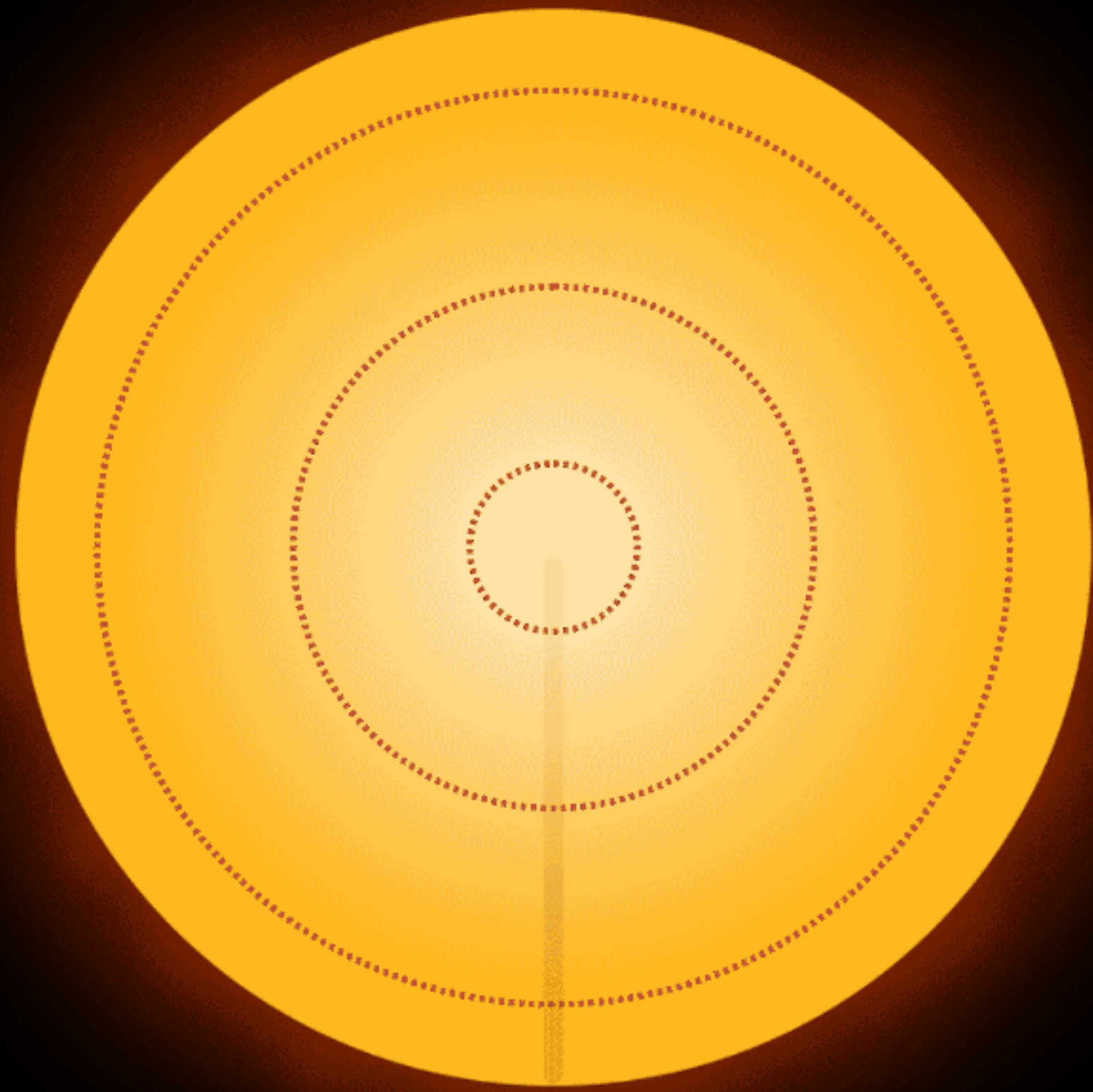


Gravitational-Wave Asteroseismology Illuminating Dense Nuclear Matter through Dynamical Tides



Fabian Gittins | INT-25-2b, Seattle | 17 Sep. 2025

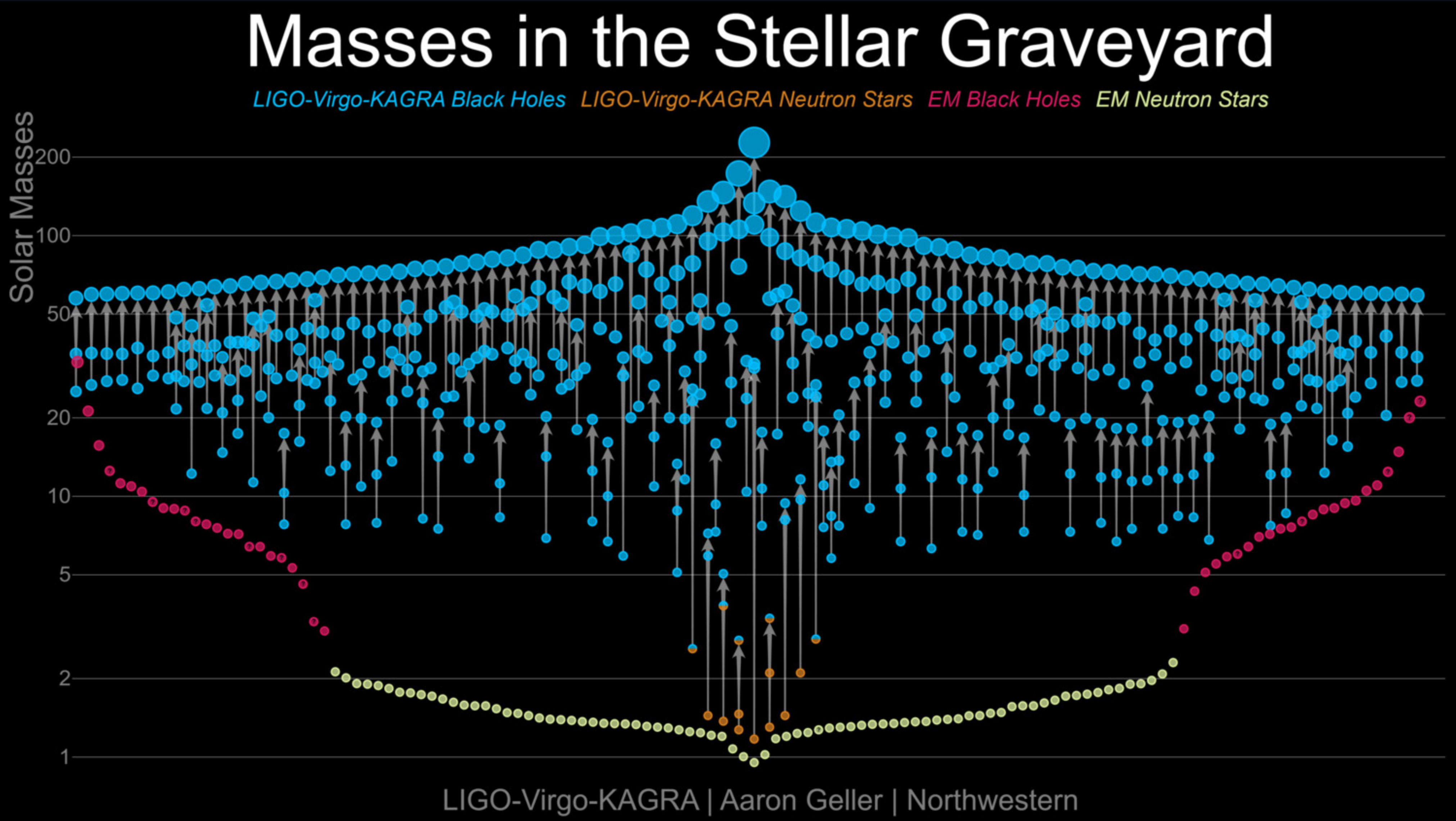


Utrecht
University

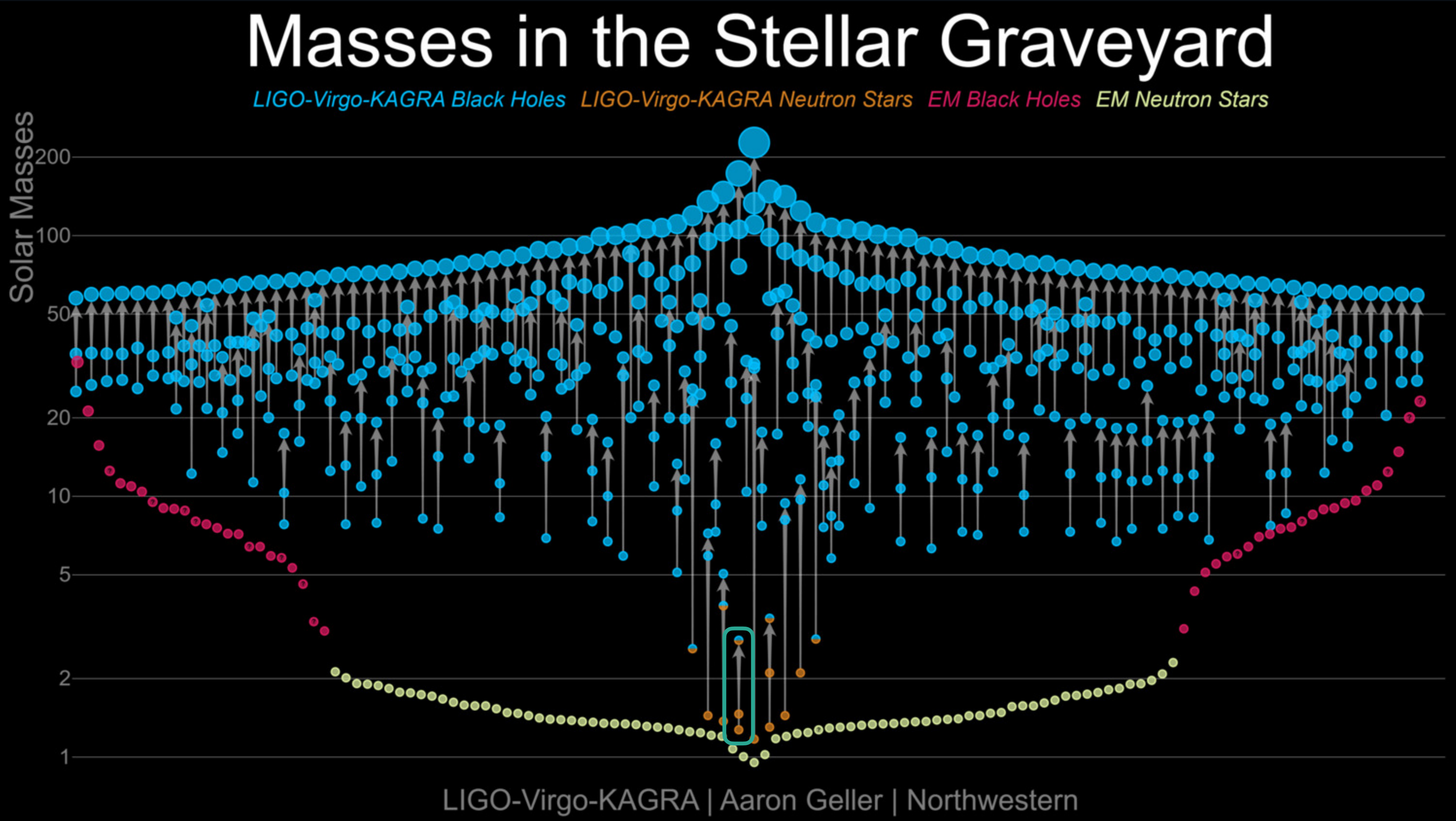


Funded by
the European Union

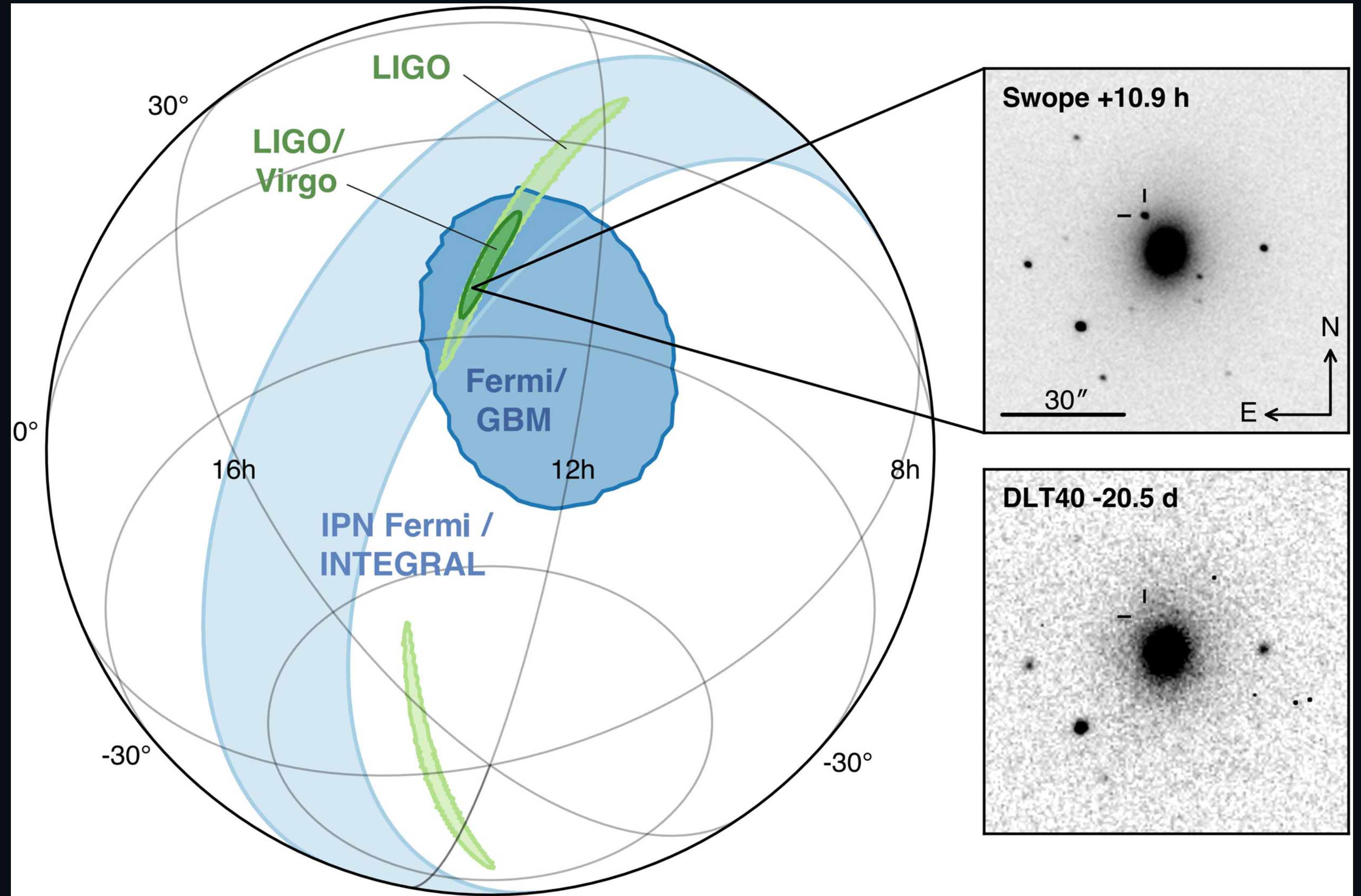
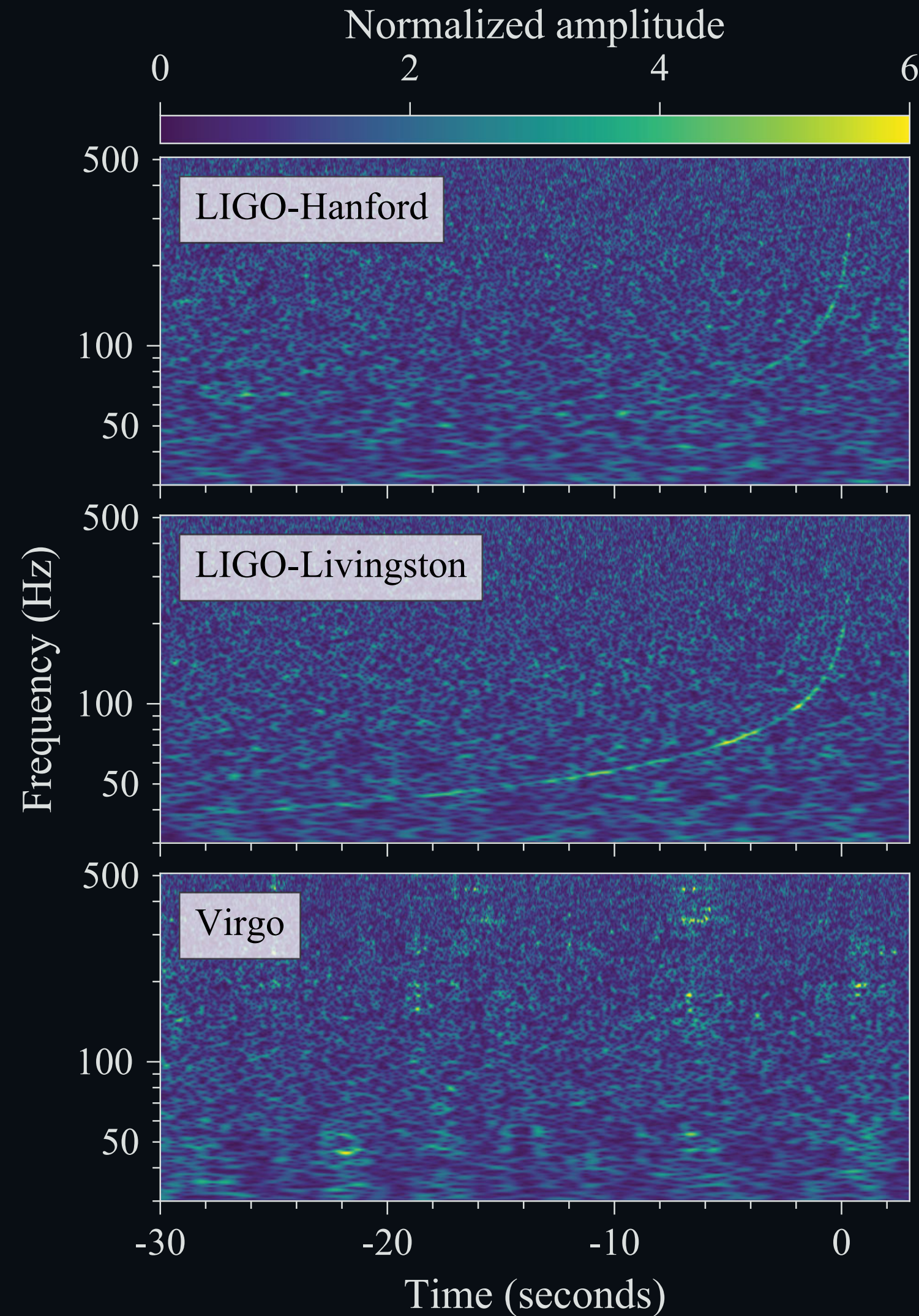
Gravitational Waves



