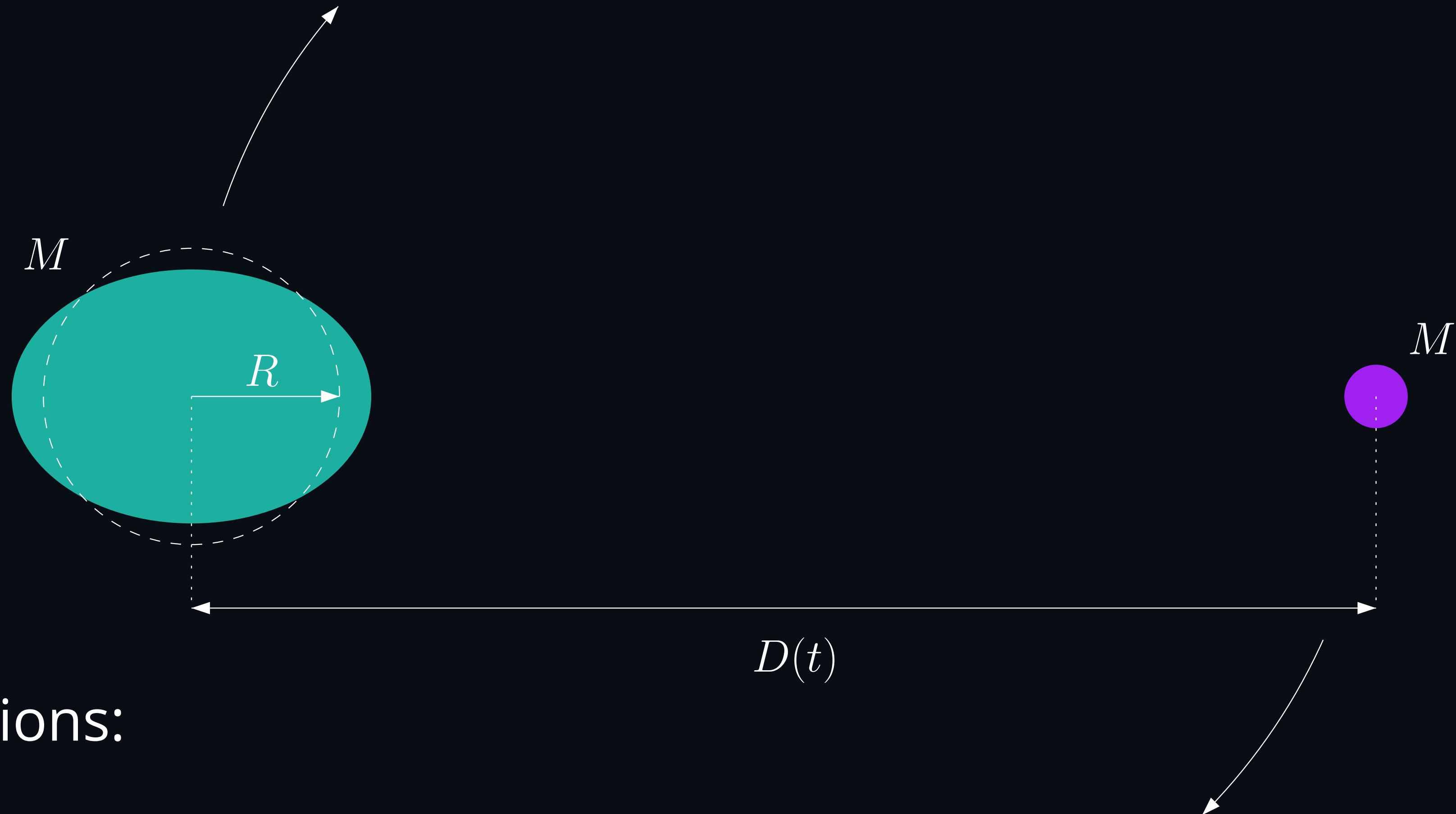
Static Tide



- Assumptions:
 - Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$
 - Slowly varying external field, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Dynamical Tide

[see talks by *S. Ghosh* (Mon); *A. Suvorov*, *A. Reboul-Salze*,
B. K. Pradhan (Tue); *Y. Gao* (Wed)]

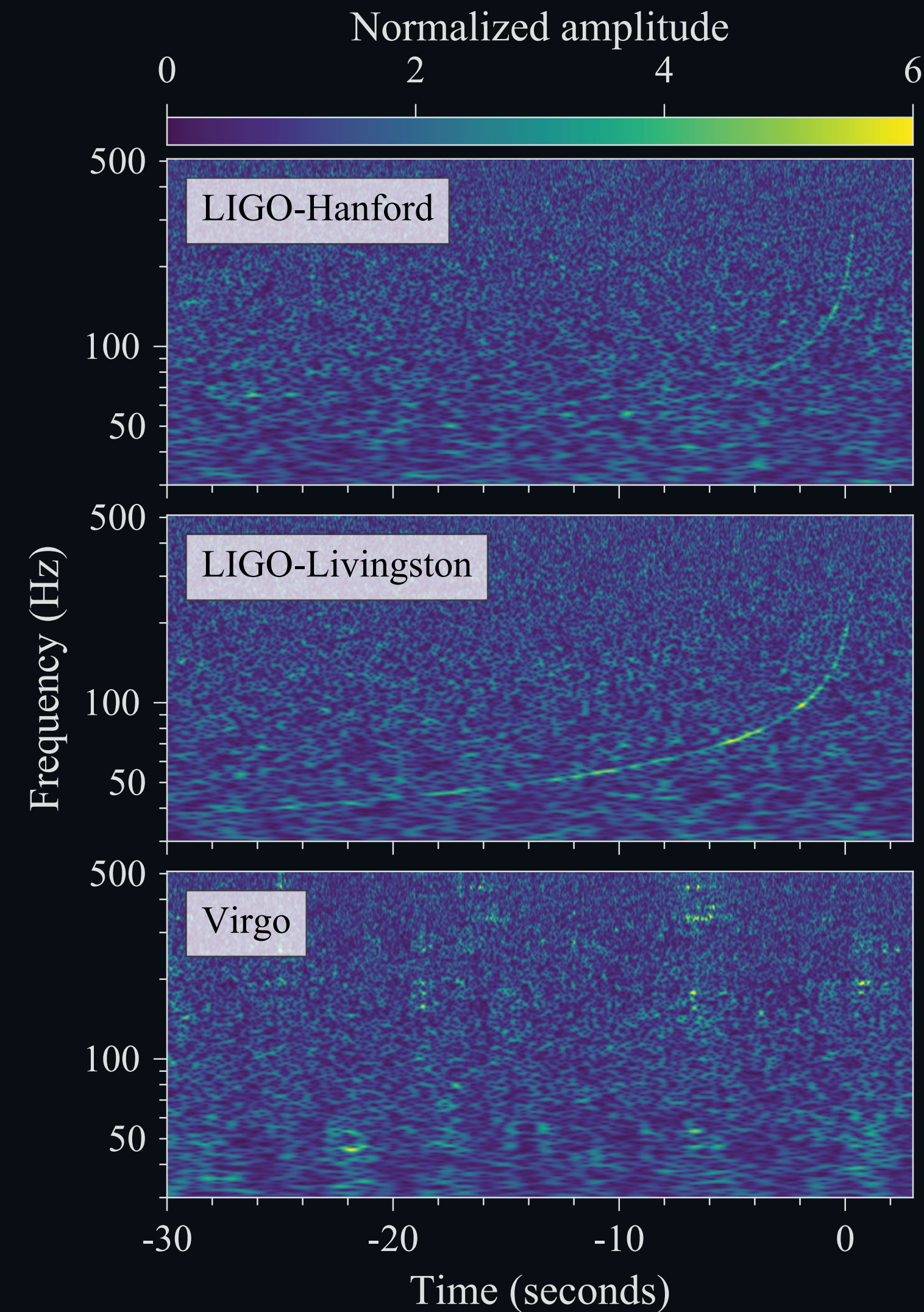


- Assumptions:

- Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$

- ~~Slowly varying external field, $\lambda \equiv \dot{\Phi}/\omega_\alpha \ll 1$~~

Inspiral



- The static approximation inevitably breaks down during the inspiral,

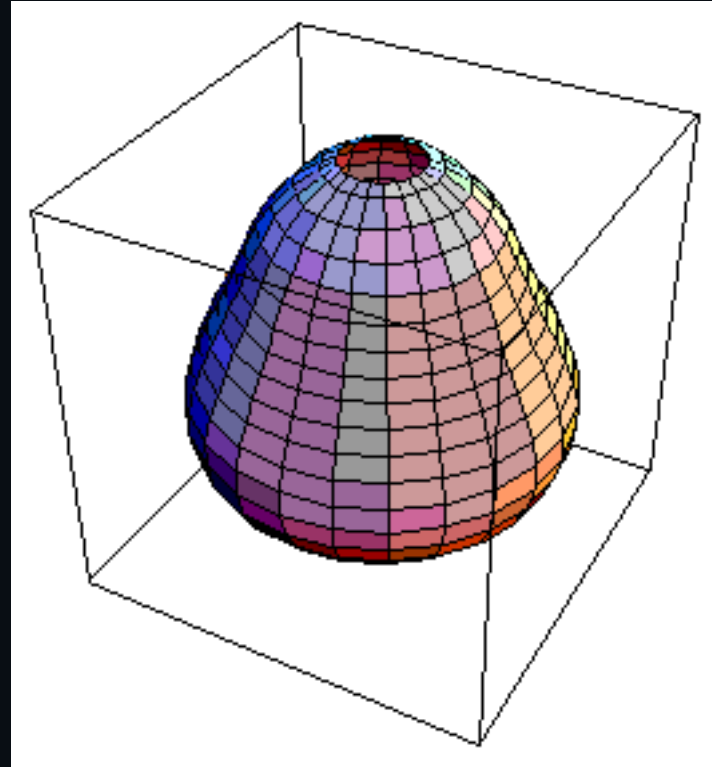
$$\lambda \sim O(1)$$

- The frequency ω_α represents a characteristic mode frequency,

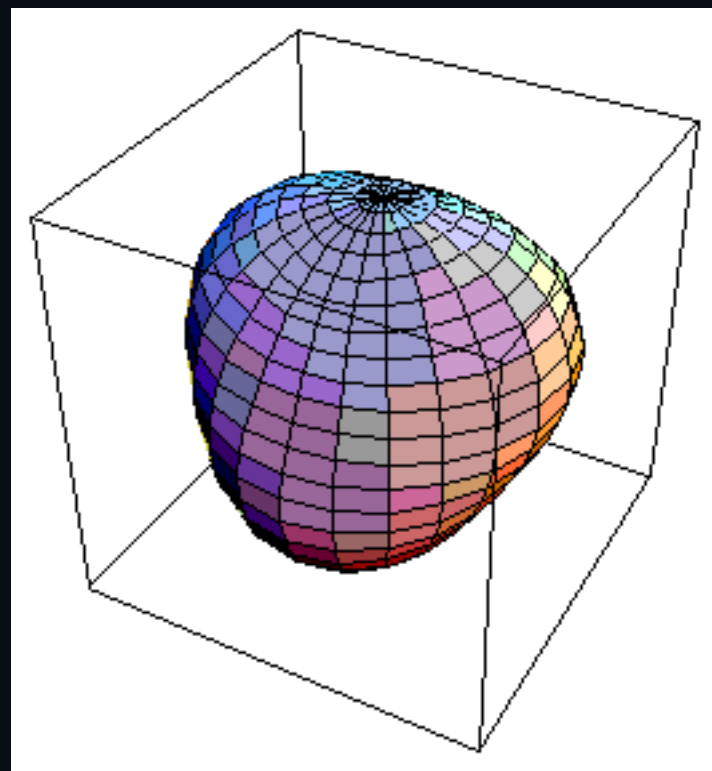
$$\omega_f \sim \sqrt{\frac{GM}{R^3}} \approx 2\pi \cdot 2.2 \text{ kHz} \left(\frac{M}{1.4M_\odot} \right)^{1/2} \left(\frac{10 \text{ km}}{R} \right)^{3/2}$$

Neutron–Star Mode Spectrum

- f -mode: scales with average density
- p -modes: sound waves in the star (overtones of the f -mode)
- g -modes: buoyancy waves from thermal/composition gradients
- inertial modes (including r -modes): associated with rotation
- i -modes: arise from phase transitions
- Also:
 - w -modes, s -modes, Alfvén modes, ...