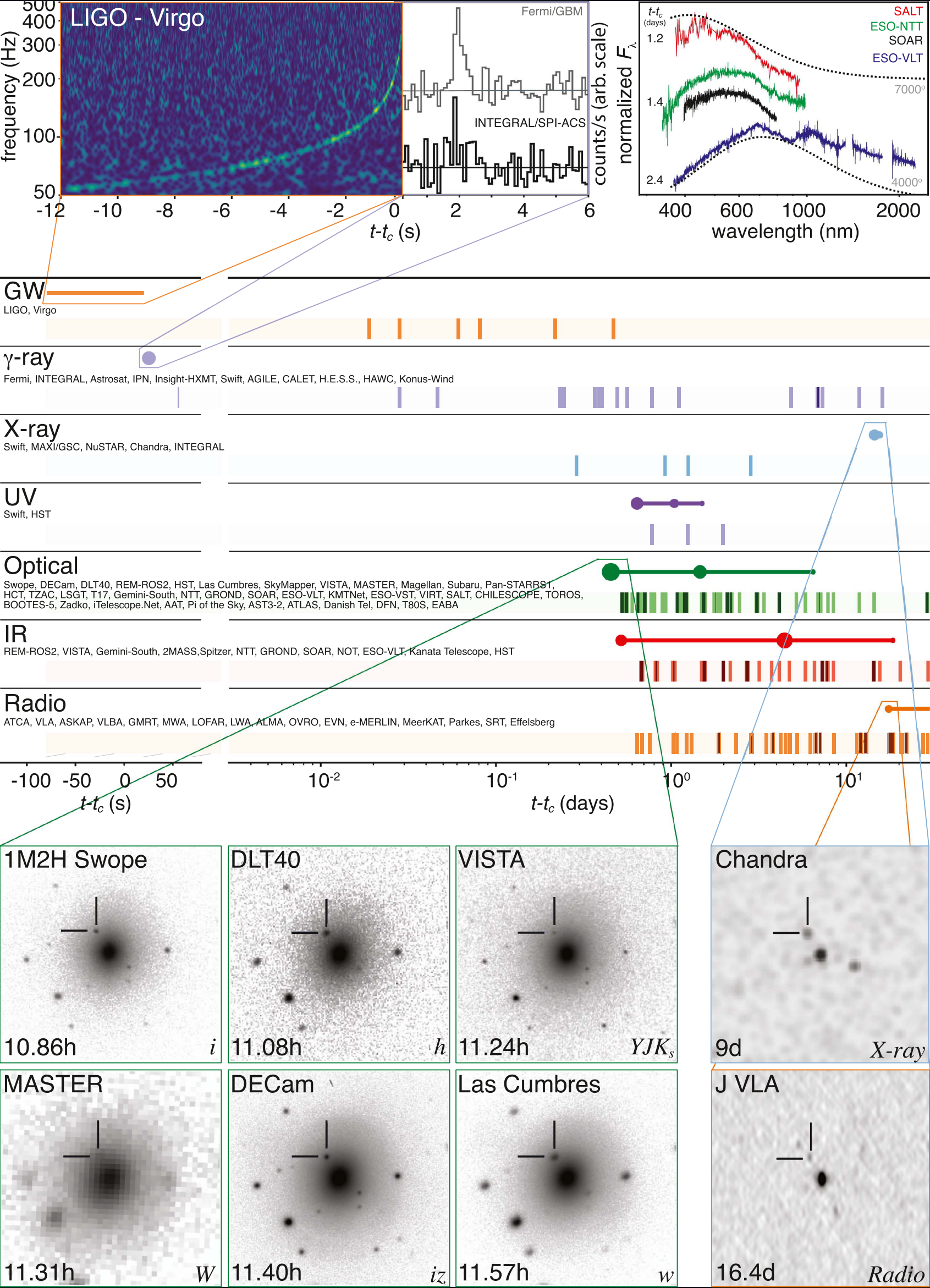
Gravitational Waves



First Binary Neutron–Star Merger



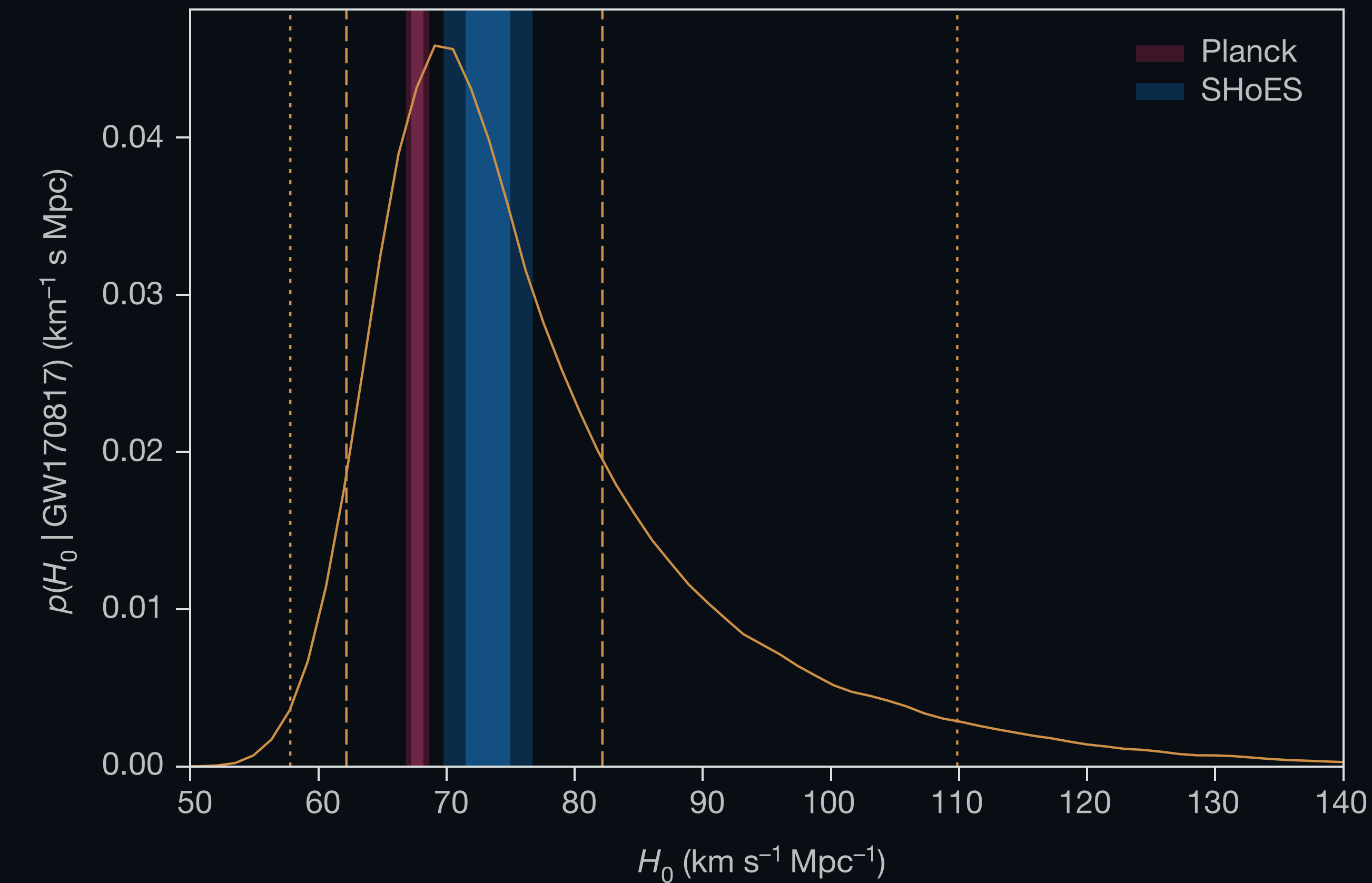
GW170817



1. Multi-messenger event

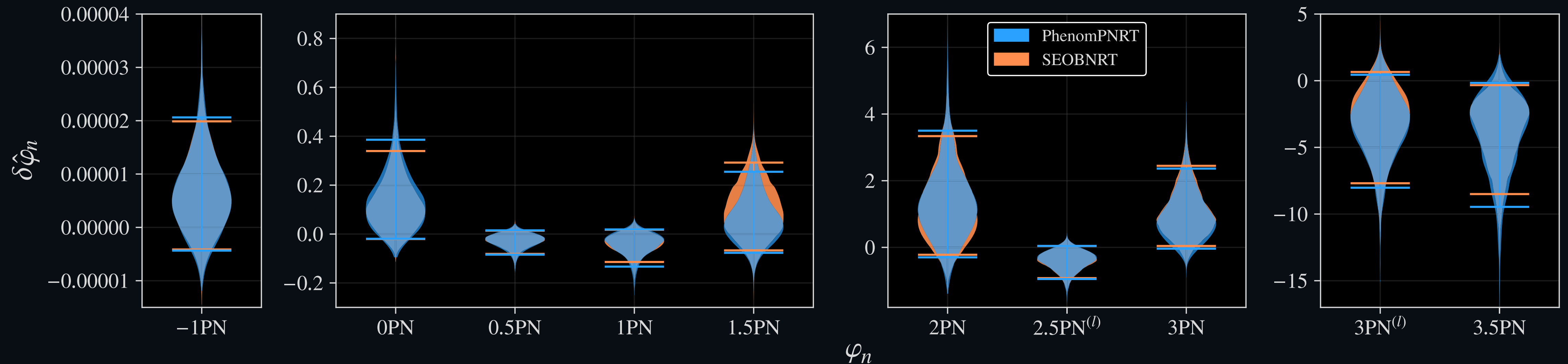
GW170817

1. Multi-messenger event
2. Cosmology



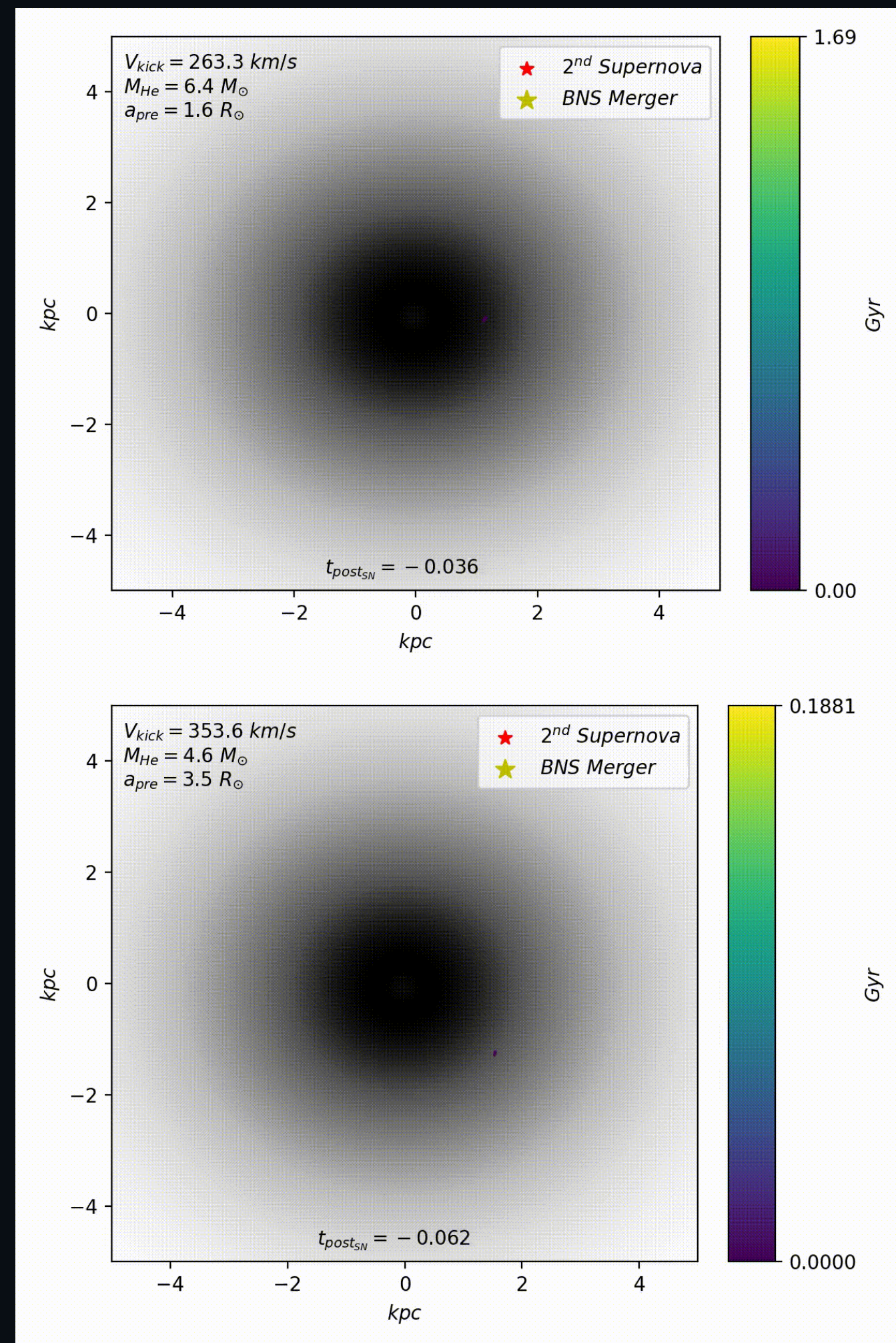
GW170817

1. Multi-messenger event
2. Cosmology
3. Tests of general relativity

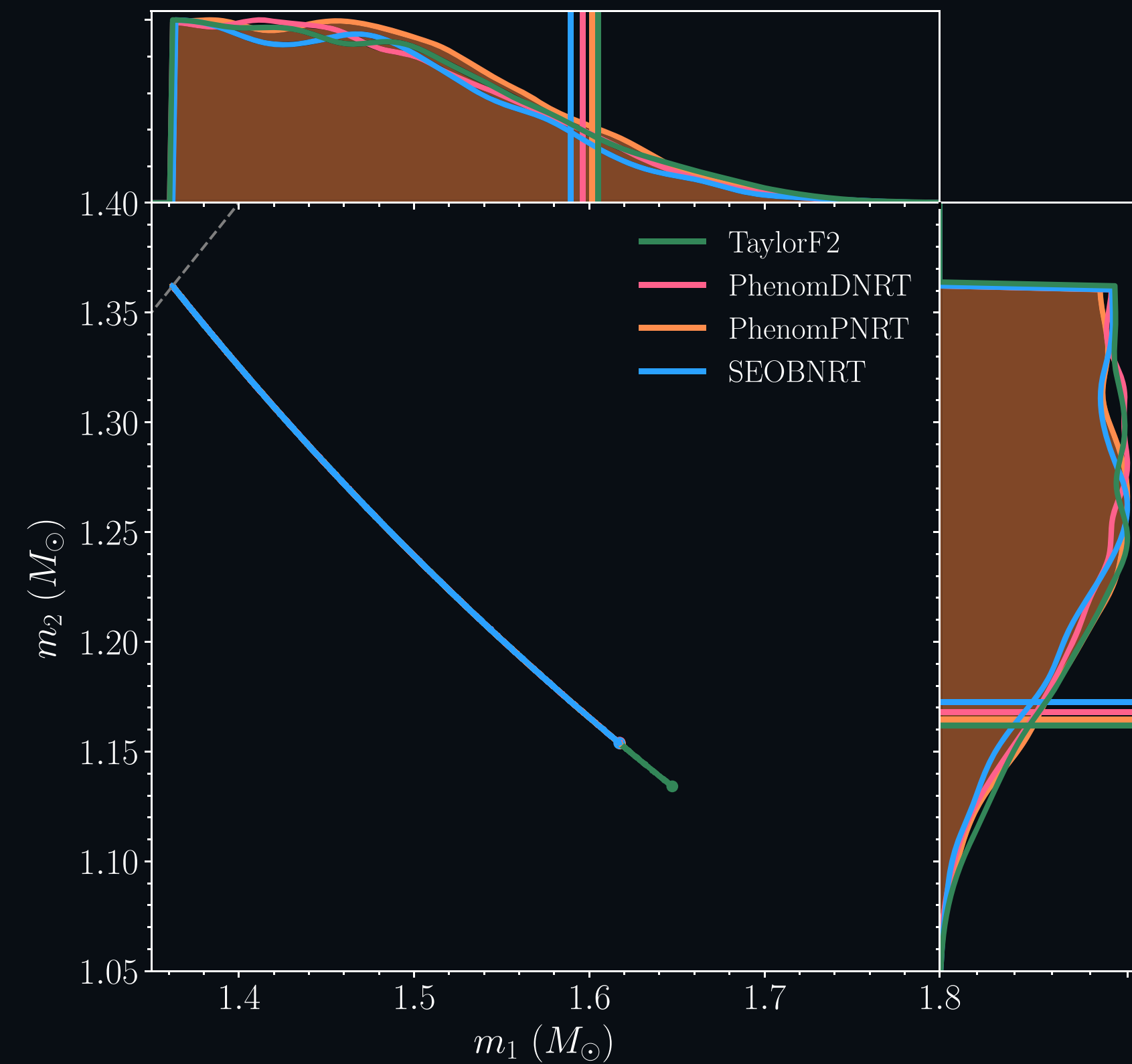


GW170817

1. Multi-messenger event
2. Cosmology
3. Tests of general relativity
4. Formation history



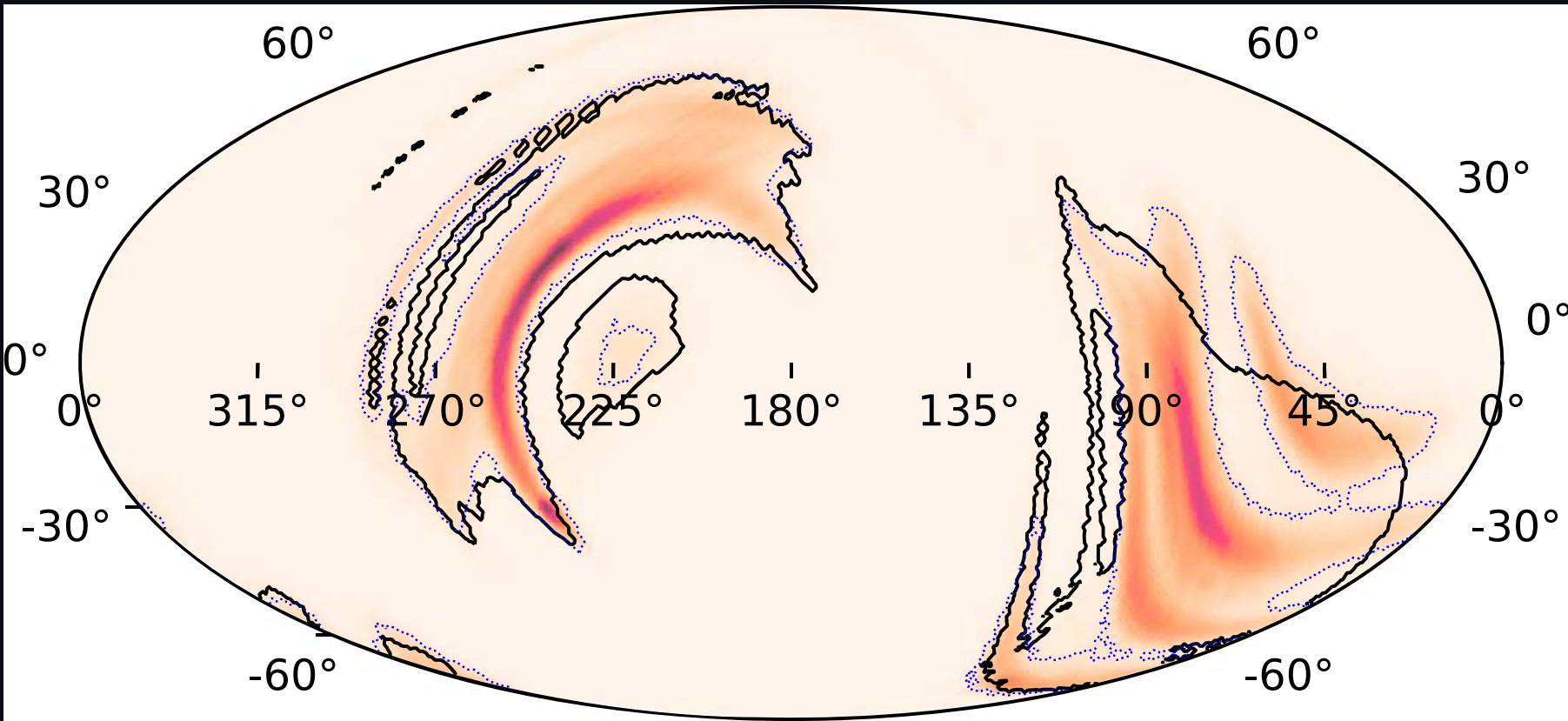
GW170817



1. Multi-messenger event
2. Cosmology
3. Tests of general relativity
4. Formation history
5. Source properties

More to Come...