$(l, m) = (3, 0)$



$(l, m) = (3, 2)$

[see talks by *D. Tseneklidou* (Mon); *S. Jaraba, J. C. Jiménez Apaza* (Tue);
G. Fernández Rodríguez, D. R. Karkevandi (Wed)]

Mode-Sum Representation

- Normal modes form a complete basis [Chandrasekhar, *Astrophys. J.* **139**, 664 (1964)],

$$\xi(t, \mathbf{x}) = \sum_{\alpha} q_{\alpha}(t) \xi_{\alpha}(\mathbf{x}), \quad \mathbf{C}(\mathbf{x}) \cdot \xi_{\alpha}(\mathbf{x}) = \omega_{\alpha}^2 \xi_{\alpha}(\mathbf{x})$$

- The tidal equation of motion simplifies to

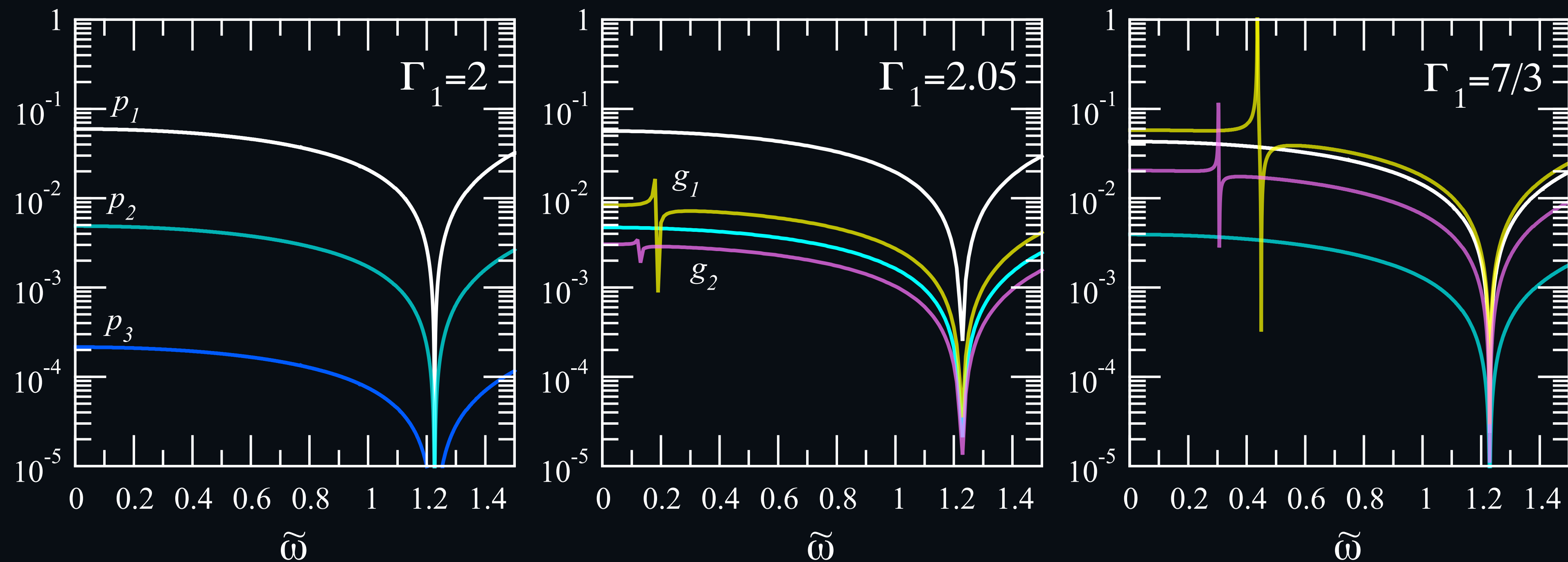
$$\ddot{q}_{\alpha}(t) + \omega_{\alpha}^2 q_{\alpha}(t) = \frac{Q_{\alpha}(t)}{\mathcal{E}_{\alpha}} \propto e^{-im\Phi(t)}$$

- **Challenge:** Can this be formulated in general relativity?

Equilibrium Tide

- For an equilibrium orbit, $\dot{\Phi} = \text{const}$,

$$q_{\alpha}(t) = \frac{Q_{\alpha}(t)}{\mathcal{E}_{\alpha}} \frac{1}{\omega_{\alpha}^2 - (m\dot{\Phi})^2}$$



Static Limit

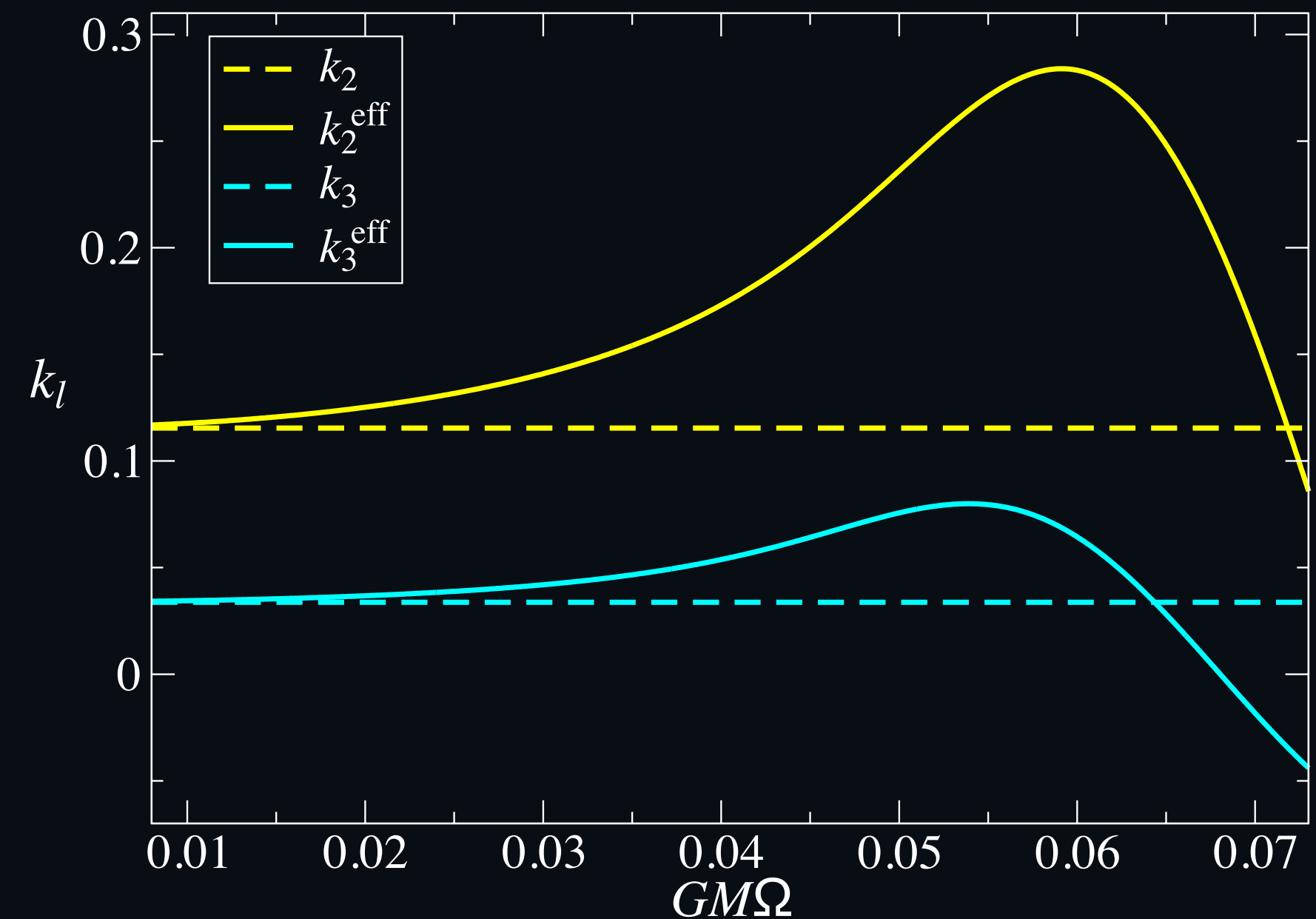
- In the static limit, $\dot{\Phi} = 0$,

$$q_{\alpha} = \frac{Q_{\alpha}}{\mathcal{E}_{\alpha}} \frac{1}{\omega_{\alpha}^2}$$

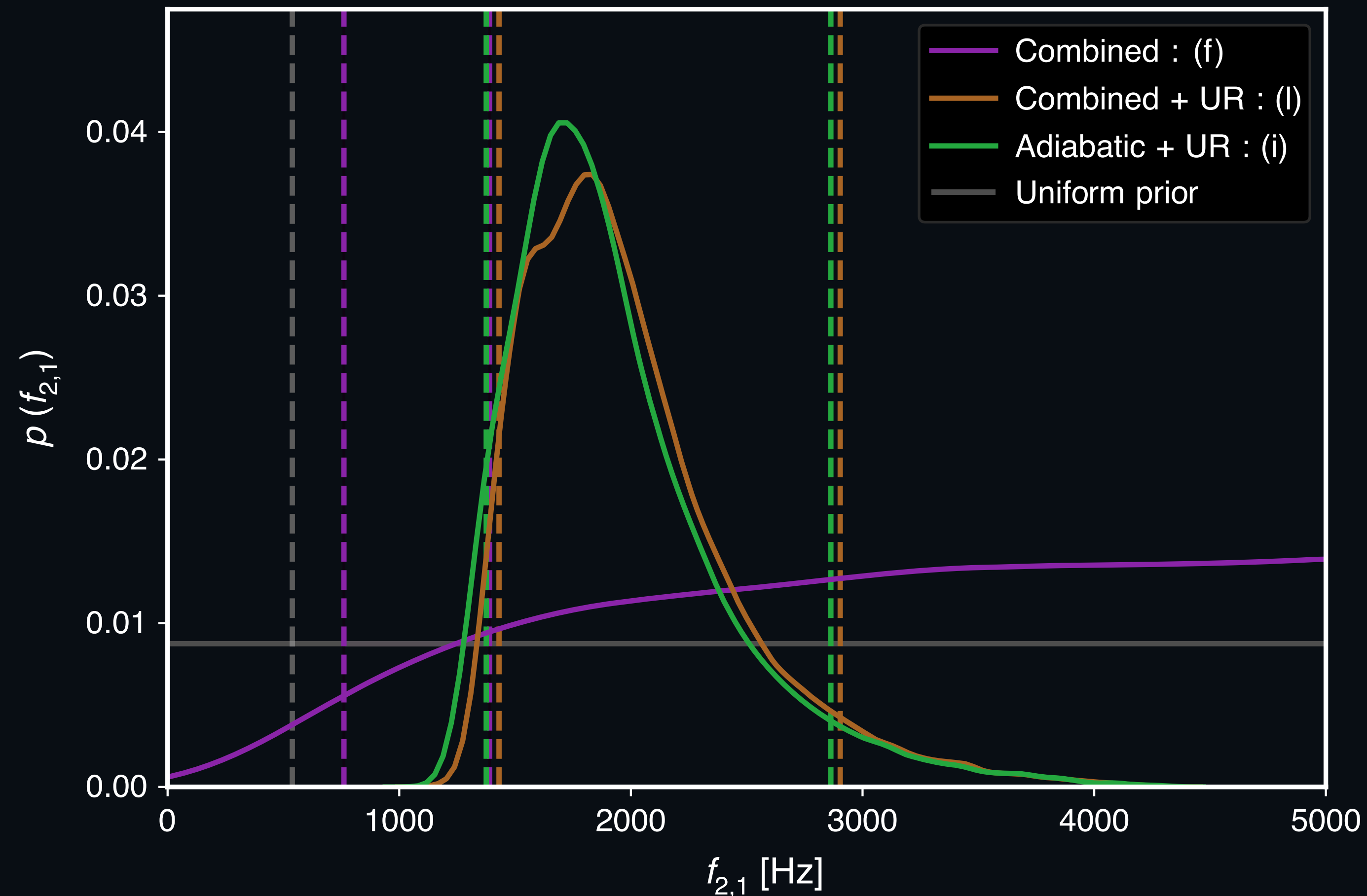
$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
Mode	k_l	Mode	k_l	Mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
9×10^{-4}		7×10^{-4}		3×10^{-4}	

f -mode Approximation

- The dynamical tide is dominated by the f -mode
- There have been models developed for the f -mode dynamical tide that use
 - *effective-one-body* [Steinhoff+, Phys. Rev. D **94**, 104028 (2016)],
 - *Newtonian* [Schmidt+Hinderer, Phys. Rev. D **100**, 021501 (2019)] and
 - *phenomenological* approaches [Abac+, Phys. Rev. D **109**, 024062 (2024)]



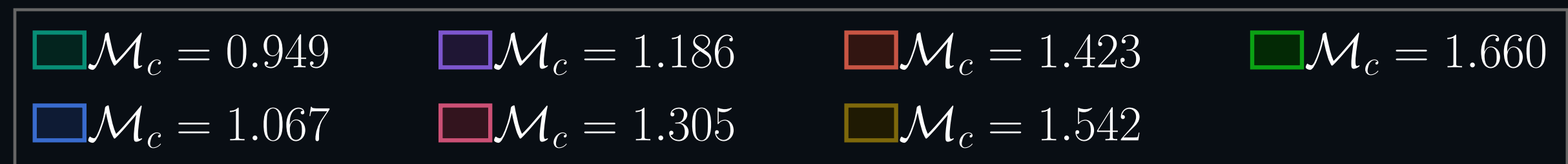
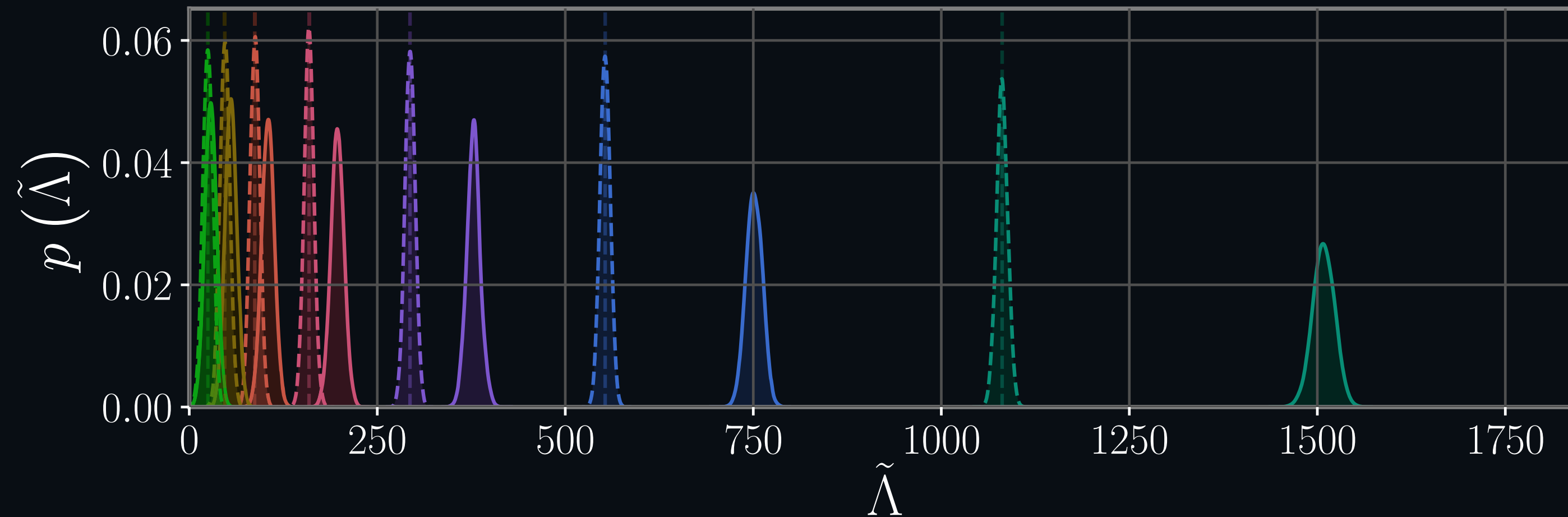
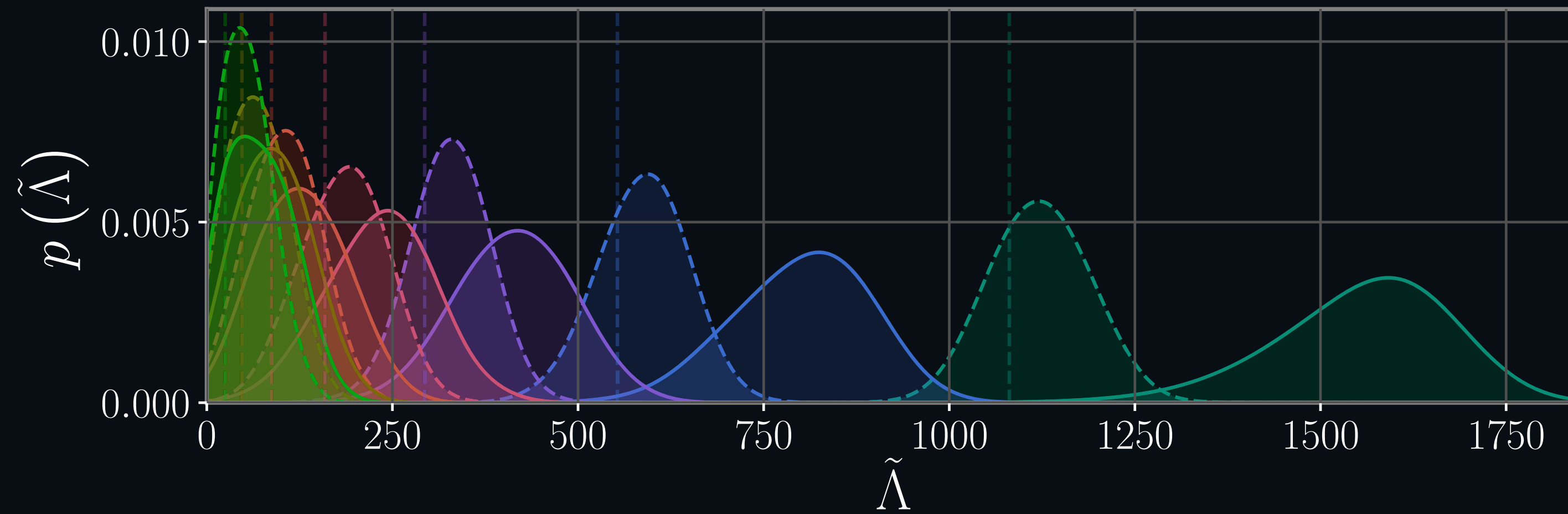
Towards Asteroseismology



[Pratten+, Nat. Commun. **11**, 2553 (2020)]

- **Challenge:** Go beyond universal relations in inference

Biases



[Pratten+, Phys. Rev. Lett. **129**, 081102 (2022)]

Sub-Dominant Modes

- Low-frequency modes (including *g*-modes, *r*-modes and *i*-modes) will become **resonant** during inspiral,

$$q_{\alpha}(t) = \frac{Q_{\alpha}(t)}{\mathcal{E}_{\alpha}} \frac{1}{\omega_{\alpha}^2 - (m\dot{\Phi})^2} \implies m\dot{\Phi} \approx \omega_{\alpha}$$