- So far, 7 observed events with neutron stars, including GW190425
- (Rough) Timeline
 - 2028–31: O5 observing run with LIGO A+
 - 2030: LIGO-India joins the network
 - Early 2030s: LIGO A# upgrade
 - Mid 2030s: Cosmic Explorer and Einstein Telescope come online



[LIGO-Virgo Collaboration, *Astrophys. J.* **892**, L3 (2020)]

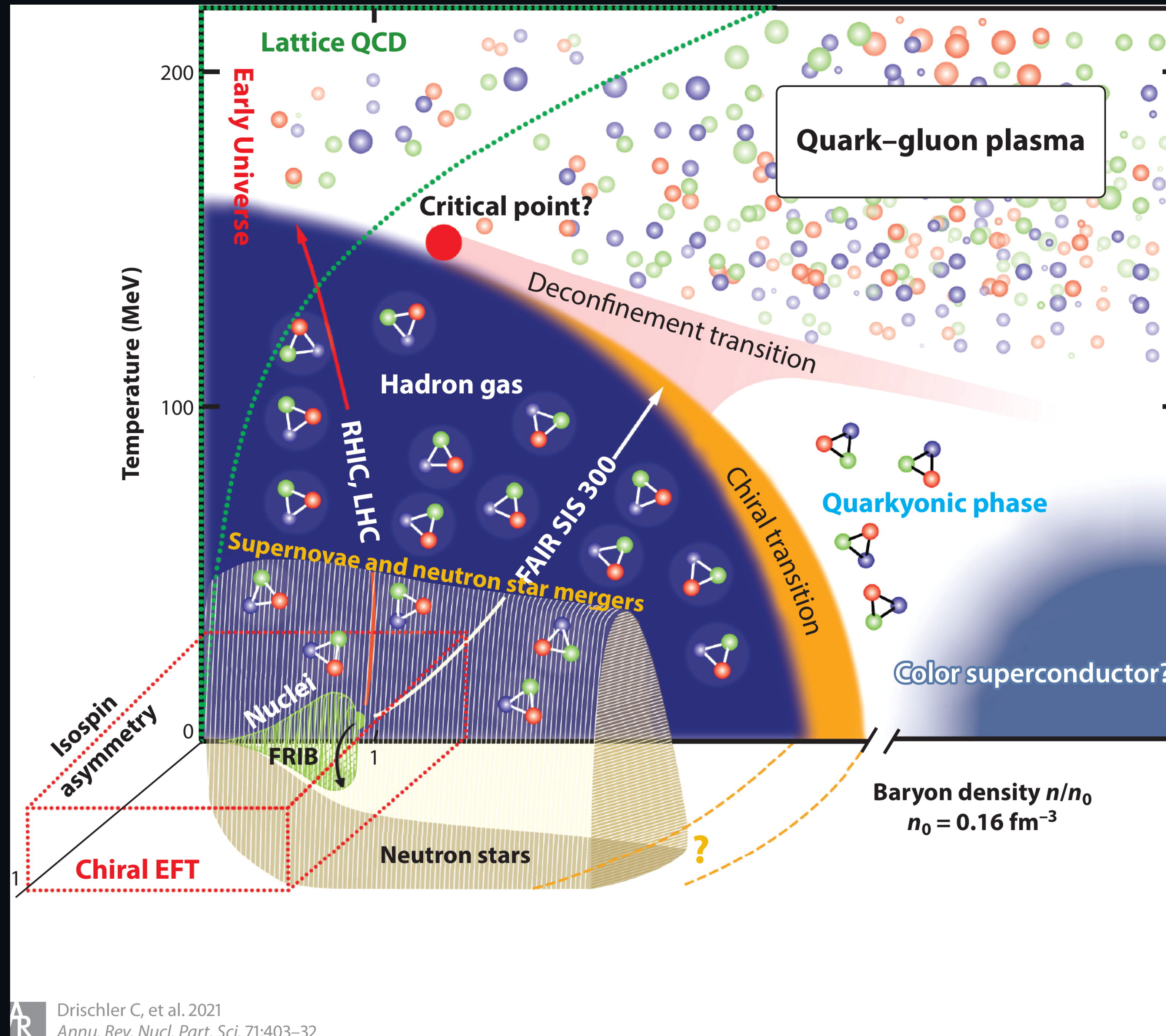
Network	$z(\rho_* = 10)$	$z(\rho_* = 100)$	$N(\rho > 10)$	$N(\rho > 30)$	$N(\rho > 100)$
BNS: cosmic merger rate is $1.2^{+2.0}_{-0.9} \times 10^6 \text{ yr}^{-1}$					
HLA	0.18	0.018	$1.3^{+1.9}_{-1.0} \times 10^3$	$2.7^{+6.6}_{-2.3} \times 10^1$	0
HLET	0.66	0.062	$8.5^{+13.0}_{-6.4} \times 10^4$	$2.5^{+3.9}_{-1.9} \times 10^3$	$4.8^{+7.4}_{-3.7} \times 10^1$
20LA	0.61	0.058	$7.1^{+11.0}_{-5.4} \times 10^4$	$2.1^{+3.1}_{-1.6} \times 10^3$	$3.9^{+6.7}_{-3.3} \times 10^1$
40LA	1.1	0.096	$2.7^{+4.1}_{-2.0} \times 10^5$	$1.1^{+1.7}_{-0.8} \times 10^4$	$2.2^{+3.3}_{-1.8} \times 10^2$
20LET	1	0.089	$1.9^{+2.9}_{-1.4} \times 10^5$	$5.9^{+9.0}_{-4.4} \times 10^3$	$1.2^{+1.9}_{-1.0} \times 10^2$
40LET	1.4	0.12	$3.9^{+5.9}_{-2.9} \times 10^5$	$1.7^{+2.6}_{-1.2} \times 10^4$	$3.5^{+5.5}_{-2.9} \times 10^2$
4020A	1.3	0.11	$3.6^{+5.5}_{-2.7} \times 10^5$	$1.7^{+2.6}_{-1.3} \times 10^4$	$3.5^{+5.6}_{-2.9} \times 10^2$
4020ET	1.7	0.13	$4.7^{+7.2}_{-3.5} \times 10^5$	$2.3^{+3.6}_{-1.8} \times 10^4$	$4.8^{+7.7}_{-3.9} \times 10^2$

[Gupta+, *Class. Quantum Gravity* **41**, 245001 (2024)]

Overview

- Gravitational waves probe *fundamental physics* through observations of neutron stars
- Tidal dynamics present opportunity to conduct asteroseismology
- Neutron-star oscillation modes are notably rich
- Opportunities and challenges ahead

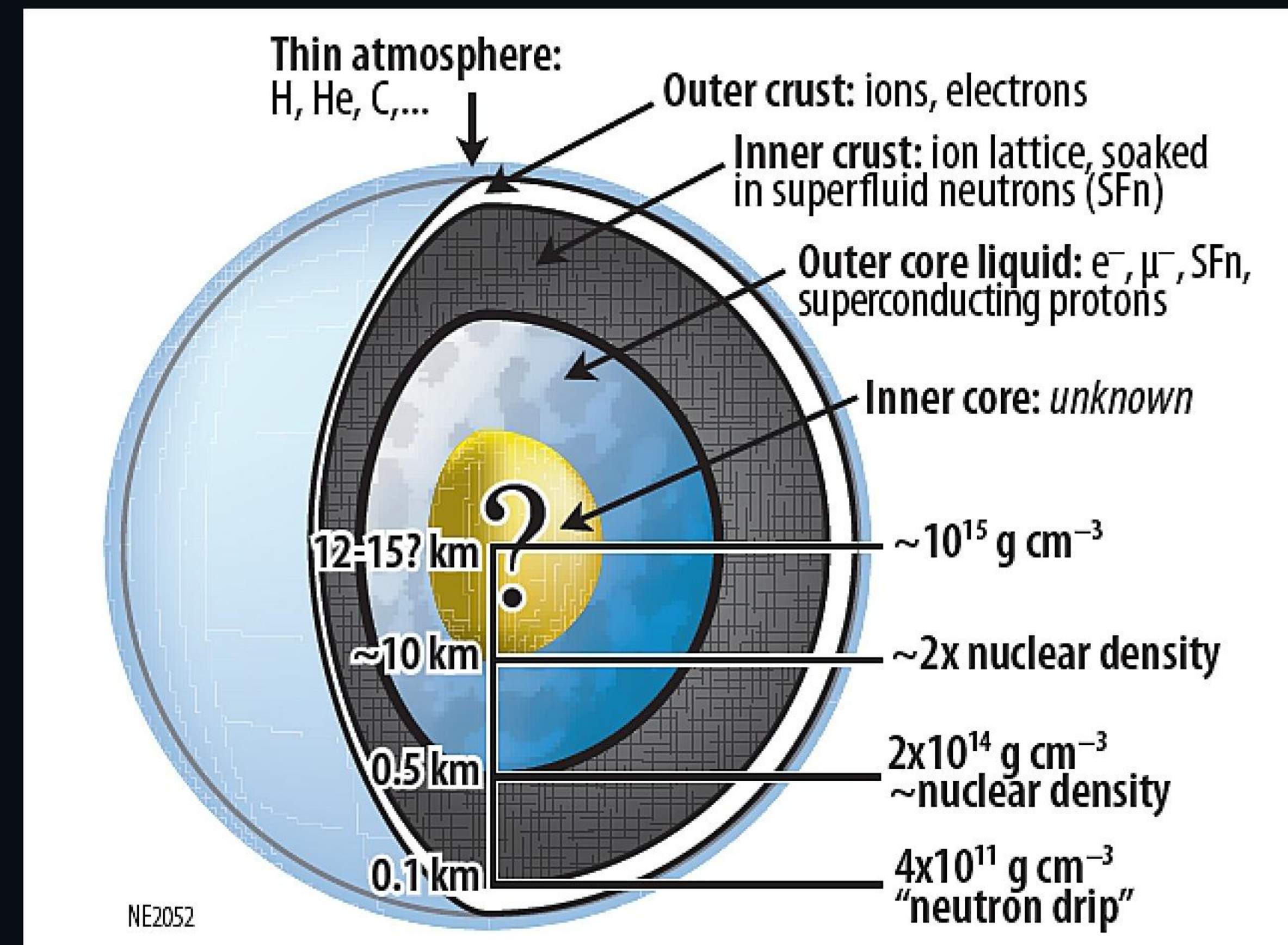
Quantum Chromodynamics (QCD)



Physics of Neutron Stars

- Neutron stars are extreme laboratories.

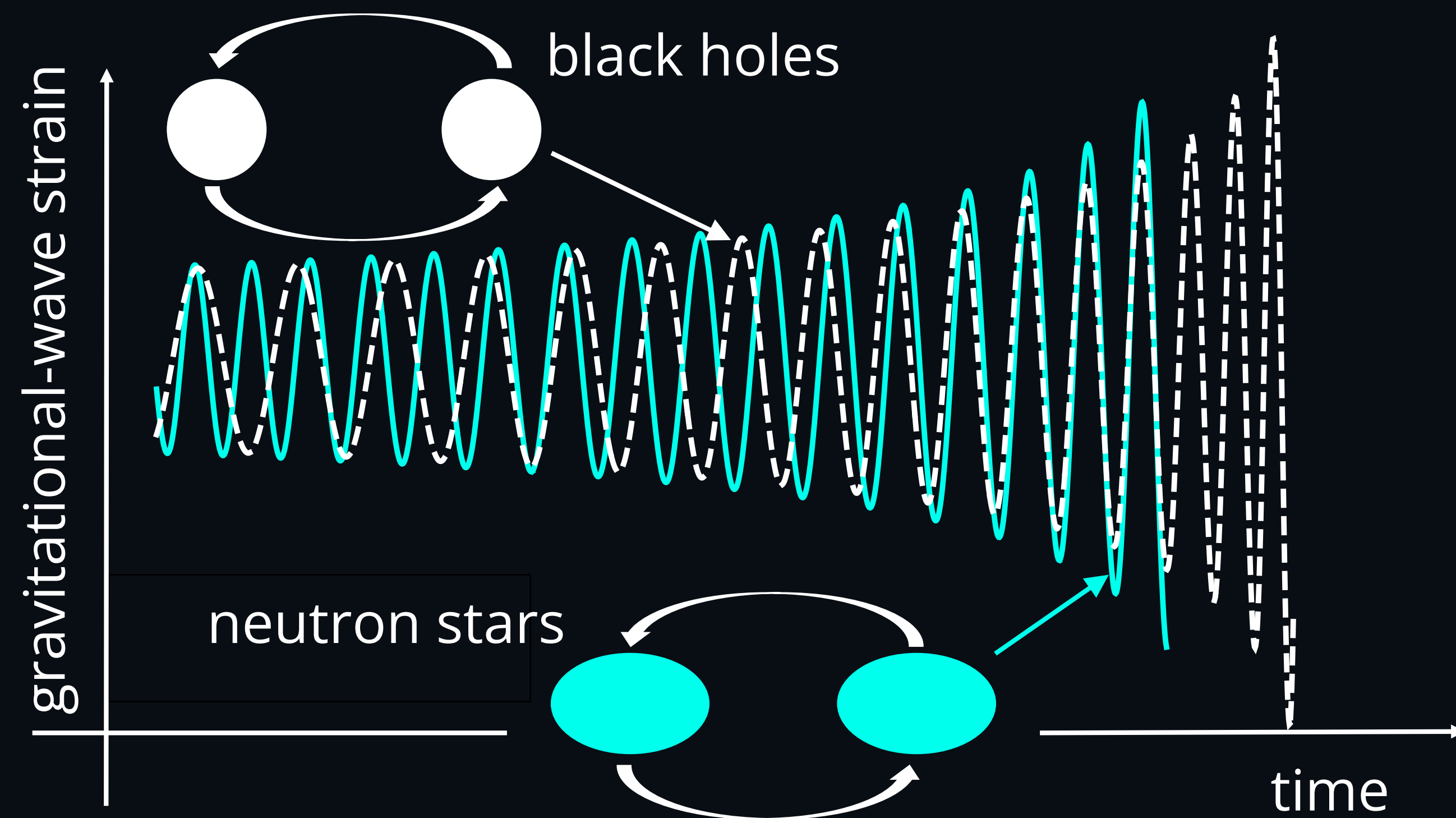
- Strong-field gravity
- Dense nuclear matter
- Rapid rotation
- Strong magnetic fields
- Superfluidity
- Solid crusts



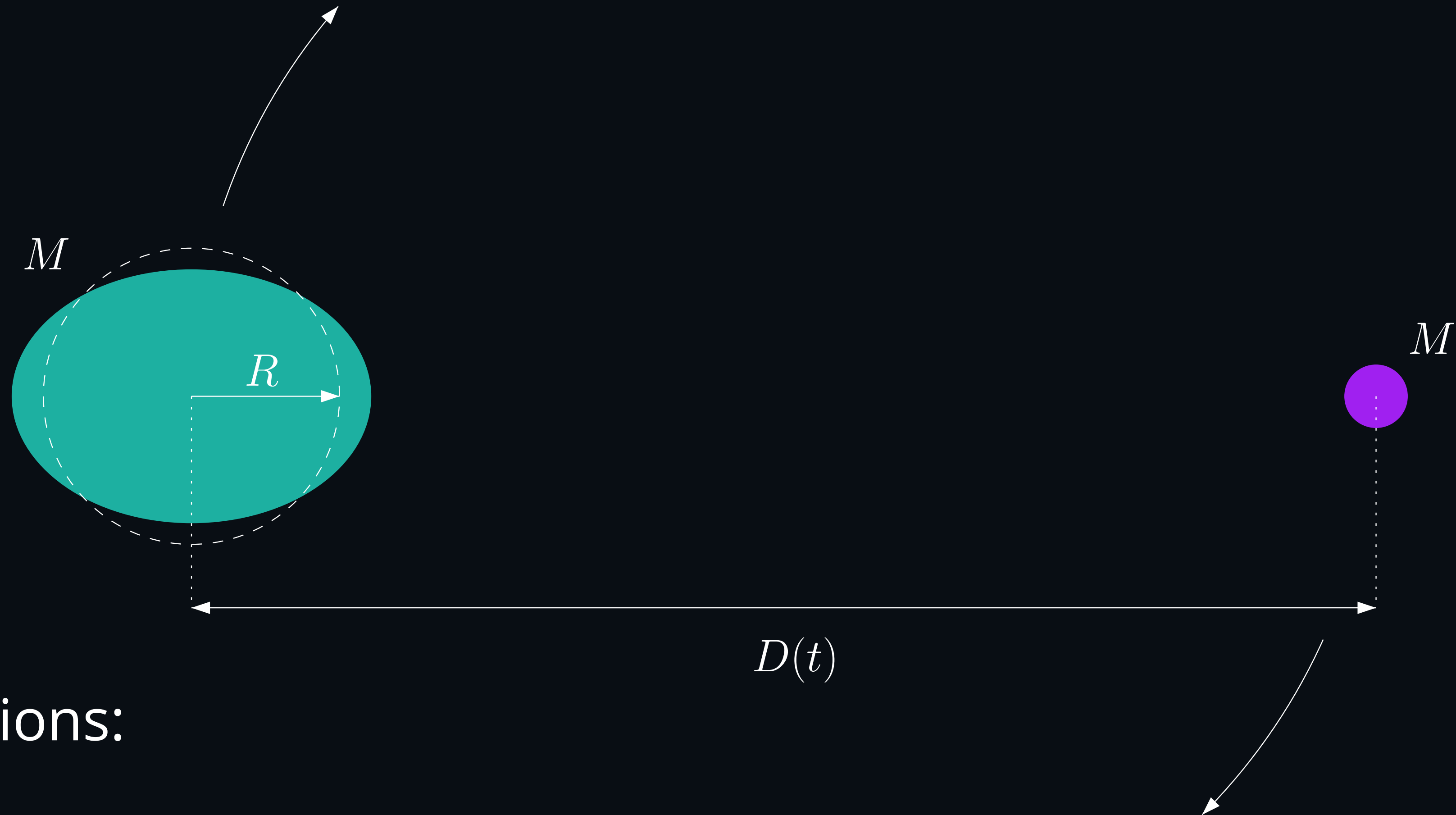
- Each of these aspects give rise to their own family of **oscillation modes**.