- Energy is extracted from the orbit,

$$\Delta E_{\alpha} \sim |q_{\alpha}|^2,$$

which results in a finite orbital phase shift $\Delta\Phi$

Composition

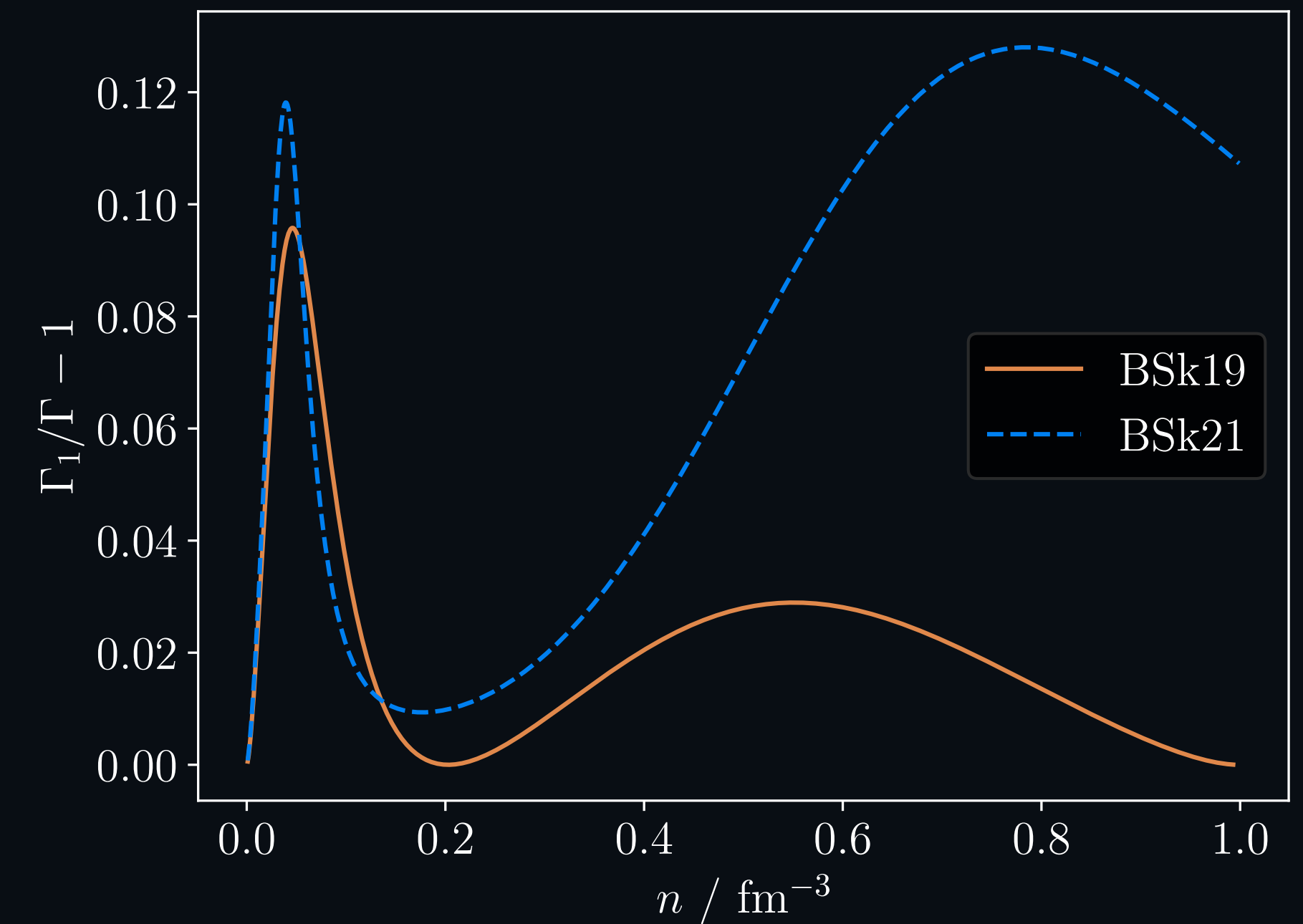
- Instead of

$$d\varepsilon = \frac{\varepsilon + p}{n_b} dn_b \implies \varepsilon = \varepsilon(n_b),$$

the first law is

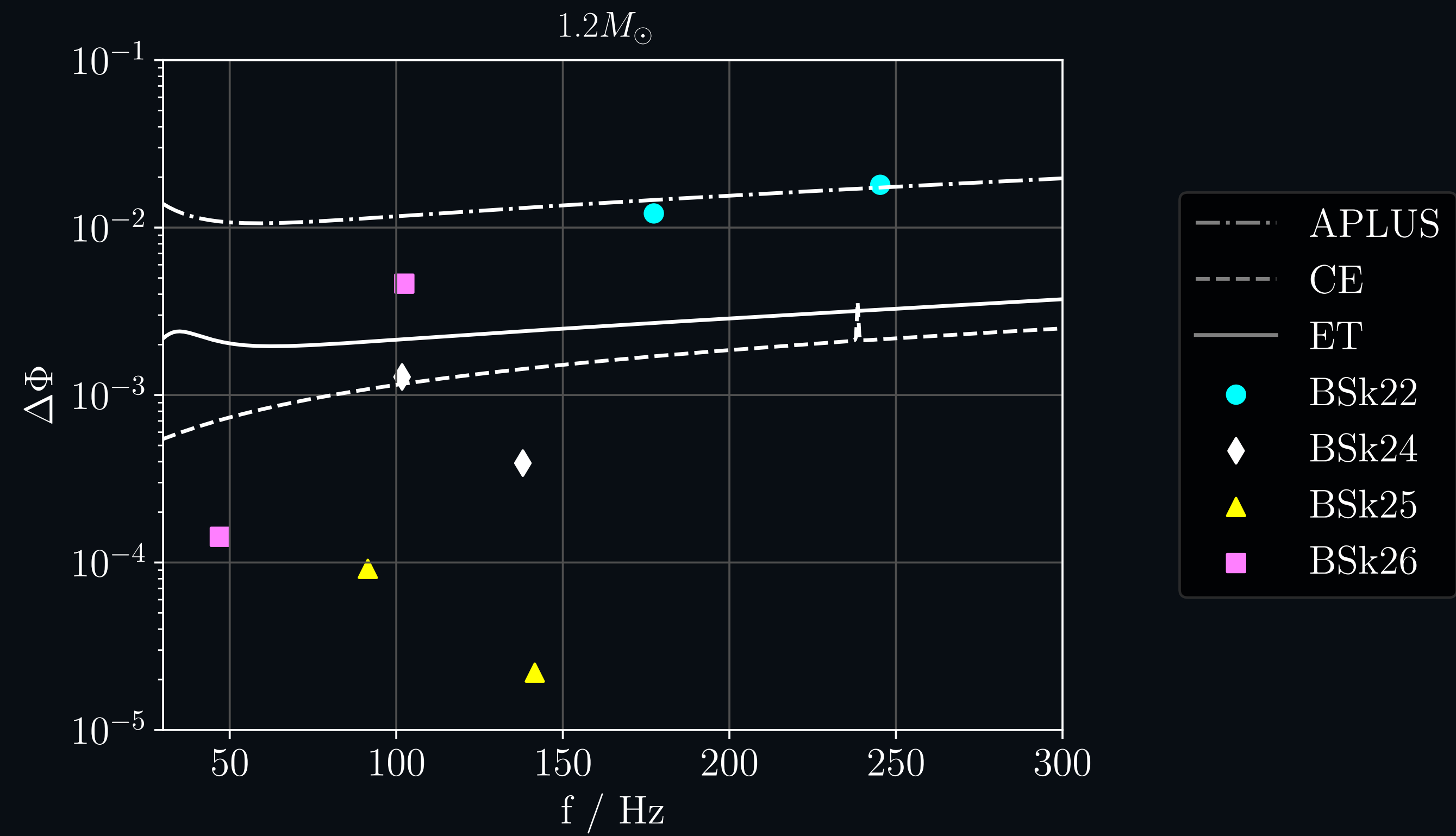
$$d\varepsilon = \frac{\varepsilon + p}{n_b} dn_b + n_b \mu_\Delta dY_e \implies \varepsilon = \varepsilon(n_b, Y_e)$$

- When there are slow weak nuclear reactions,
 $\mu_\Delta \neq 0$



[FG + Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

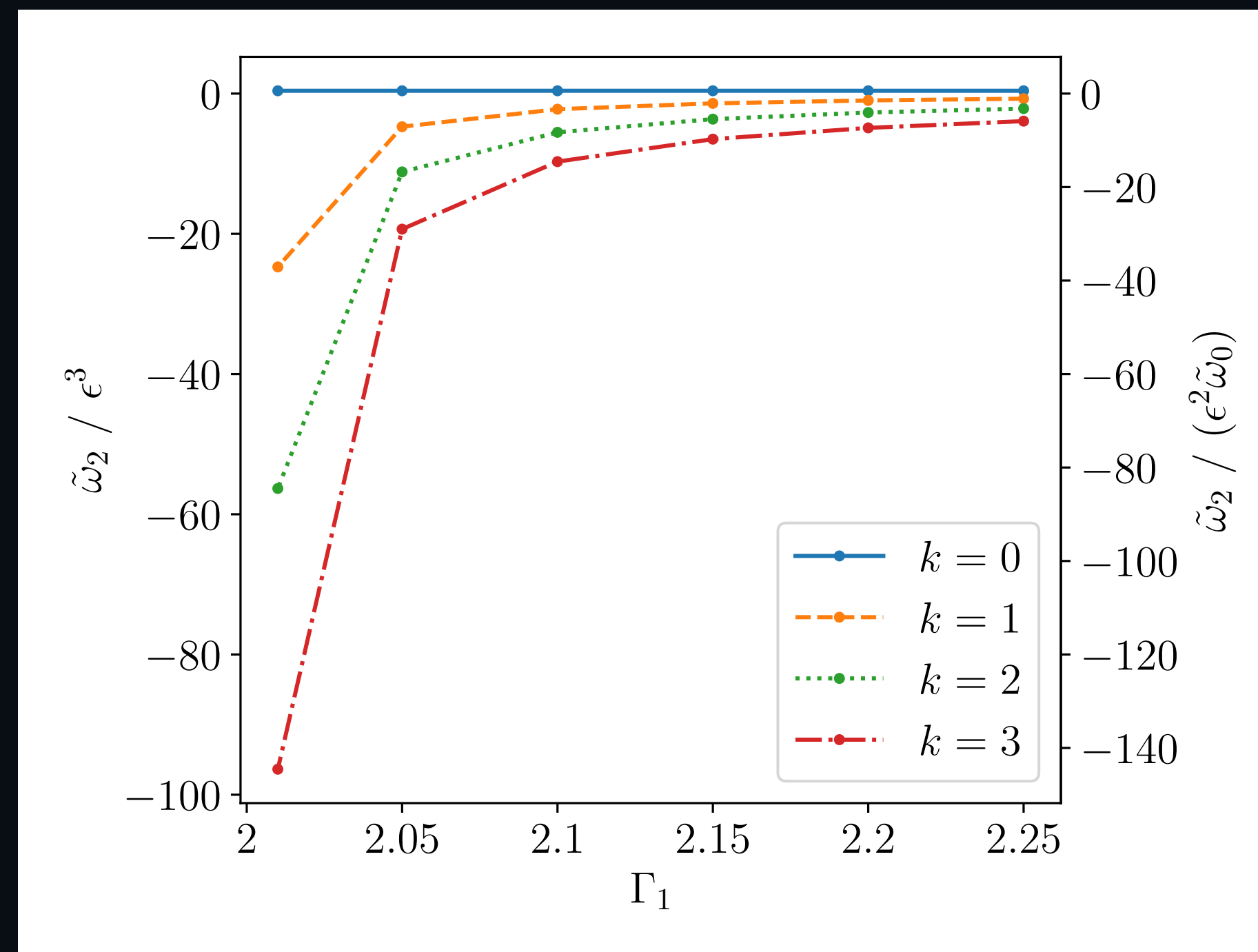
g -modes



[Counsell, FG + Andersson, Mon. Not. R. Astron. Soc. **536**, 1967 (2025)]