Matter Effects

- Consider two compact binaries: one with **black holes**, while the other comprises **neutron stars**
- The binaries are otherwise identical; same component *masses*, *spins*, *binary orientation* and *position* with respect to the detectors

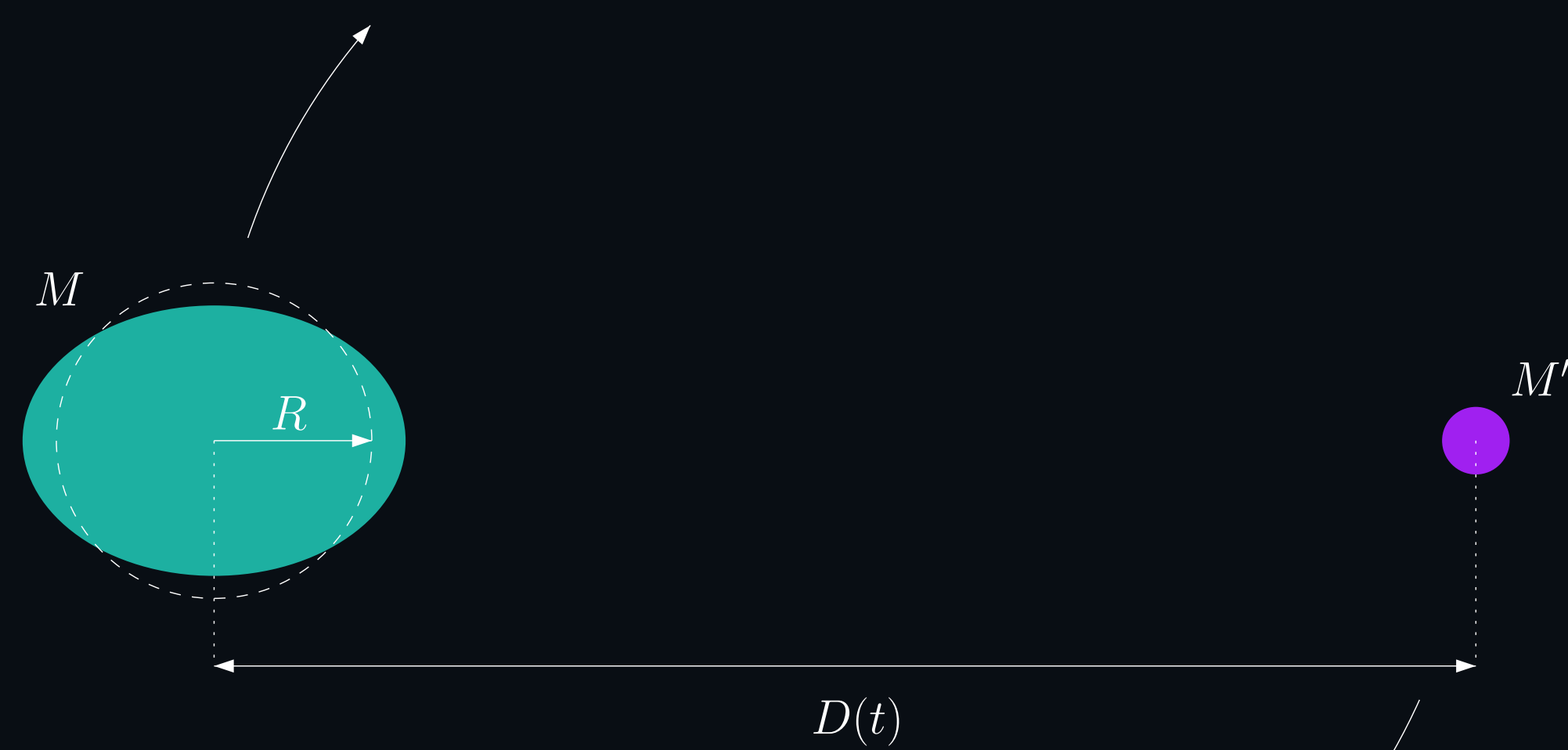


Static Tide



- Assumptions:
 - Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$
 - Slowly varying external field, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Static Tide



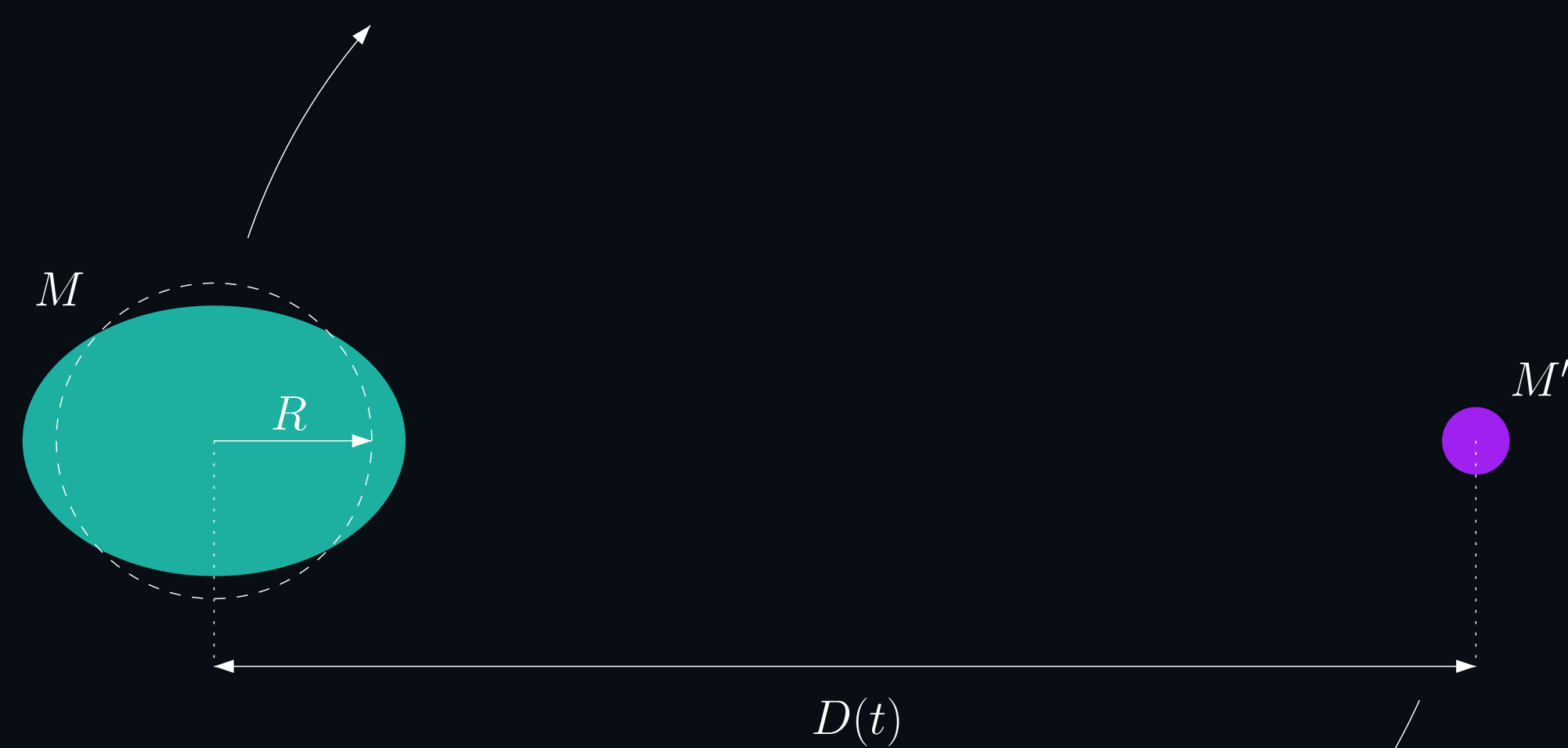
- Shape is quantified by the star's *tidal Love numbers* k_l , which are extracted from the exterior potential (metric),

$$U_l(r) \equiv \delta\Phi_l(r) + \chi_l(r) = \left[2k_l \left(\frac{R}{r} \right)^{2l+1} + 1 \right] \left(\frac{r}{R} \right)^l \chi_l(R),$$

where the potential satisfies

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dU_l}{dr} \right) - \frac{l(l+1)}{r^2} U_l = - \frac{4\pi G \rho}{dp/d\rho} U_l$$

Static Tide



- At lowest order, the tide enters the gravitational-wave phase as
[Flanagan+Hinderer, Phys. Rev. D **77**, 021502 (2008)]

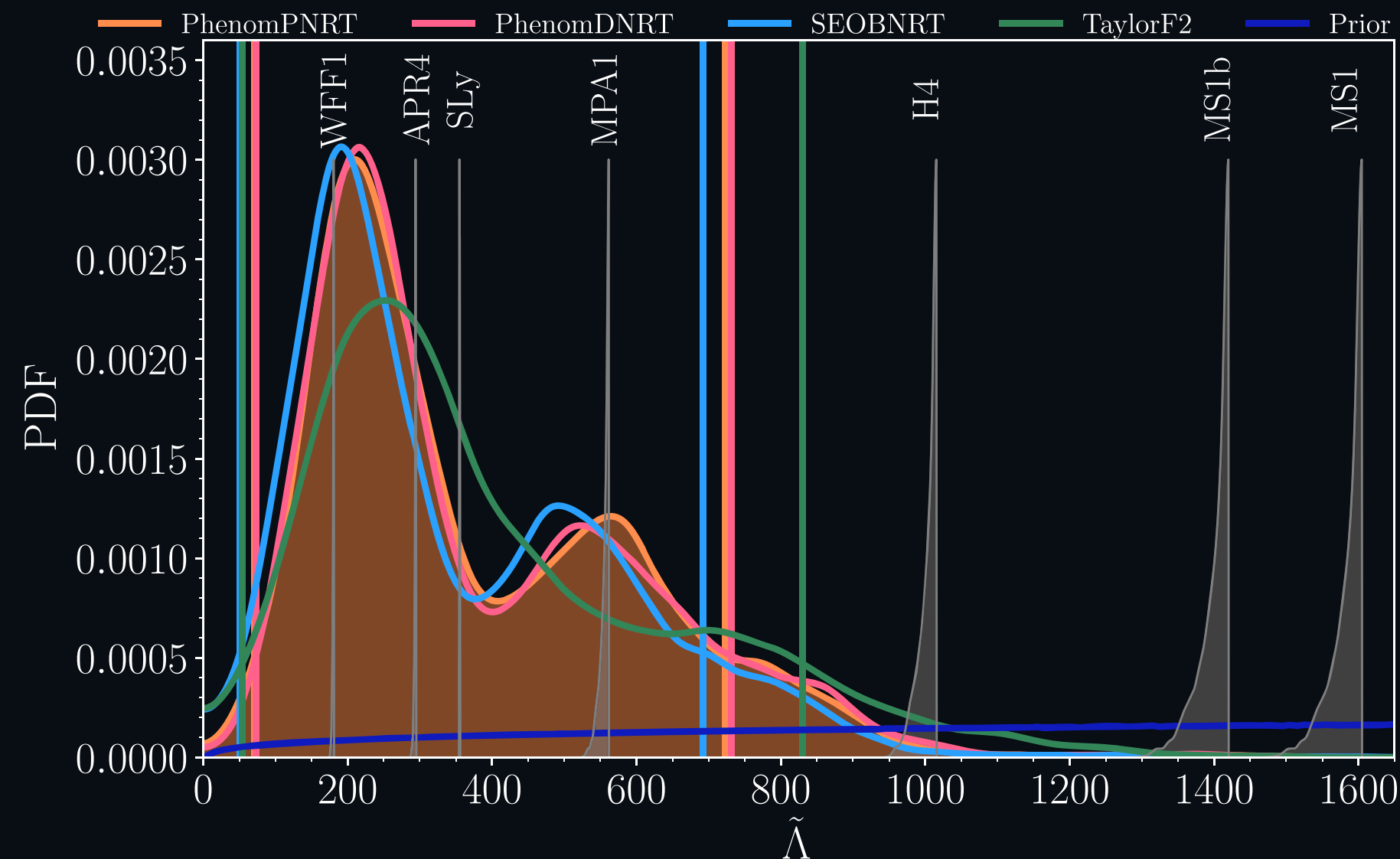
$$\delta\Psi(\nu) = -\frac{3}{128} \frac{M_{\text{total}}}{\mu} \frac{1}{\nu^5} \cdot \frac{39}{2} \tilde{\Lambda} \nu^{10},$$

where

$$\tilde{\Lambda} = \frac{16}{13} \frac{1}{M_{\text{total}}} \left[(M + 12M') M^4 \Lambda + (M' + 12M) M'^4 \Lambda' \right], \quad \Lambda = \frac{2}{3} \left(\frac{c^2 R}{GM} \right)^5 k_2$$

Matter Constraints

GW170817

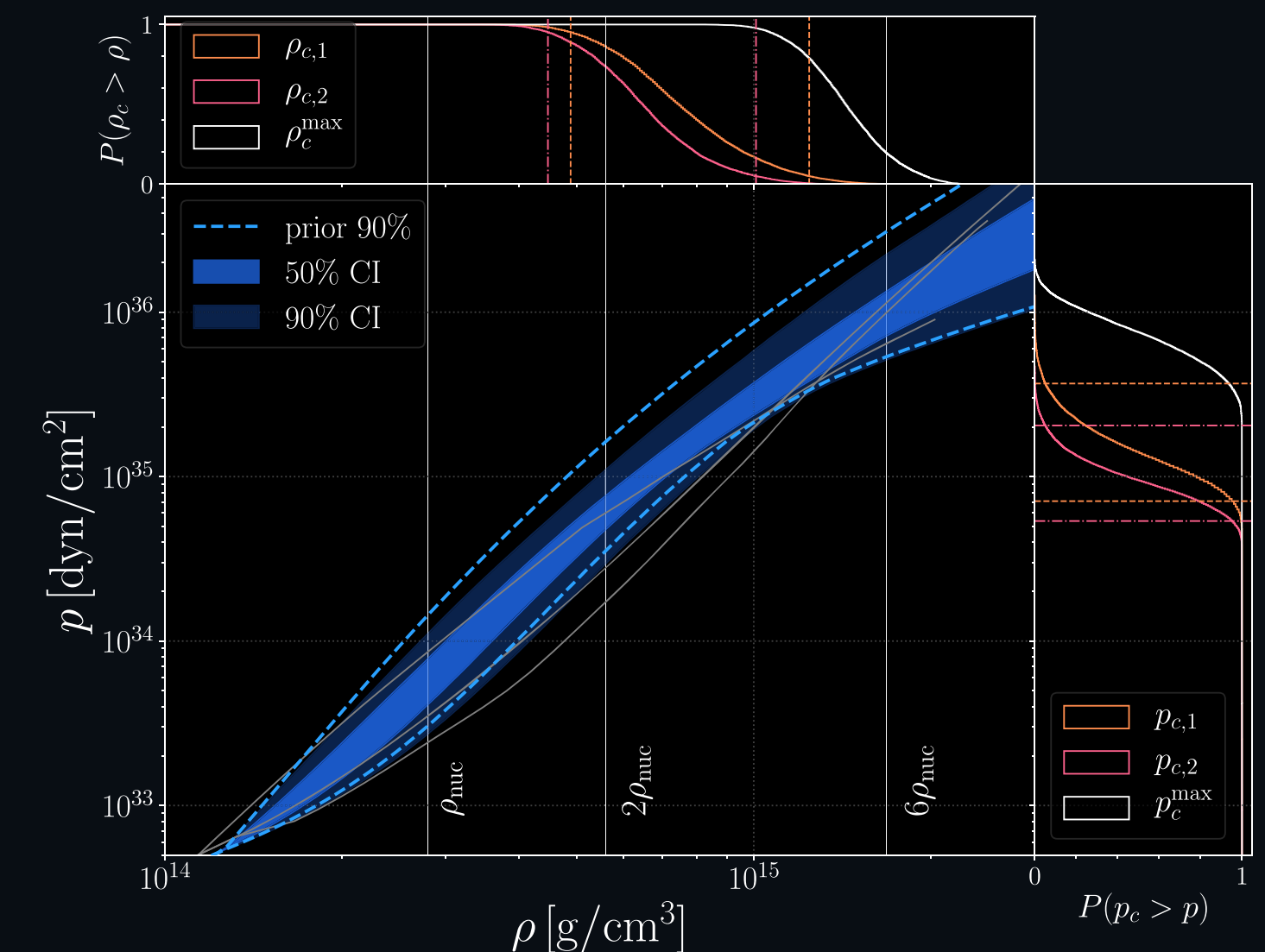
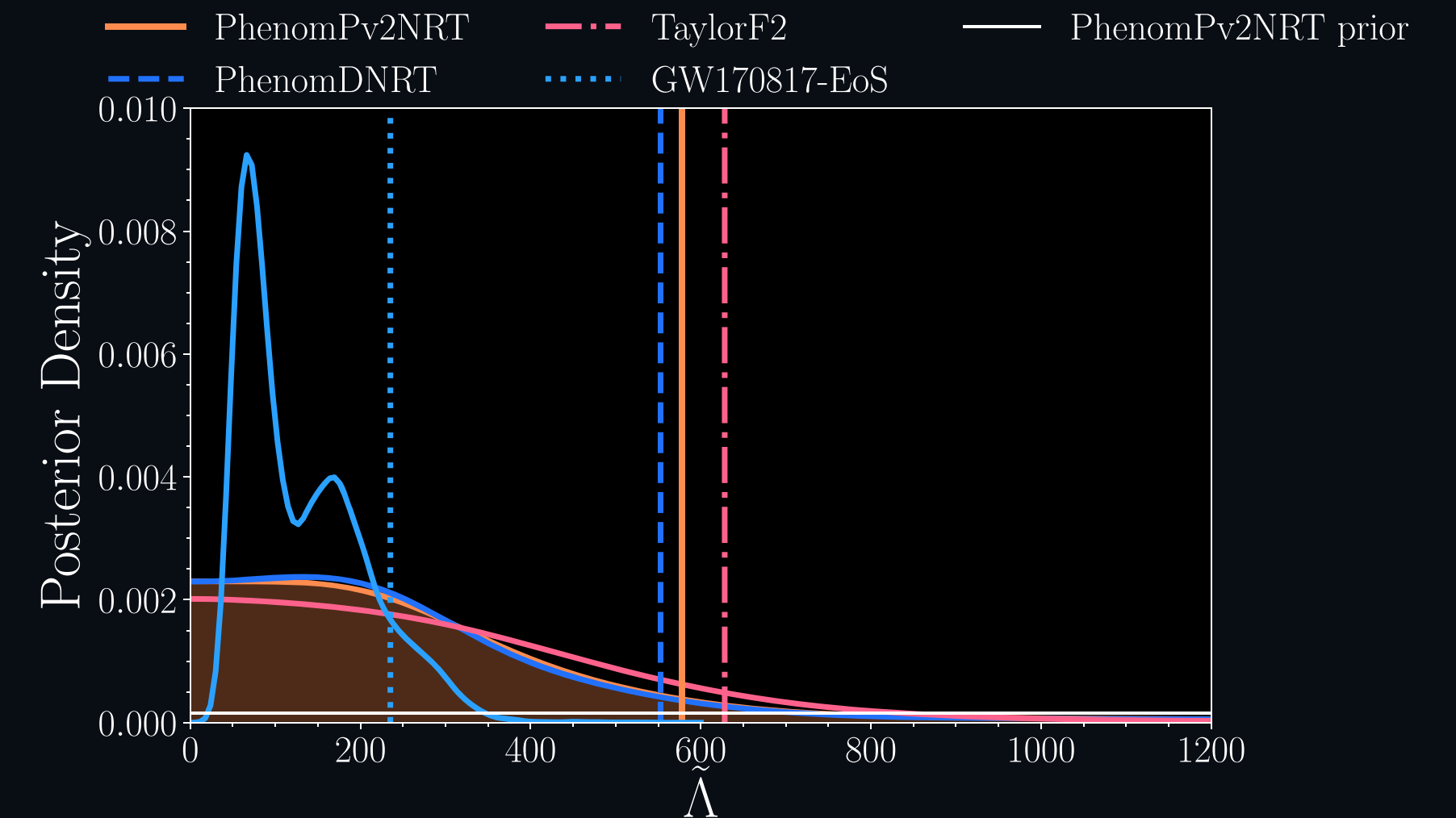


[LIGO-Virgo Collaboration, Phys. Rev. X **9**, 011001 (2019)]

➔ Provided $\rho = \rho(p)$, one can solve for M and Λ
[see Khadkikar's talk yesterday afternoon]

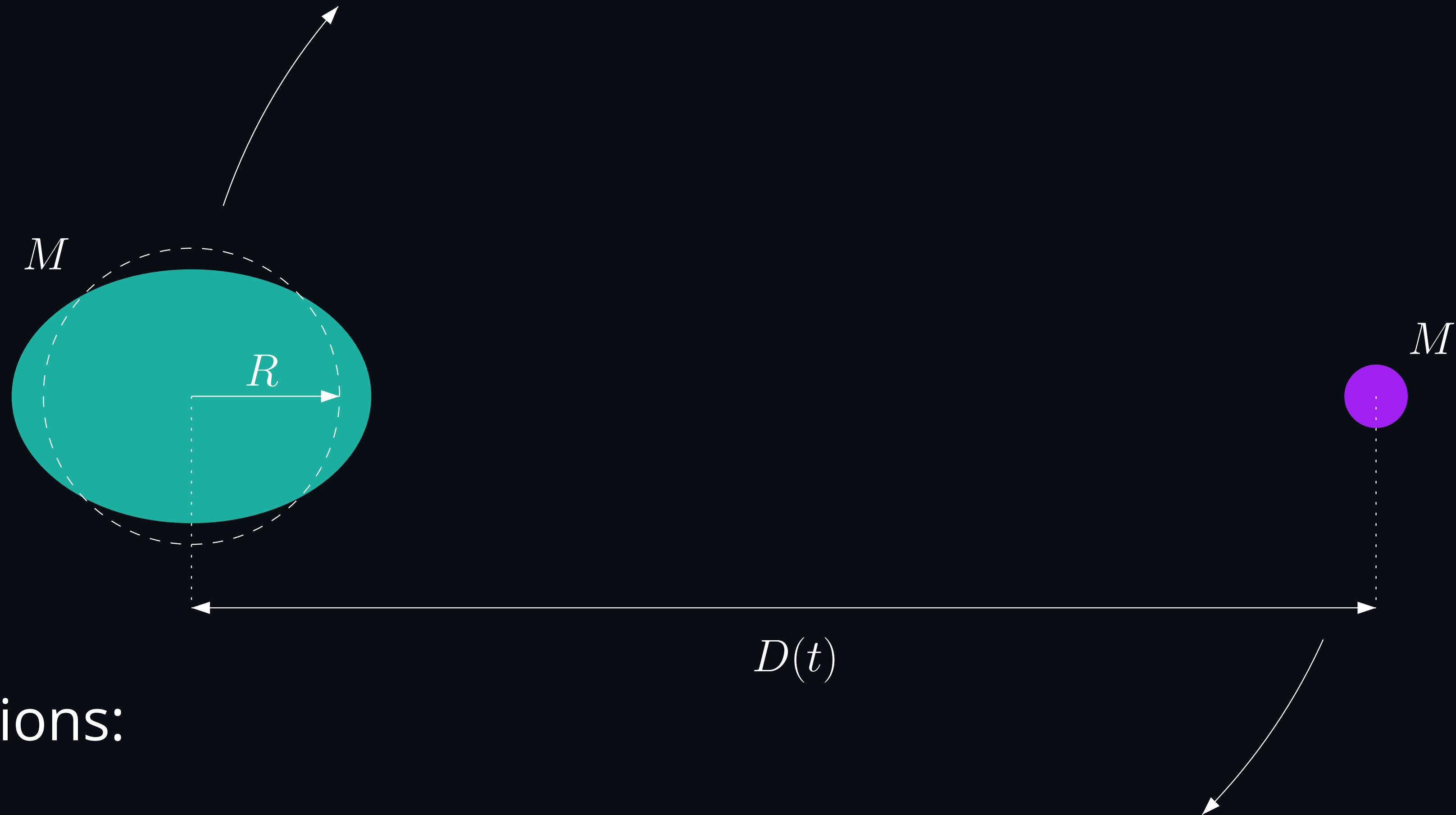
- However, there is more physics in the tide...

GW190425



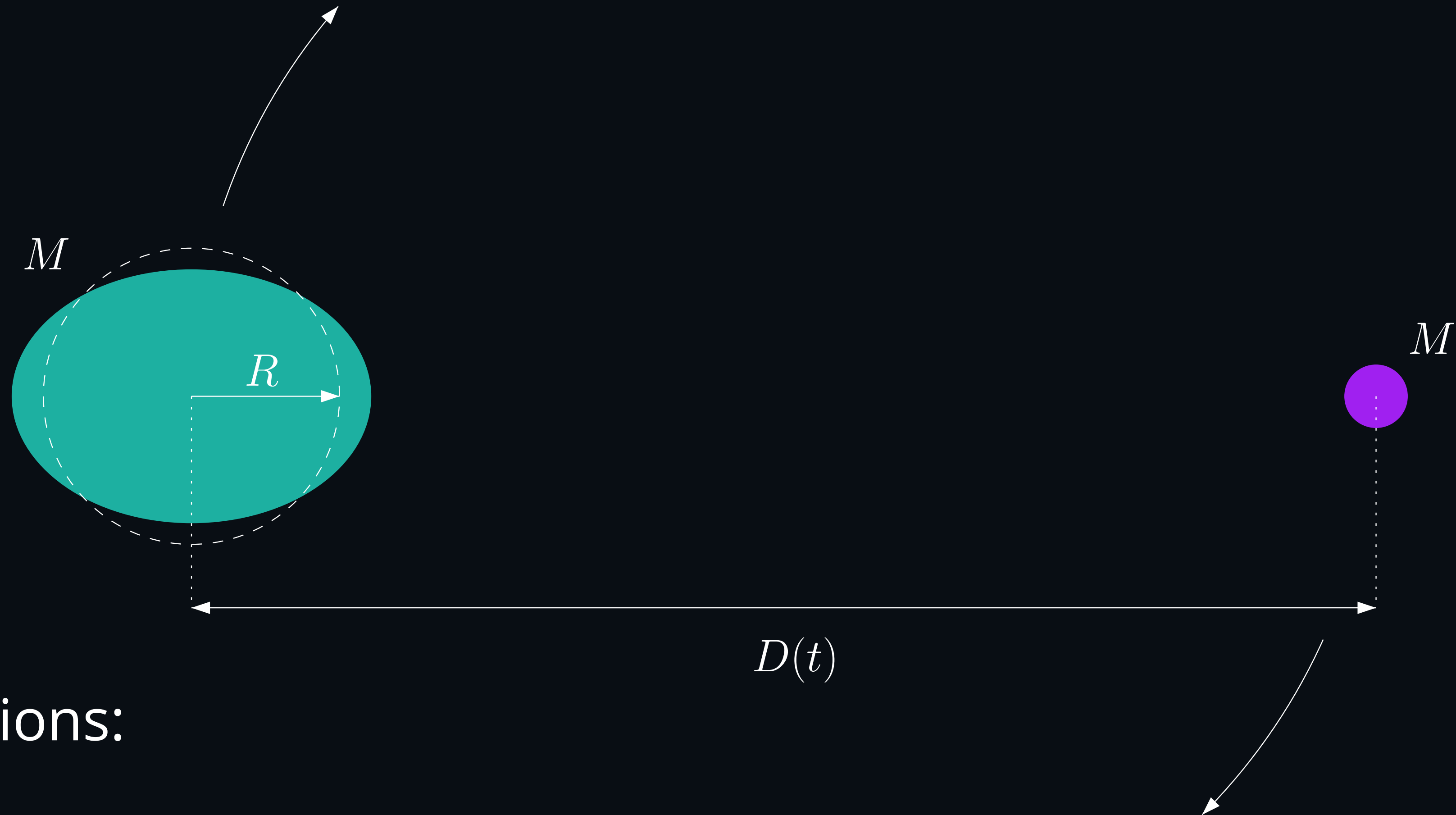
[LIGO-Virgo Collaboration, Astrophys. J. **892**, L3 (2020)]

Static Tide



- Assumptions:
 - Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$
 - Slowly varying external field, $\lambda = \dot{\Phi}/\omega_\alpha \ll 1$

Dynamical Tide

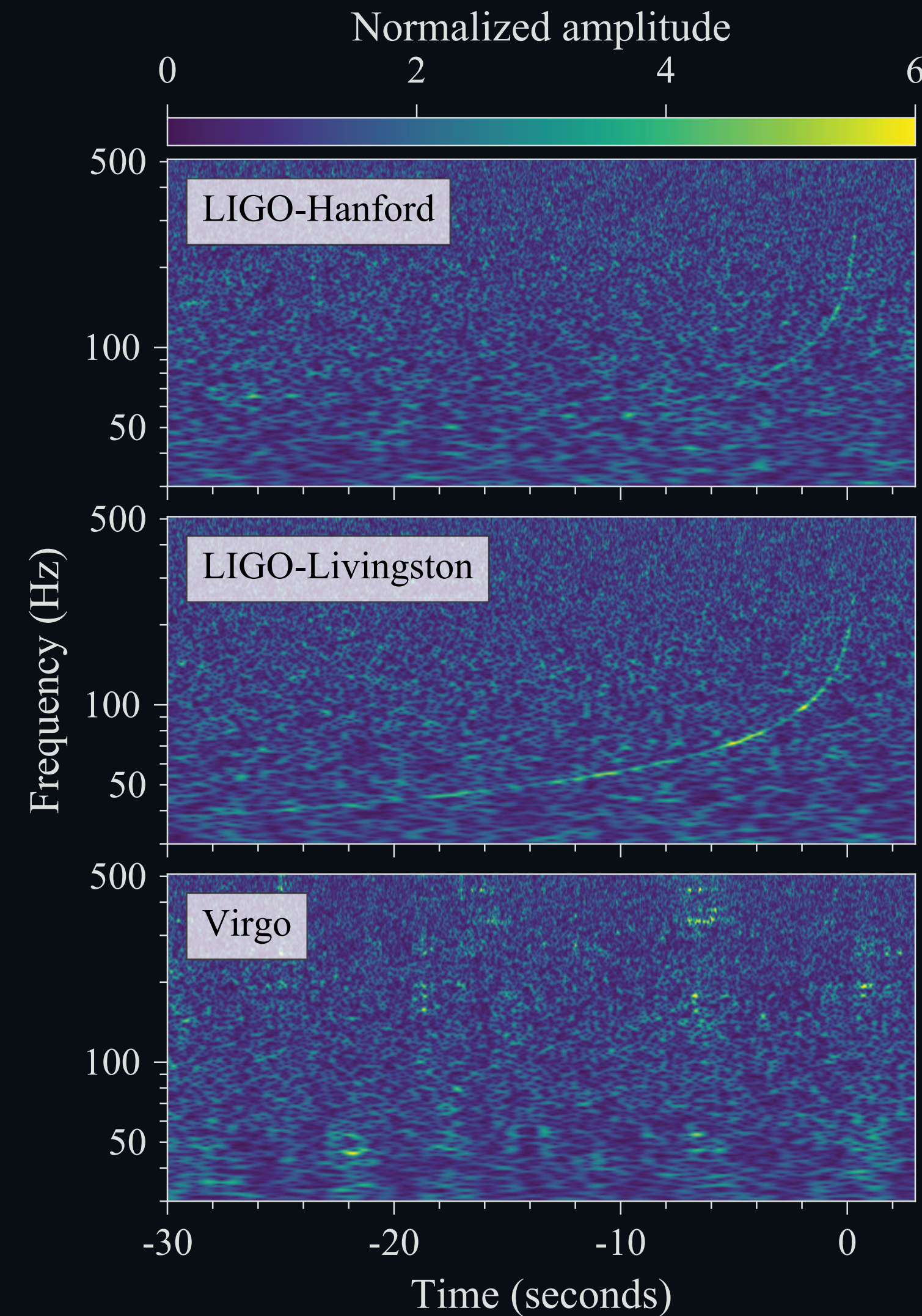


- Assumptions:

- Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$

- ~~Slowly varying external field, $\lambda \equiv \dot{\Phi}/\omega_\alpha \ll 1$~~

Inspiral



- The static approximation inevitably breaks down during the inspiral,

$$\lambda \sim O(1)$$

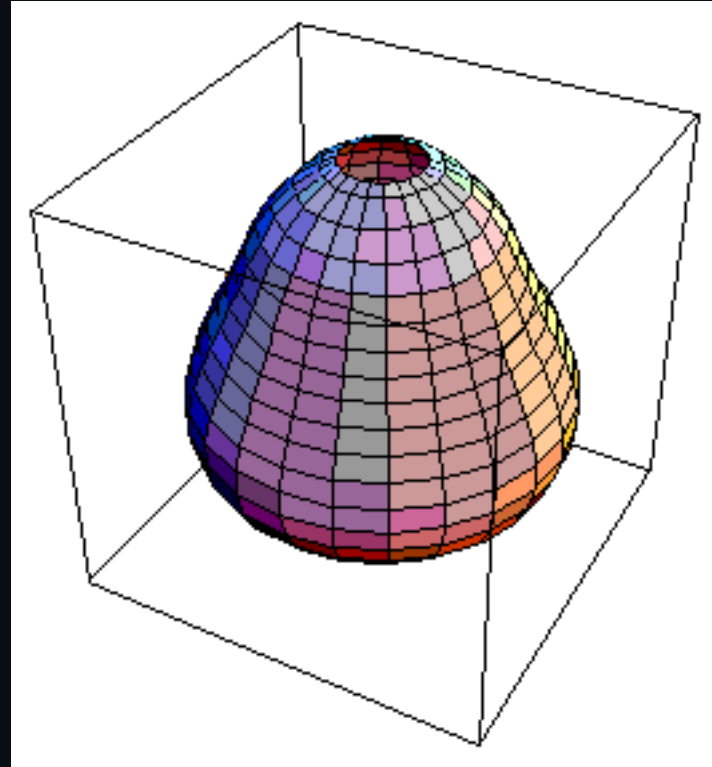
- The frequency ω_α represents a characteristic mode frequency,

$$\omega_f \sim \sqrt{\frac{GM}{R^3}} \approx 2\pi \cdot 2.2 \text{ kHz} \left(\frac{M}{1.4M_\odot} \right)^{1/2} \left(\frac{10 \text{ km}}{R} \right)^{3/2}$$

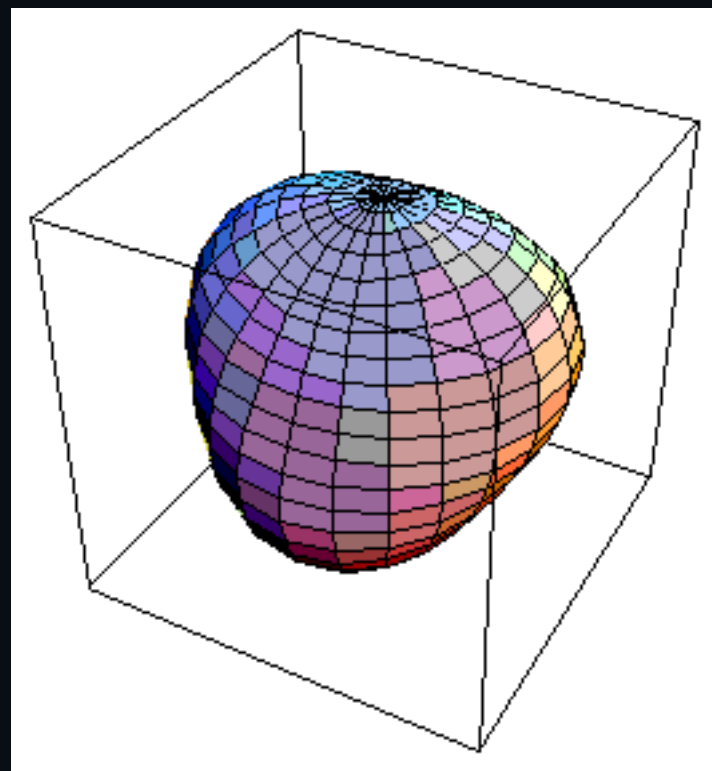
Neutron-Star Mode Spectrum

- f -mode: scales with average density
- p -modes: sound waves in the star (overtone of the f -mode)
- g -modes: buoyancy waves from thermal/composition gradients
- inertial modes (including r -modes): associated with rotation
- i -modes: arise from phase transitions
- Also:
 - w -modes, s -modes, Alfvén modes, ...