Rotation

- A special class of inertial modes have *axial* parity: the *r*-modes
- The *r*-modes are famous for their gravitational-wave-driven instability
- They also probe composition gradients



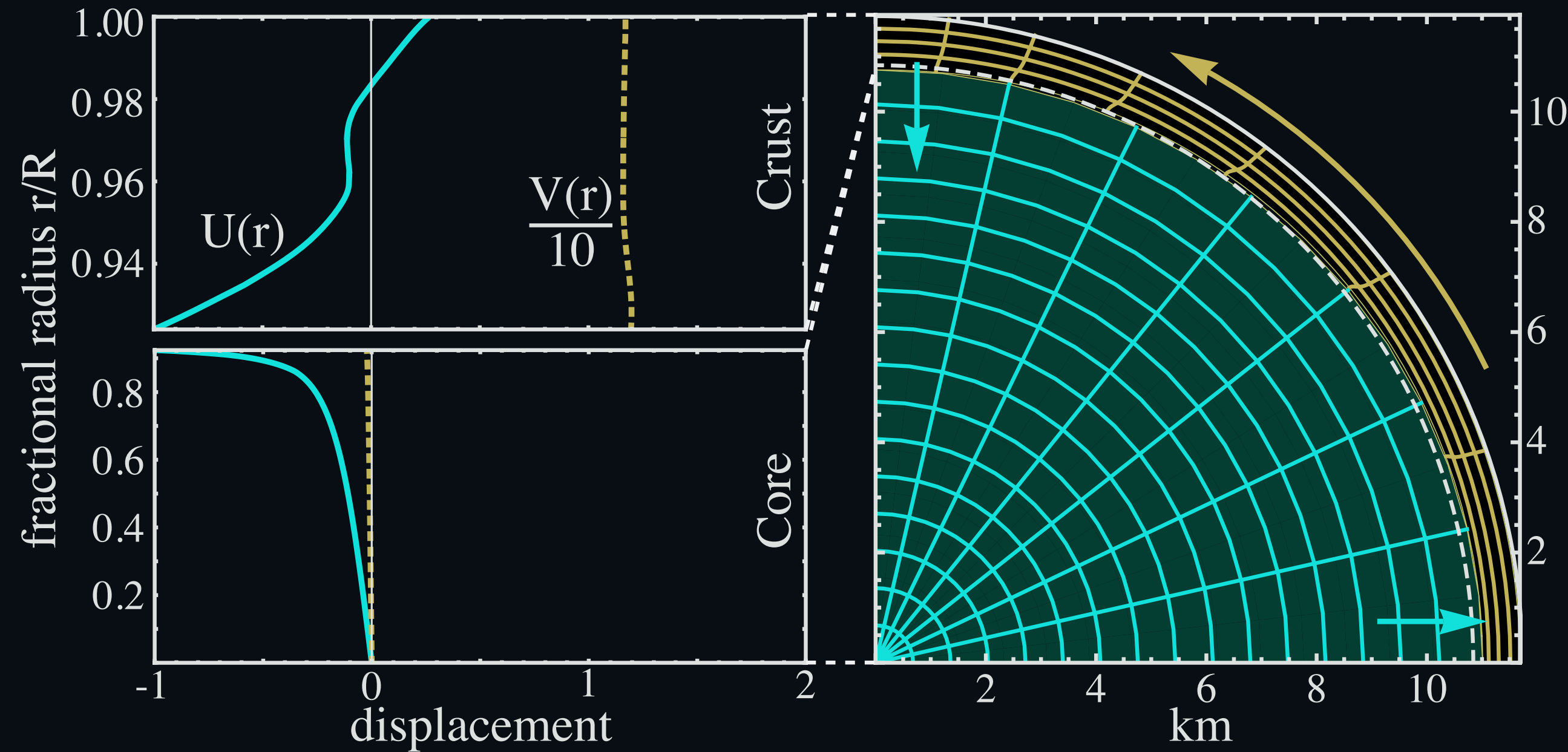
[FG + Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

r-modes

- The *r*-mode couples strongly to the gravito-magnetic tide
- Estimates give [\[Flanagan+Racine, Phys. Rev. D **75**, 044001 \(2007\)\]](#)

$$\Delta\Phi \approx -0.03 \left(\frac{R}{10 \text{ km}} \right)^4 \left(\frac{f_{\text{spin}}}{100 \text{ Hz}} \right)^{2/3} \left(\frac{1.4M_{\odot}}{M} \right)^{10/3}$$

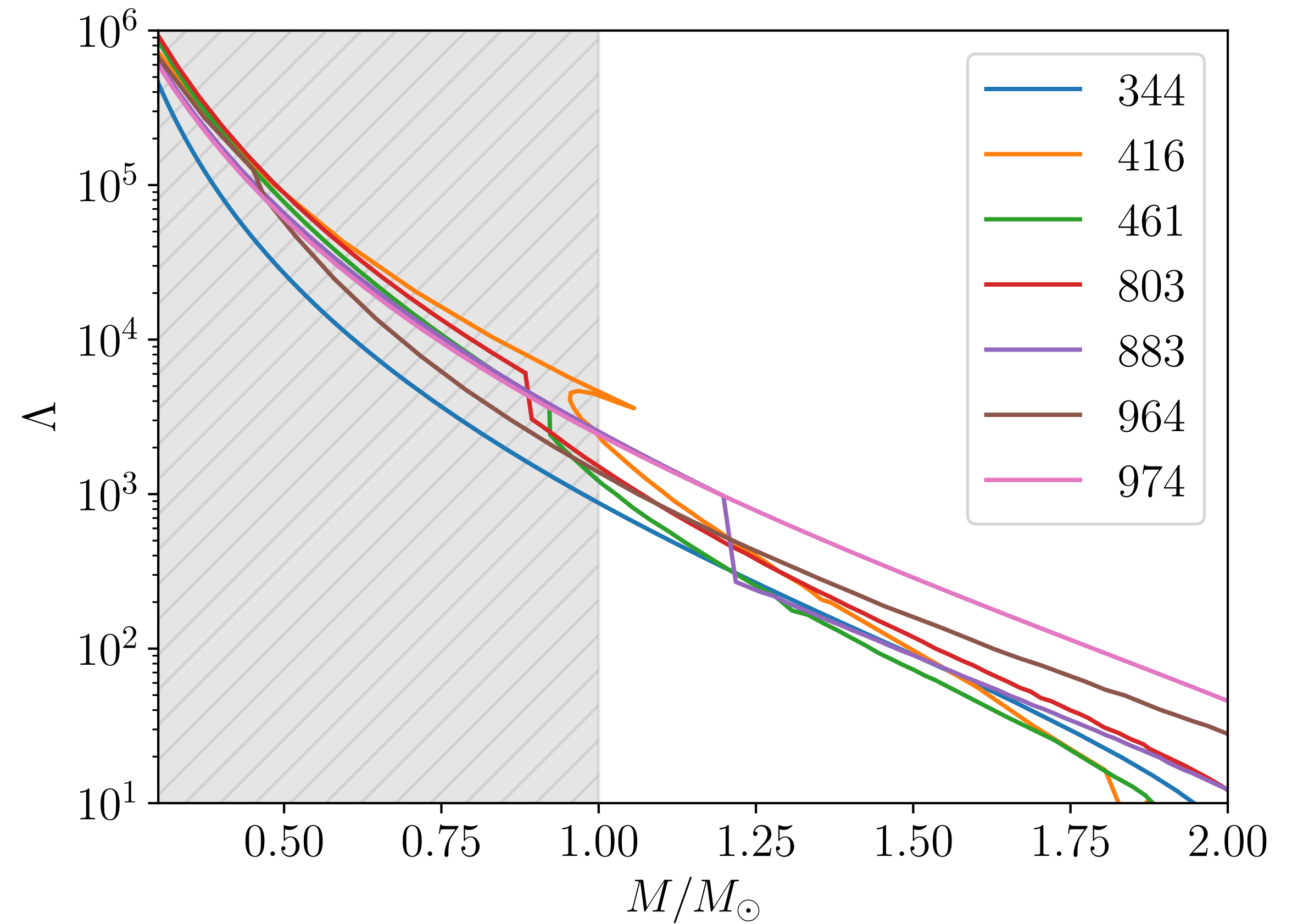
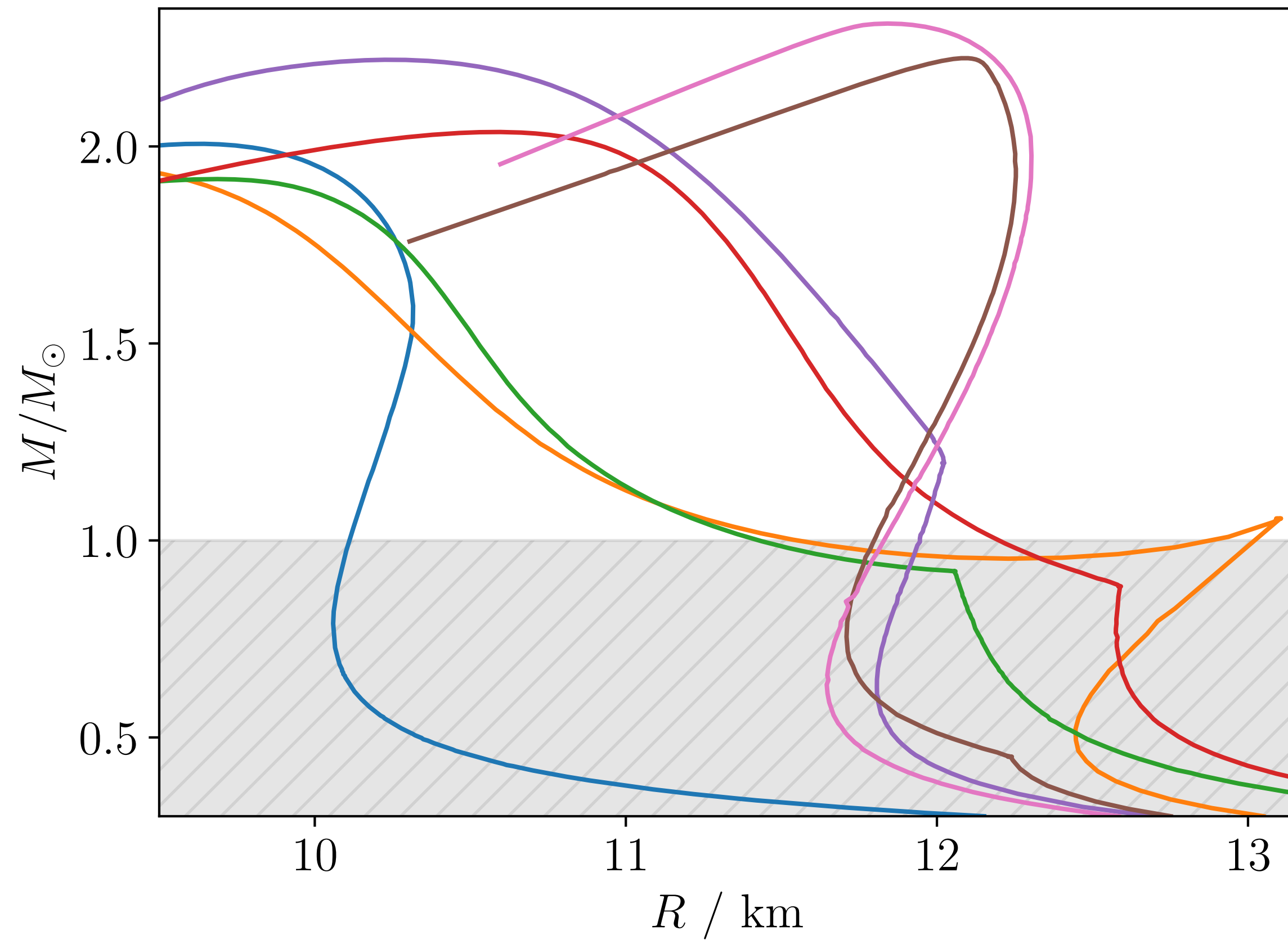
Phase Transitions