➡ Modes provide *unprecedented* access to neutron-star physics



$(l, m) = (3, 0)$



$(l, m) = (3, 2)$

Mode-Sum Representation

- Normal modes form a complete basis [Chandrasekhar, *Astrophys. J.* **139**, 664 (1964)],

$$\xi(t, \mathbf{x}) = \sum_{\alpha} q_{\alpha}(t) \xi_{\alpha}(\mathbf{x}), \quad \mathbf{C}(\mathbf{x}) \cdot \xi_{\alpha}(\mathbf{x}) = \omega_{\alpha}^2 \xi_{\alpha}(\mathbf{x})$$

- The tidal equation of motion simplifies to

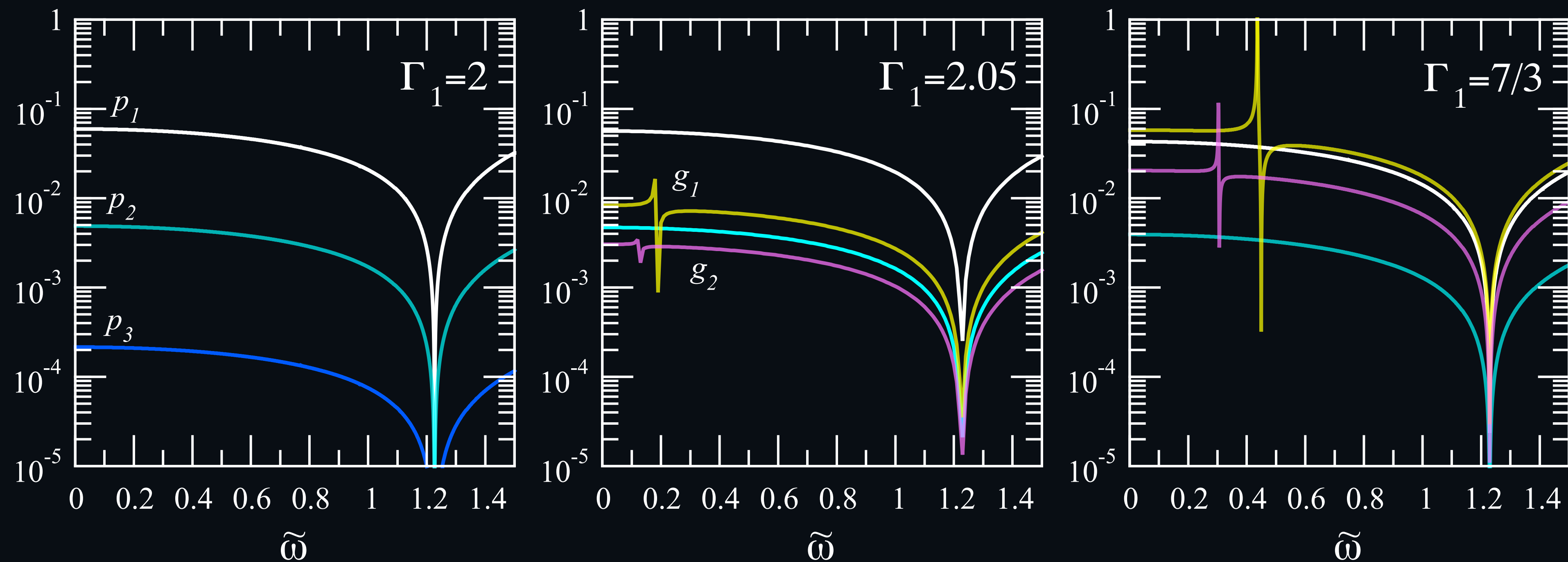
$$\ddot{q}_{\alpha}(t) + \omega_{\alpha}^2 q_{\alpha}(t) = \frac{Q_{\alpha}(t)}{\mathcal{E}_{\alpha}} \propto e^{-im\Phi(t)}$$

- **Challenge:** Can this be formulated in general relativity [see Hegade K R's talk this afternoon]?

Equilibrium Tide

- For an equilibrium orbit, $\dot{\Phi} = \text{const}$,

$$q_{\alpha}(t) = \frac{Q_{\alpha}(t)}{\mathcal{E}_{\alpha}} \frac{1}{\omega_{\alpha}^2 - (m\dot{\Phi})^2}$$



Static Limit

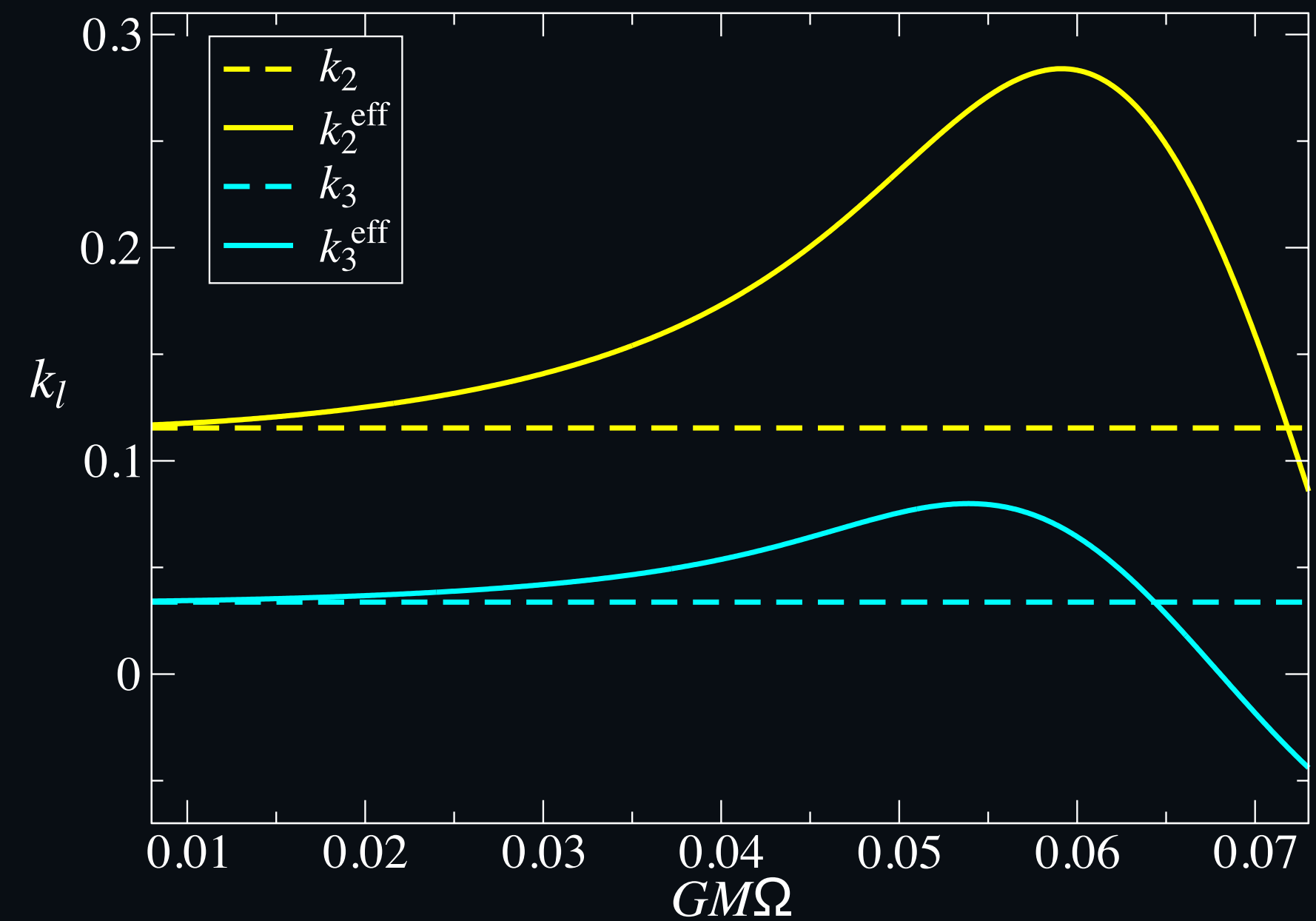
- In the static limit, $\dot{\Phi} = 0$,

$$q_{\alpha} = \frac{Q_{\alpha}}{\mathcal{E}_{\alpha}} \frac{1}{\omega_{\alpha}^2}$$

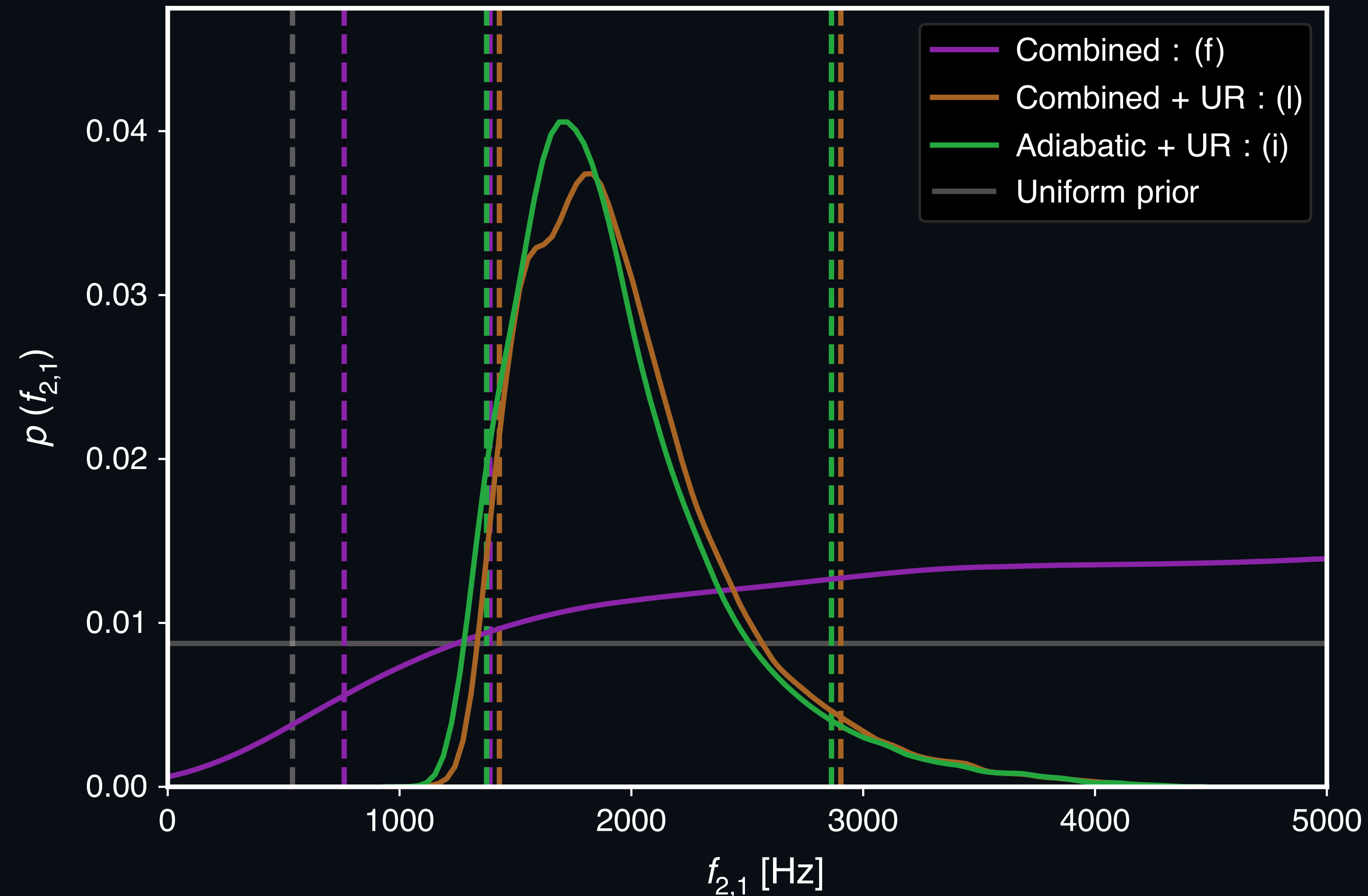
$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
Mode	k_l	Mode	k_l	Mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
9×10^{-4}		7×10^{-4}		3×10^{-4}	

f -mode Approximation

- The dynamical tide is dominated by the f -mode
- There have been models developed for the f -mode dynamical tide that use
 - *effective-one-body* [Steinhoff+, Phys. Rev. D **94**, 104028 (2016)],
 - *Newtonian* [Schmidt+Hinderer, Phys. Rev. D **100**, 021501 (2019)] and
 - *phenomenological* approaches [Abac+, Phys. Rev. D **109**, 024062 (2024)]



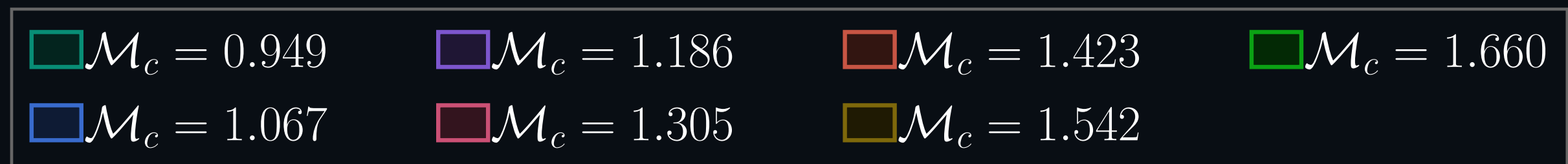
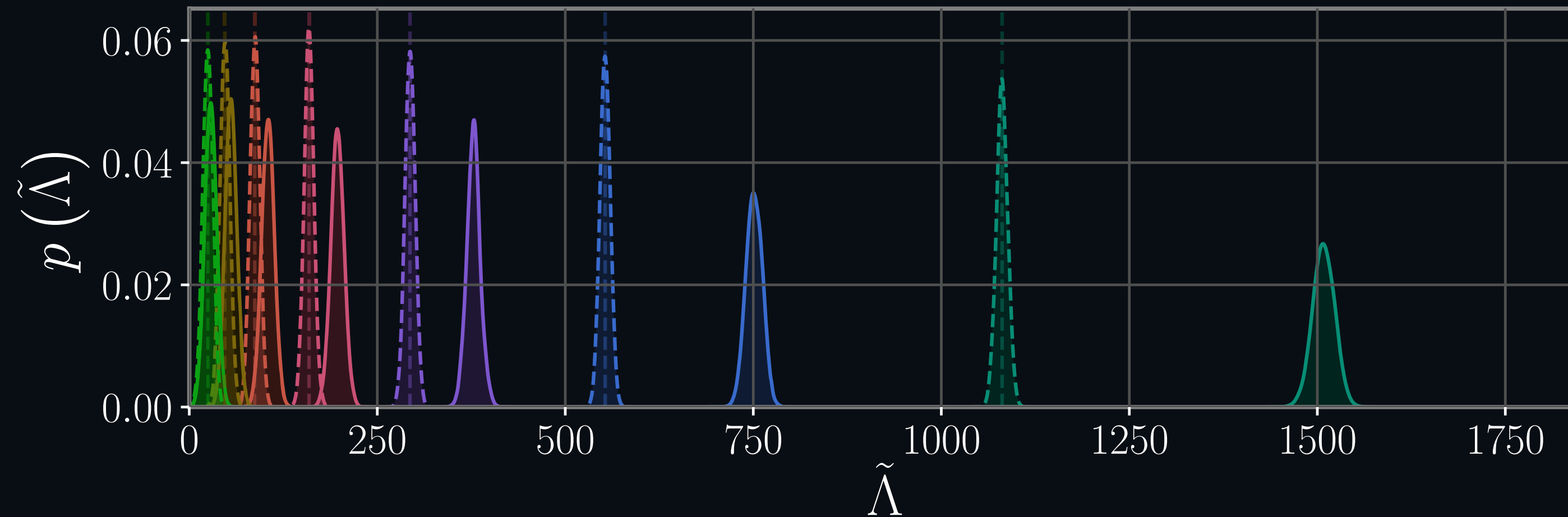
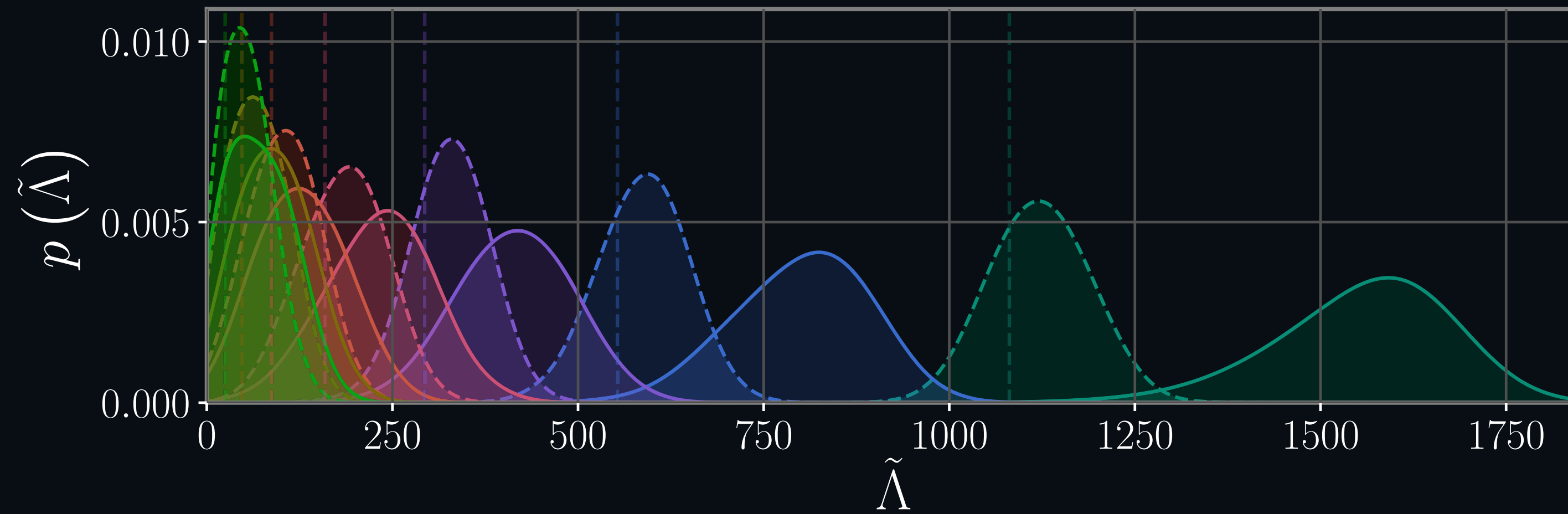
Towards Asteroseismology



[Pratten+, Nat. Commun. **11**, 2553 (2020)]

- **Challenge:** Can we go beyond universal relations in inference?

Biases



[Pratten+, Phys. Rev. Lett. **129**, 081102 (2022)]

Sub-Dominant Modes

- Low-frequency modes (including *g*-modes, *r*-modes and *i*-modes) will become **resonant** during inspiral,

$$m\dot{\Phi} \approx \omega_{\alpha}$$

- Energy is extracted from the orbit,

$$\Delta E_{\alpha} \sim |q_{\alpha}|^2,$$

which results in a finite orbital phase shift $\Delta\Phi$

Origin of g -modes

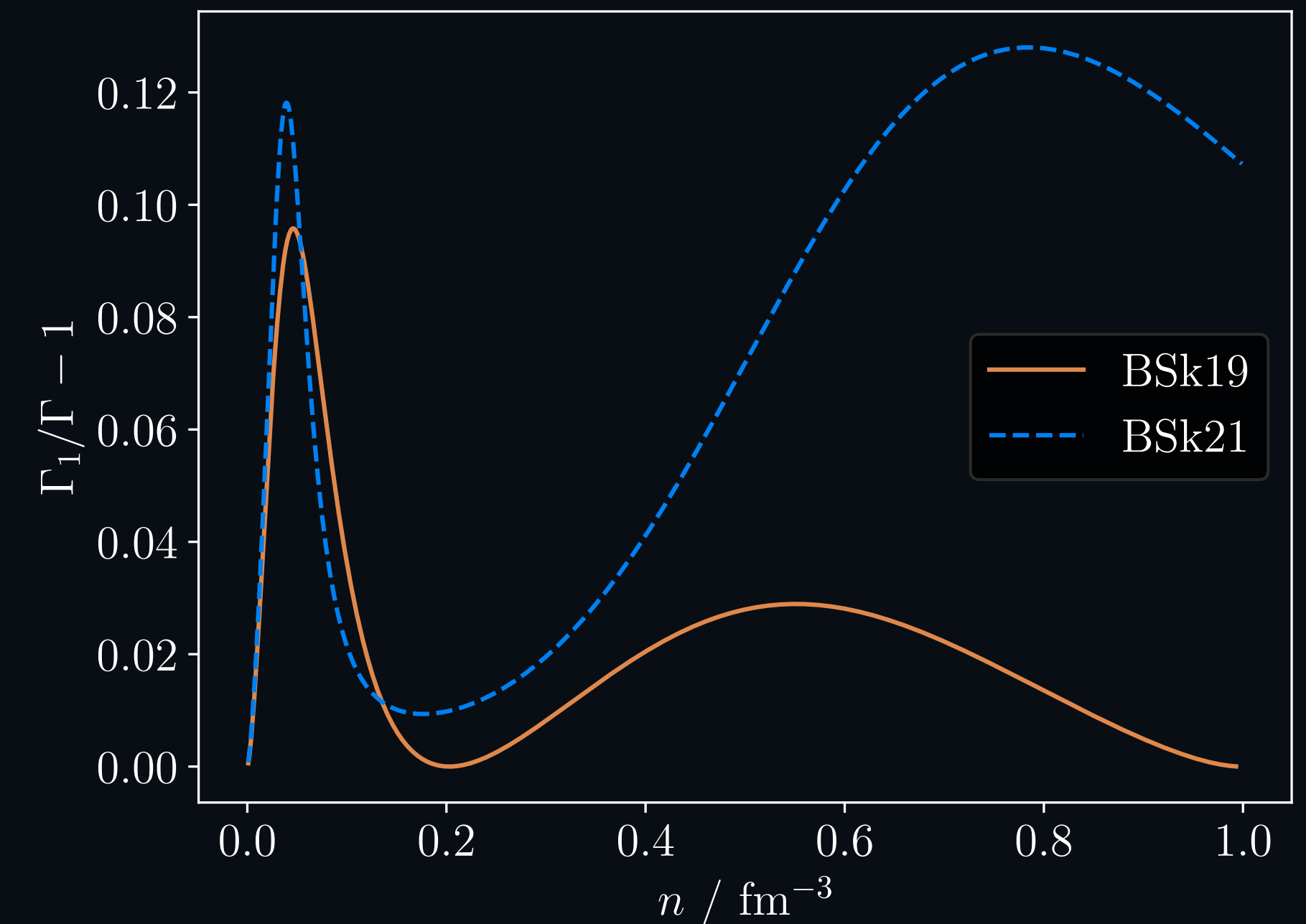
- Instead of

$$d\varepsilon = \frac{\varepsilon + p}{n_b} dn_b \implies \varepsilon = \varepsilon(n_b),$$

the first law is

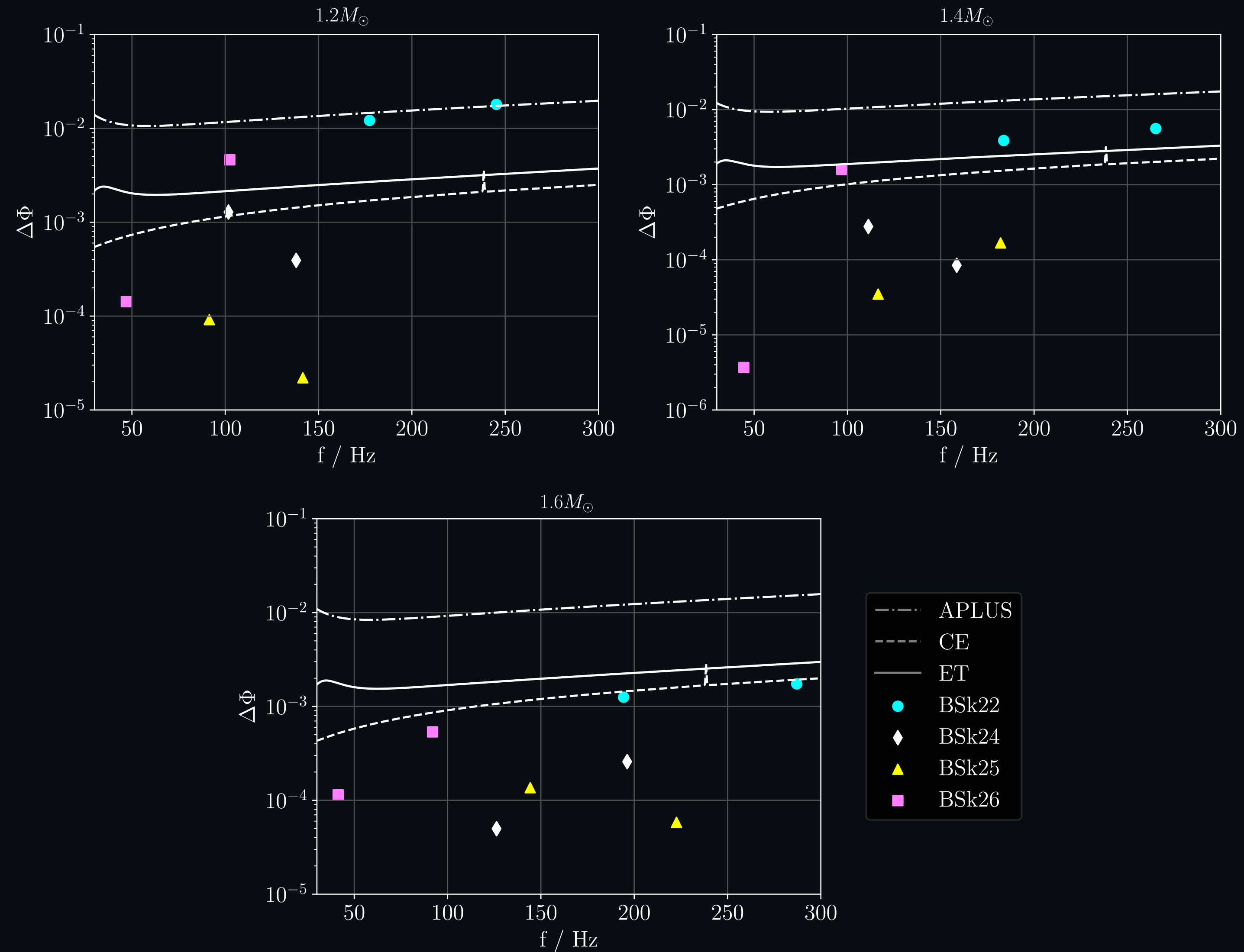
$$d\varepsilon = \frac{\varepsilon + p}{n_b} dn_b + n_b \mu_\Delta dY_e \implies \varepsilon = \varepsilon(n_b, Y_e)$$

- When there are slow weak nuclear reactions,
 $\mu_\Delta \neq 0$
- They are also sensitive to superfluidity
[Yu+Weinberg, Mon. Not. R. Astron. Soc. **464**, 2622 (2017)]



[Gittins+Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

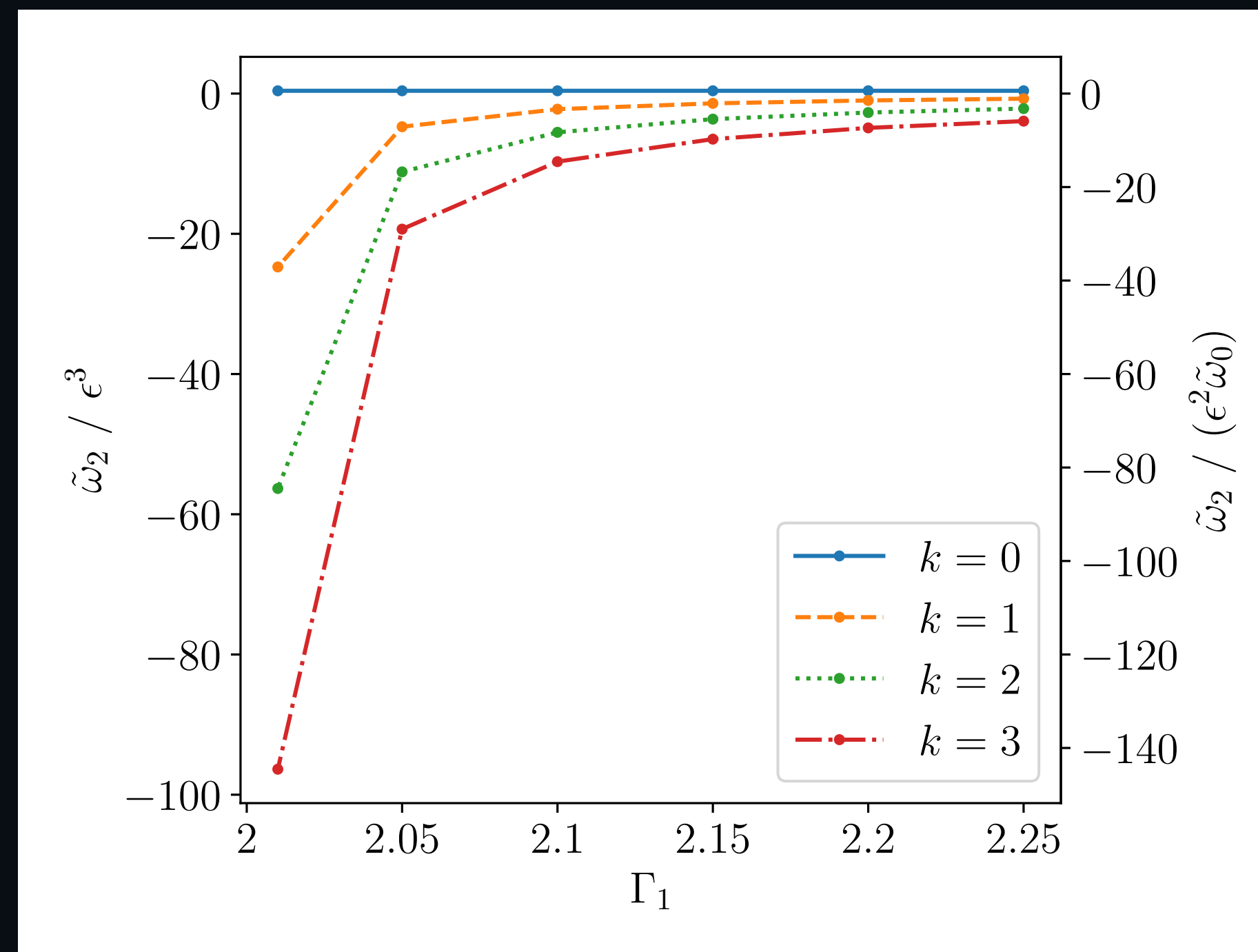
g -mode Resonances



[Counsell+, Mon. Not. R. Astron. Soc. **536**, 1967 (2025)]

r -modes

- A special class of inertial modes have *axial* parity: the r -modes
- The r -modes are famous for their gravitational-wave-driven instability
- They also probe composition gradients



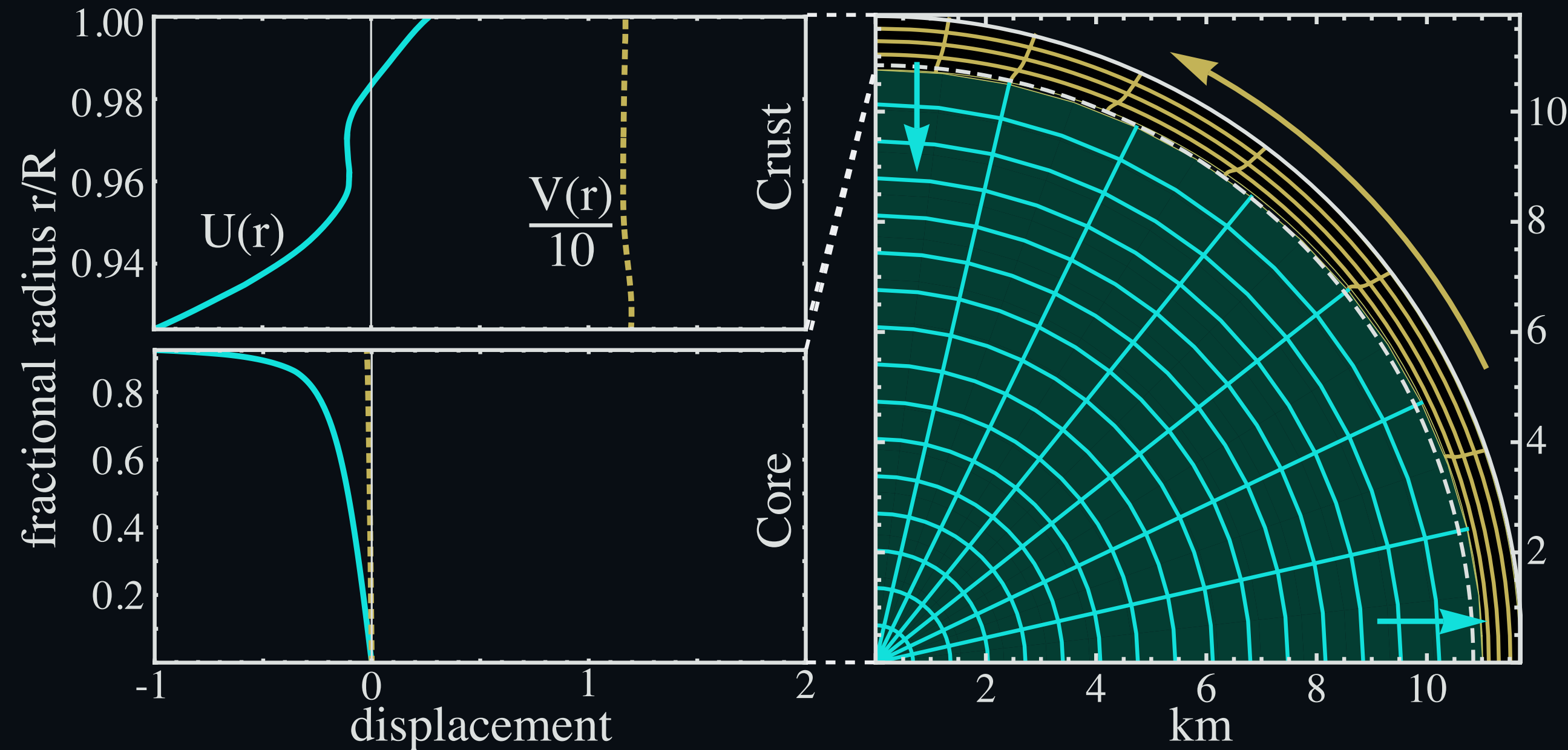
[Gittins+Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

r-mode Resonances

- The *r*-mode couples strongly to the gravito-magnetic tide
- Estimates give [\[Flanagan+Racine, Phys. Rev. D **75**, 044001 \(2007\)\]](#)

$$\Delta\Phi \approx -0.03 \left(\frac{R}{10 \text{ km}} \right)^4 \left(\frac{f_{\text{spin}}}{100 \text{ Hz}} \right)^{2/3} \left(\frac{1.4M_{\odot}}{M} \right)^{10/3}$$

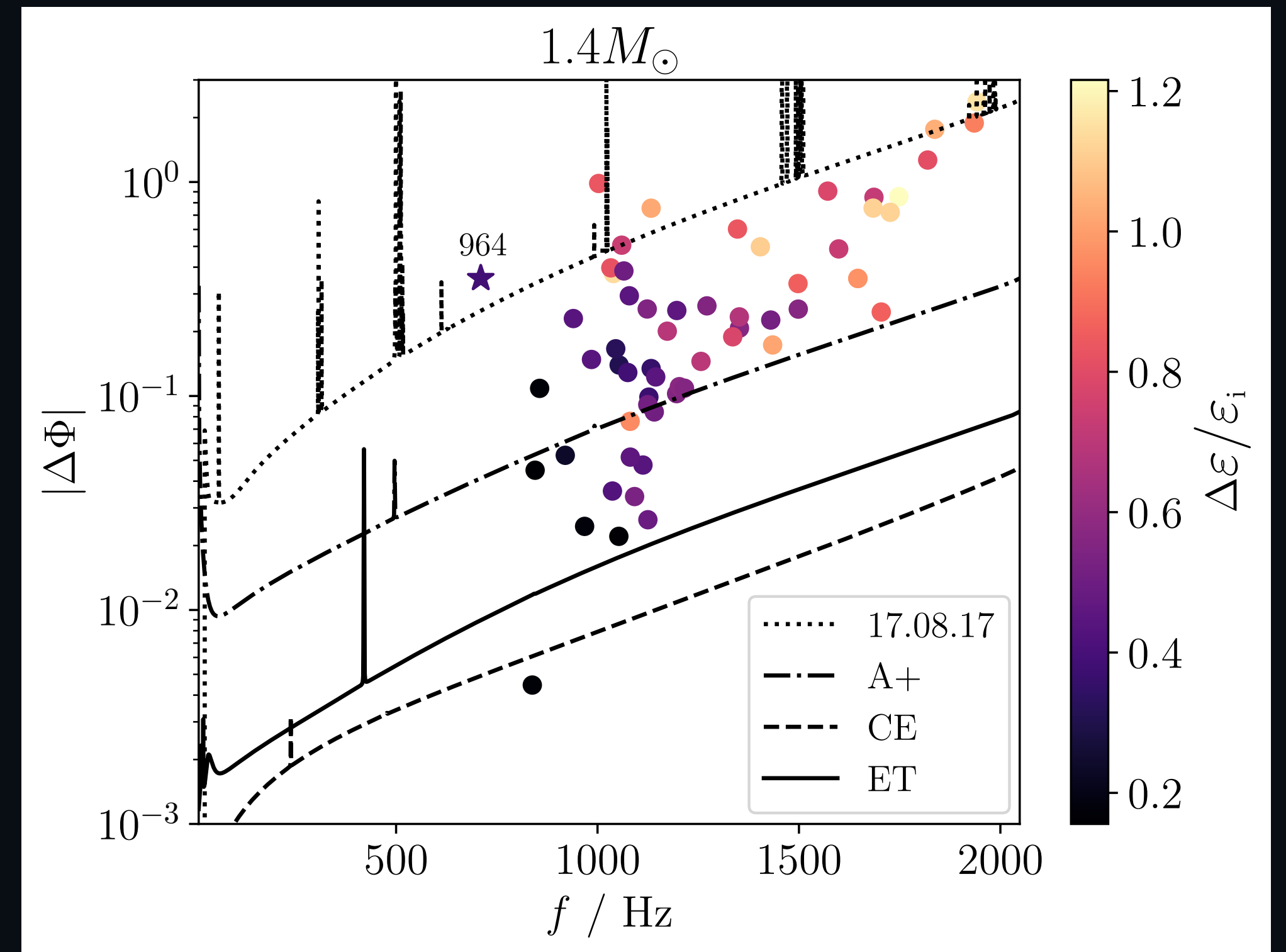
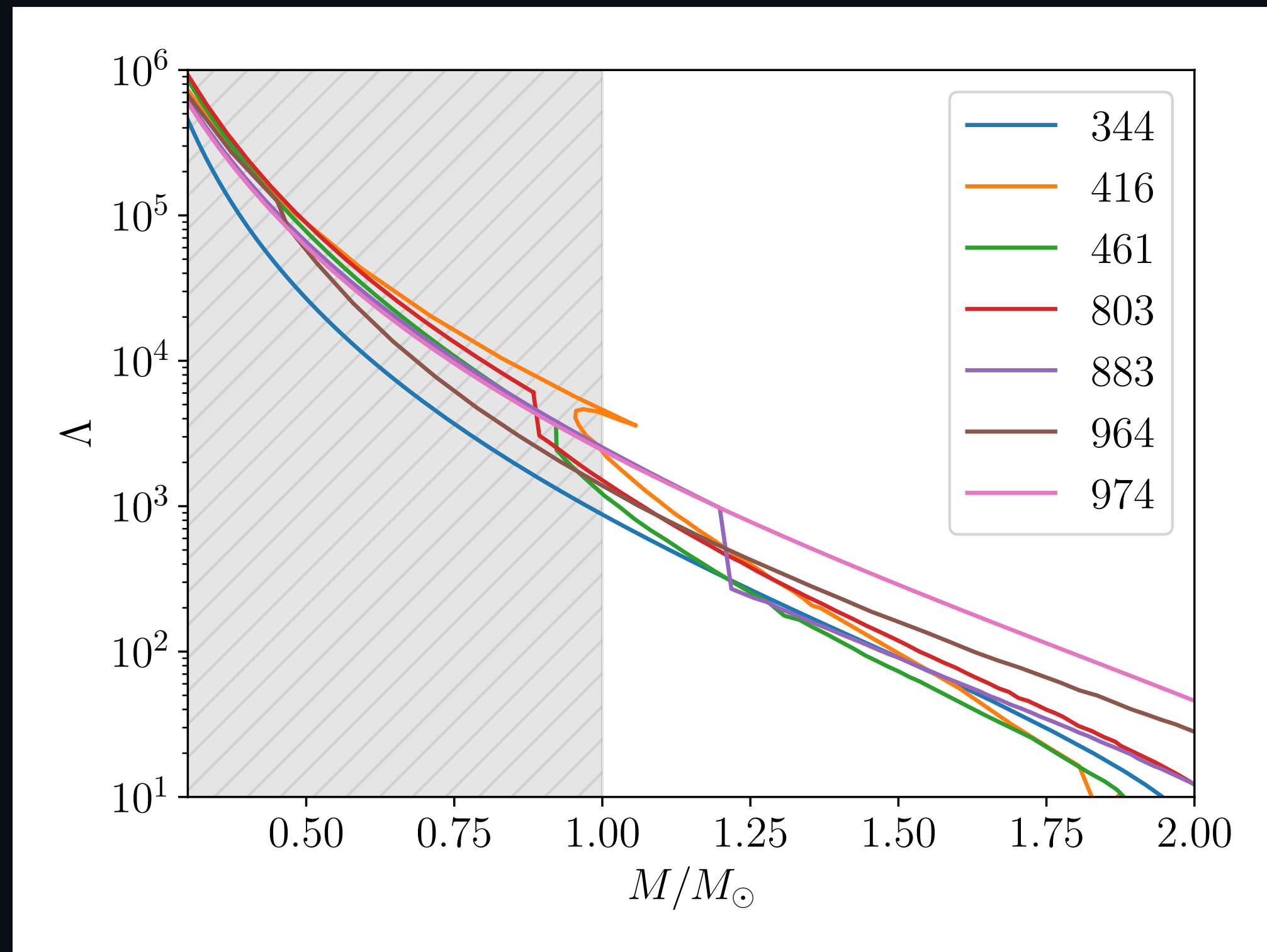
i-modes from Phase Transitions