[Tsang+, Phys. Rev. Lett. **108**, 011102 (2012)]

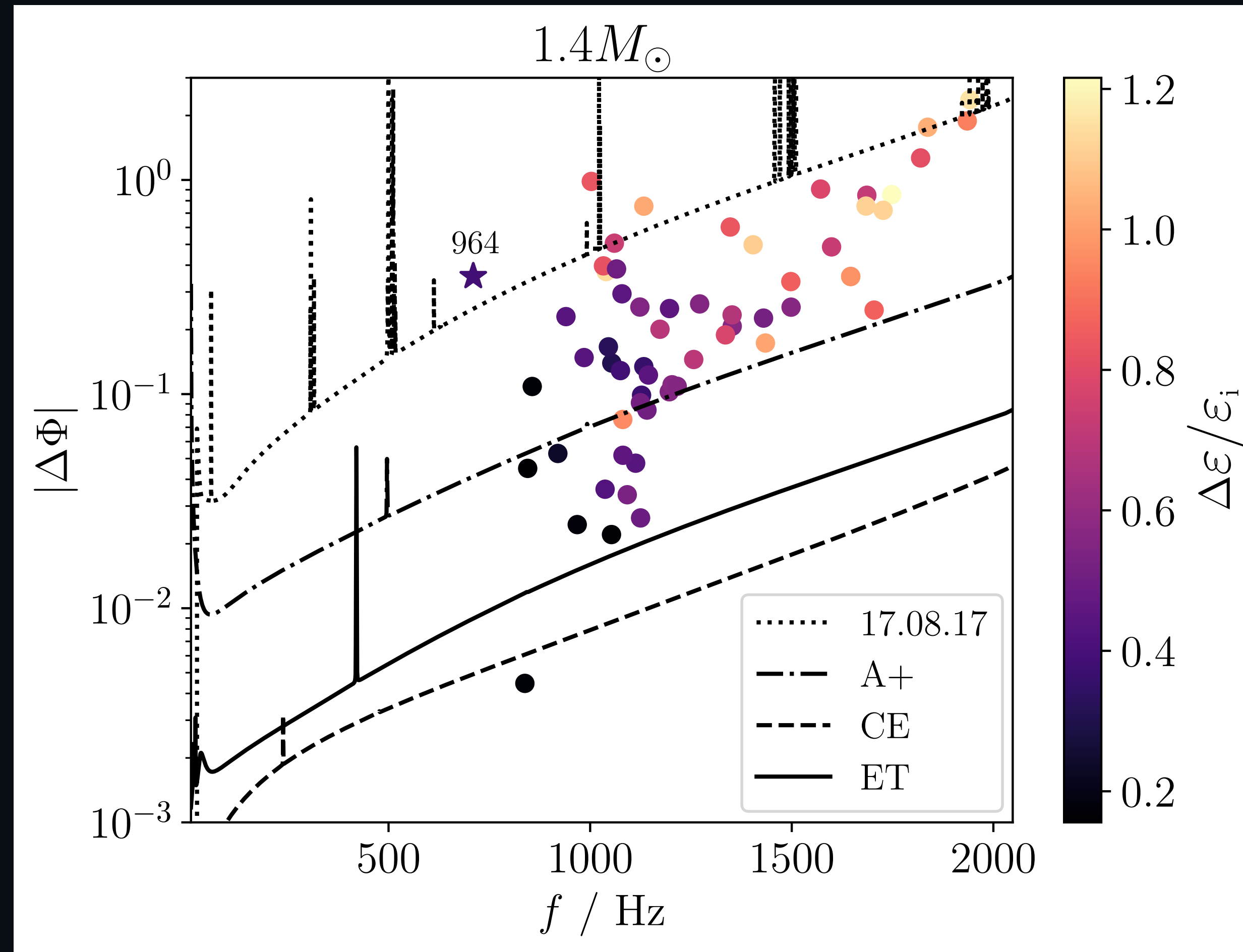
- The interfacial *i*-mode arises when there is a first-order phase transition in the star
- This may occur at the core-crust interface or (possibly) at a transition to deconfined quark matter in the core

Masquerade Problem



[Counsell, FG+, Phys. Rev. Lett. **135**, 081402 (2025)]

i-modes

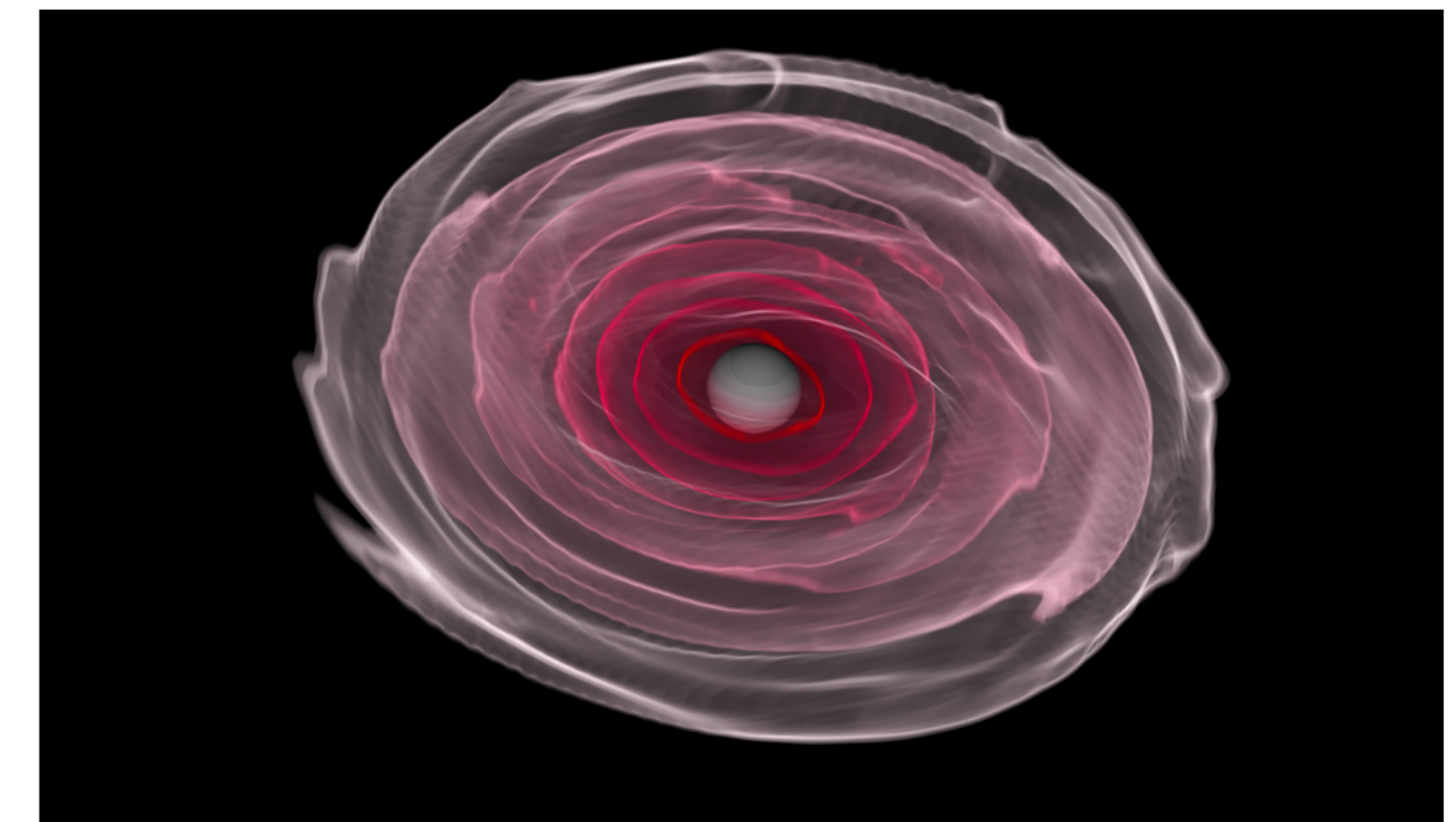
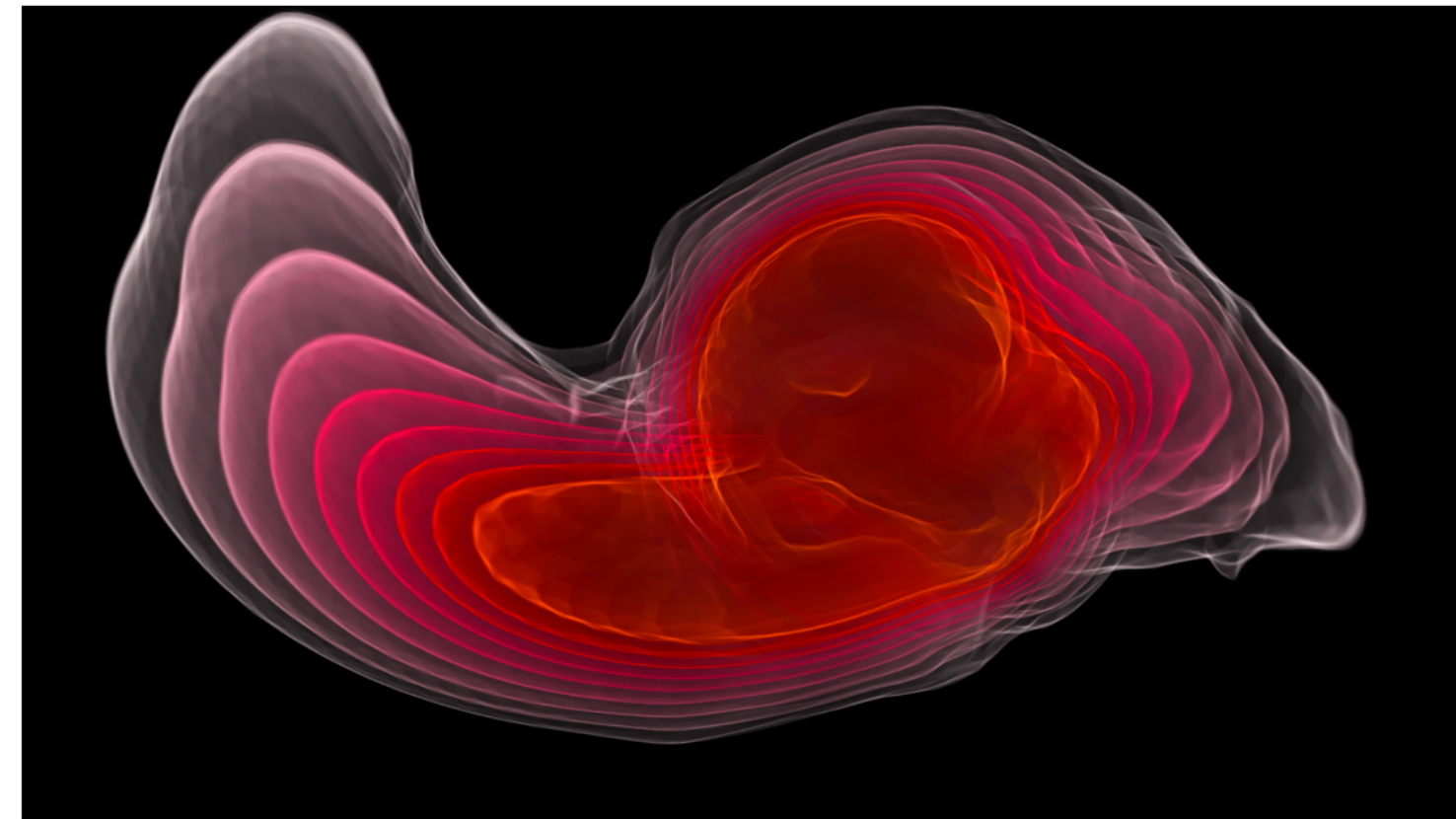
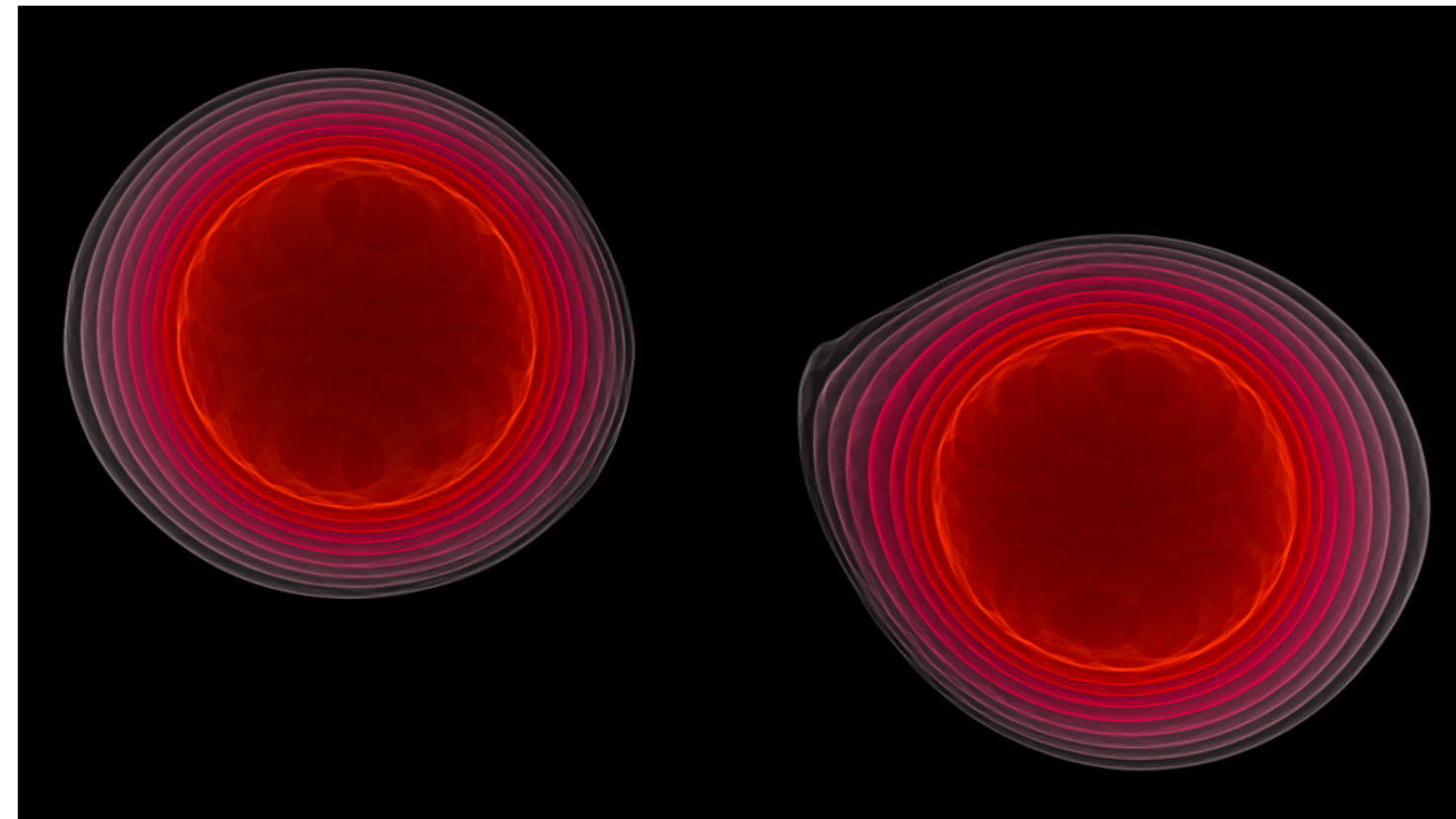
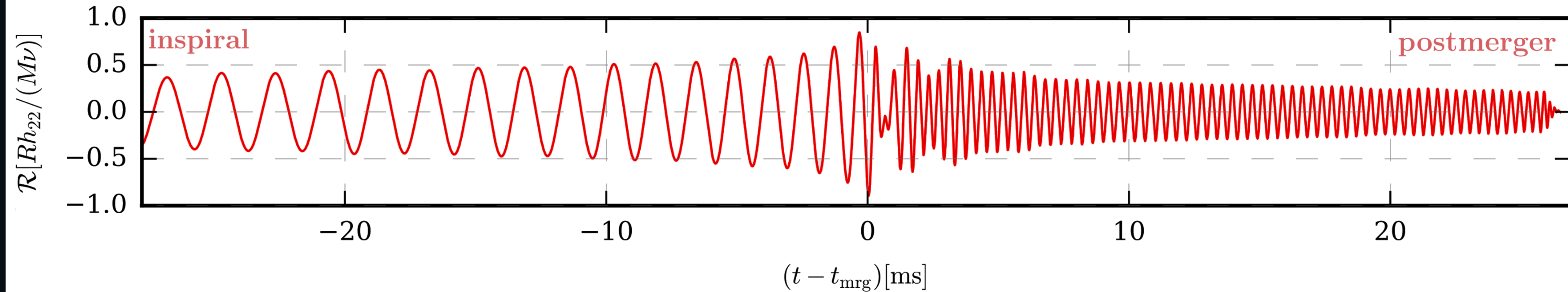


[Counsell, FG+, Phys. Rev. Lett. **135**, 081402 (2025)]

- **Challenge:** Develop models of this resonant behaviour

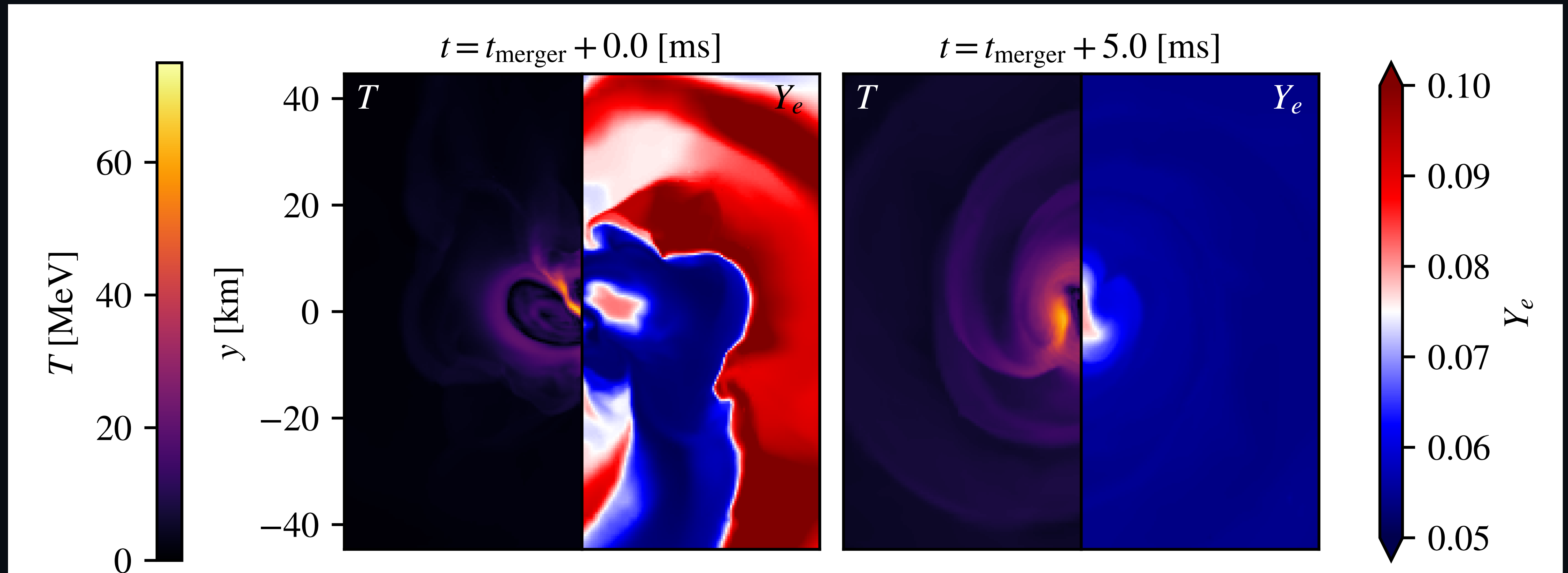
Role of Simulations

[see talks by *M. Miravet-Tenés* (Mon); *A. Harutyunyan*,
F. H. Nouri (Wed)]