[Tsang+, Phys. Rev. Lett. **108**, 011102 (2012)]

- The interfacial *i*-mode arises when there is a first-order phase transition in the star
- This may occur at the core-crust interface or (possibly) at a transition to deconfined quark matter in the core

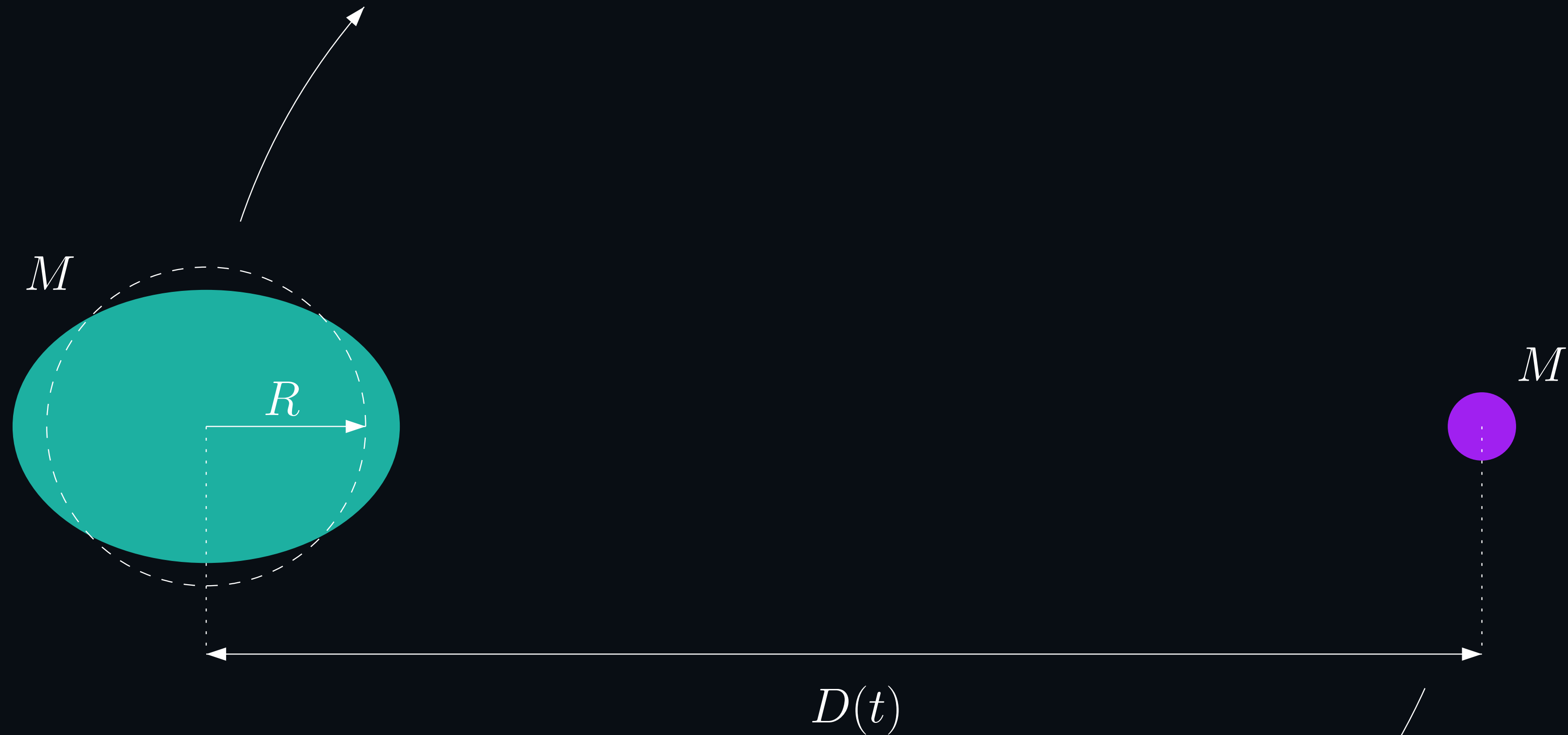
i-mode Resonances



[Counsell+, Phys. Rev. Lett. **135**, 081402 (2025)]

- **Challenge:** Can we develop models of this resonant behaviour?

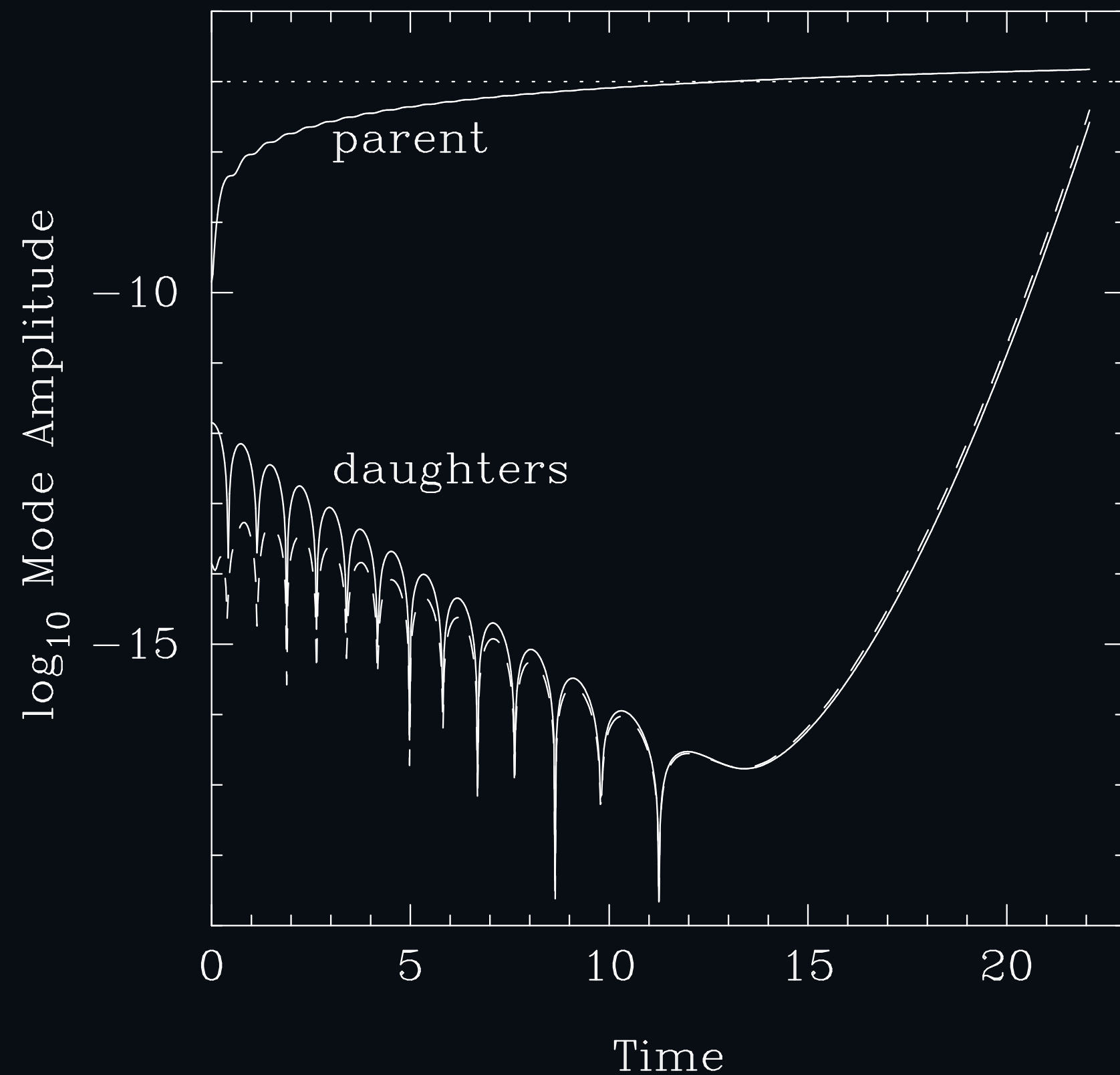
Non-Linear Tide



- Assumptions:

- Perturbative regime, $\epsilon = (M'/M)(R/D)^3 \ll 1$ — go to higher orders [\[see Yu's talk tomorrow afternoon\]](#)

p - g -mode Instability

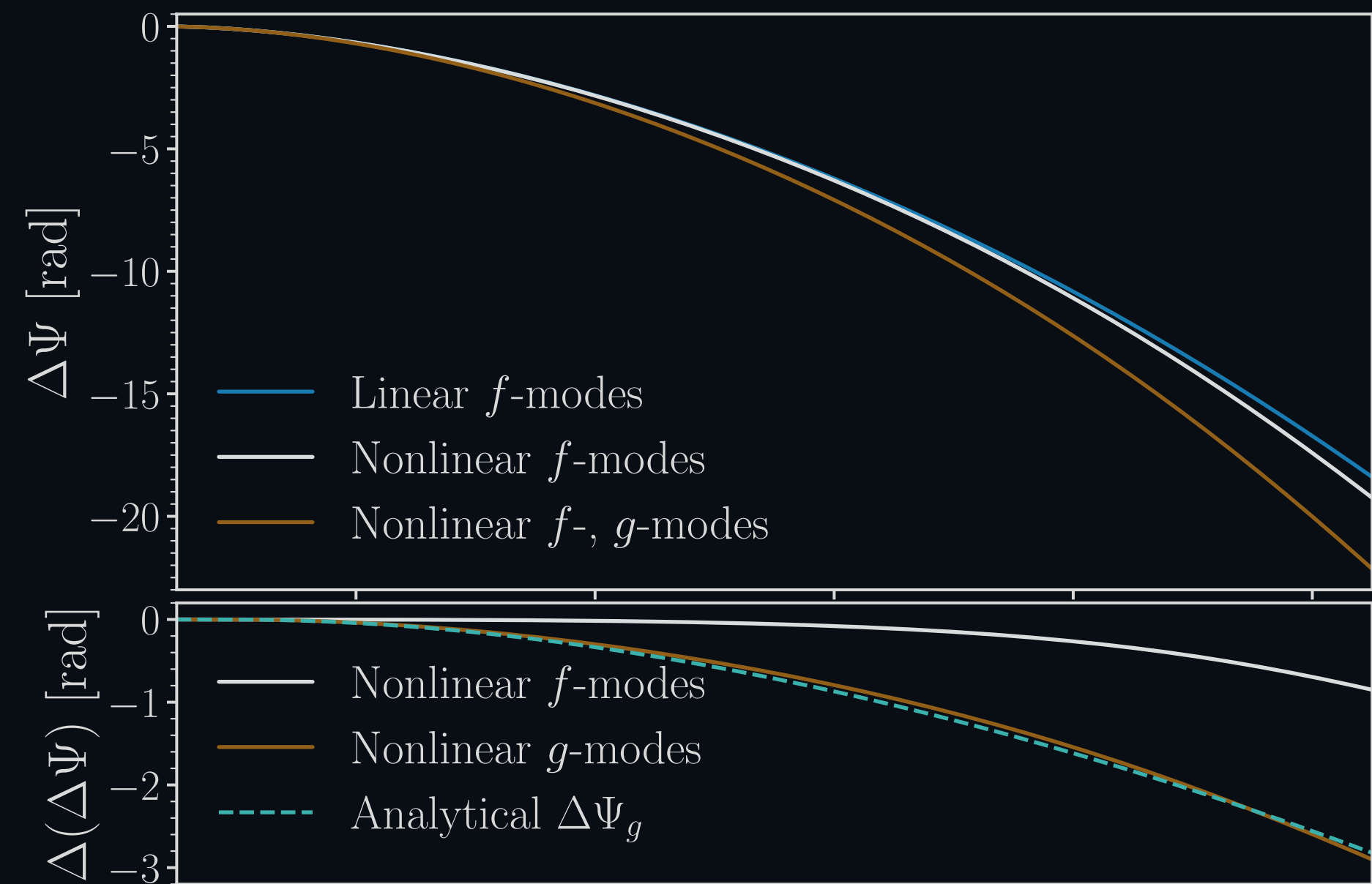
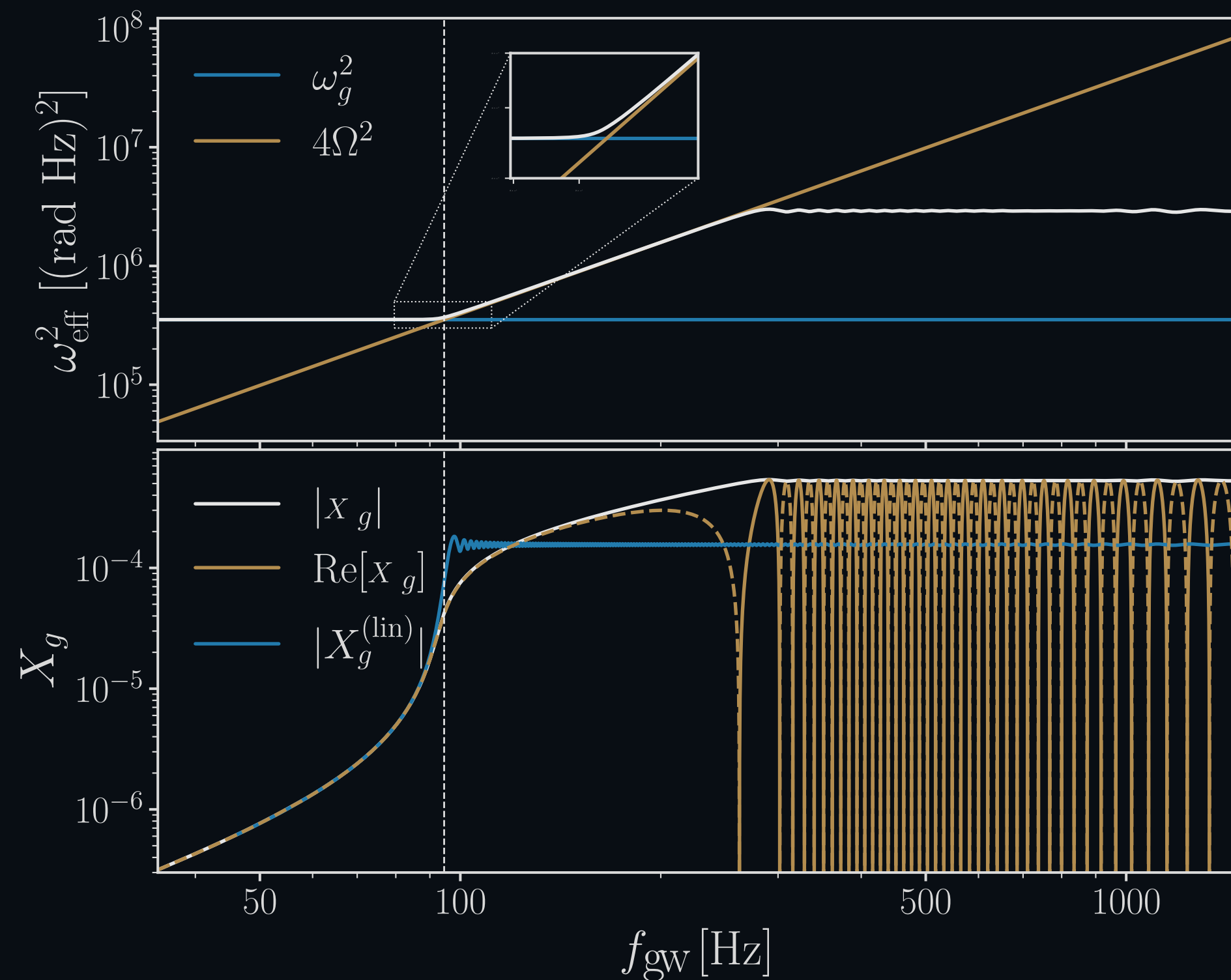


[Weinberg+, *Astrophys. J.* **769**, 121 (2013)]

- Tide couples to a low-frequency g -mode and a high-frequency p -mode
- May lead to substantial phase shifts
- No support found in GW170817 [LIGO-Virgo Collaboration+Weinberg, *Phys. Rev. Lett.* **122**, 061104 (2019)]
- **Challenge:** Is this prediction robust? When does it saturate?

Resonance Locking

- At non-linear order, g -modes are *anharmonic*; their oscillation frequencies depend on mode energies
- This gives rise to **resonance locking**: the frequency can shift to match the tidal driving [Kwon+, arXiv:2410.03831; arXiv:2503.11837]



Conclusions

Opportunities	Challenges
Gravitational waves probe dense nuclear matter by encoding fine tidal deformations	Can the mode-sum be formulated in general relativity?
The tide presents the opportunity to conduct neutron-star seismology	Can we go beyond universal relations in inference?
Oscillation modes grant access to rich physics	Can we develop gravitational-waveform models of resonant oscillation modes?
Gravitational-wave interferometers are rapidly increasing in sensitivity to these features	Are the non-linear tidal predictions robust?