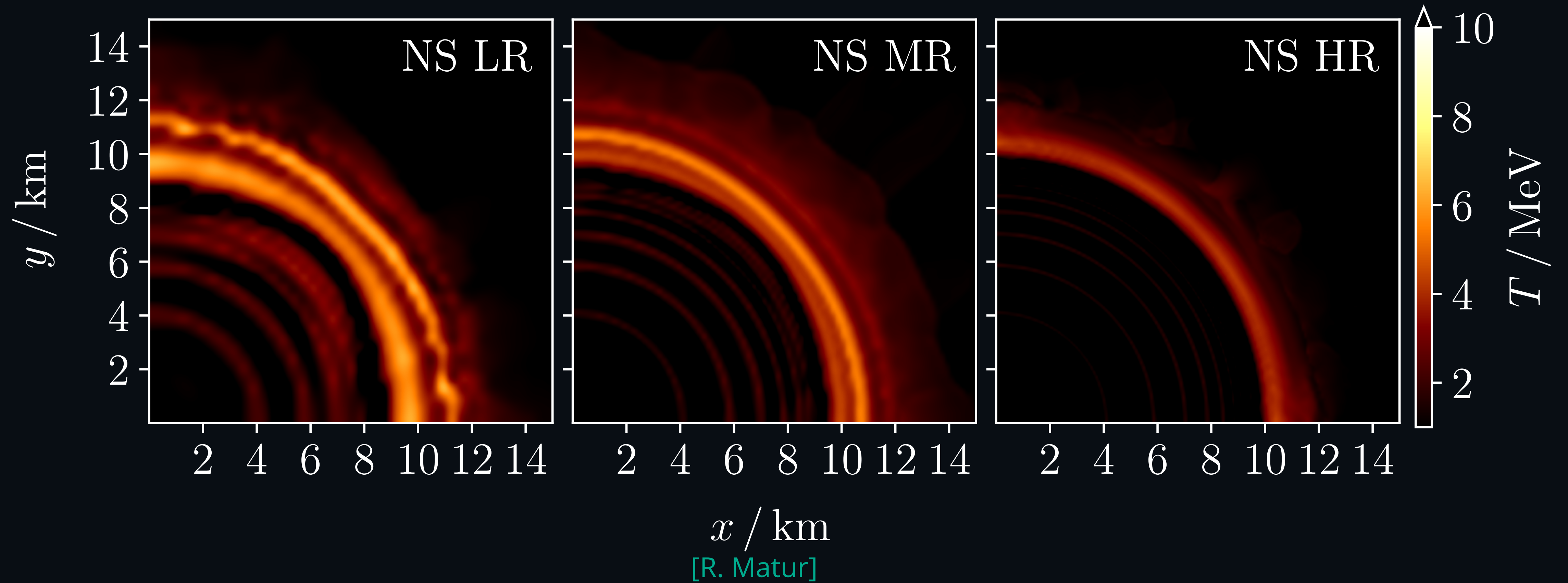
[Dietrich+, Gen. Relativ. Gravit. **53**, 27 (2021)]

Way too hot!

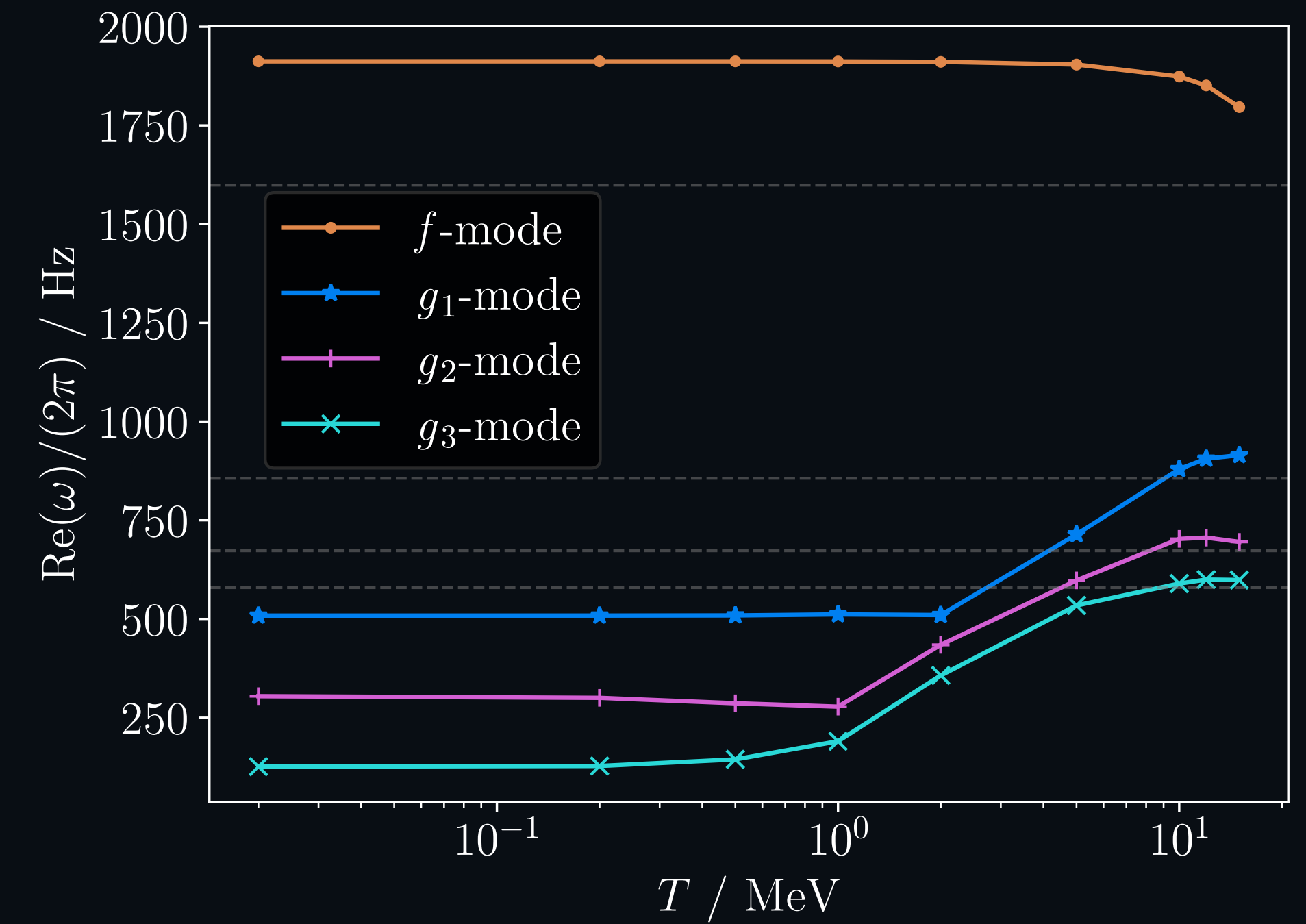
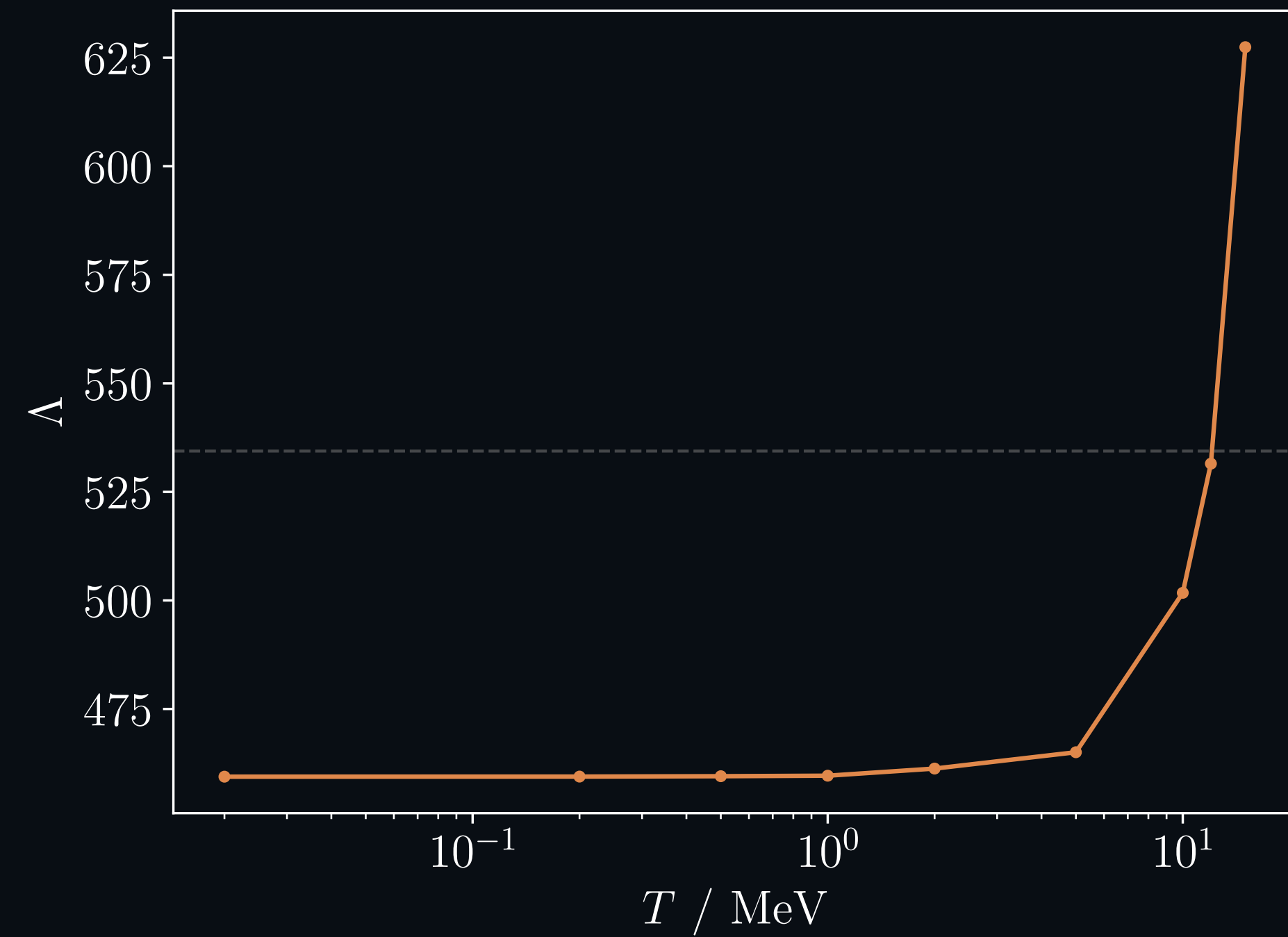


[Hammond+, Phys. Rev. D. **104**, 103006 (2021)]

Way too hot!



Tidal Systematics



[FG+, Phys. Rev. D. **104**, 103006 (2021)]

Conclusions

Opportunities	Challenges
Gravitational waves probe dense nuclear matter by encoding fine tidal deformations	Can the mode-sum be formulated in general relativity?
The tide presents the opportunity to conduct neutron-star seismology	Go beyond universal relations in inference
Oscillation modes grant access to rich physics: composition and phase transitions	Develop gravitational-waveform models of resonant oscillation modes