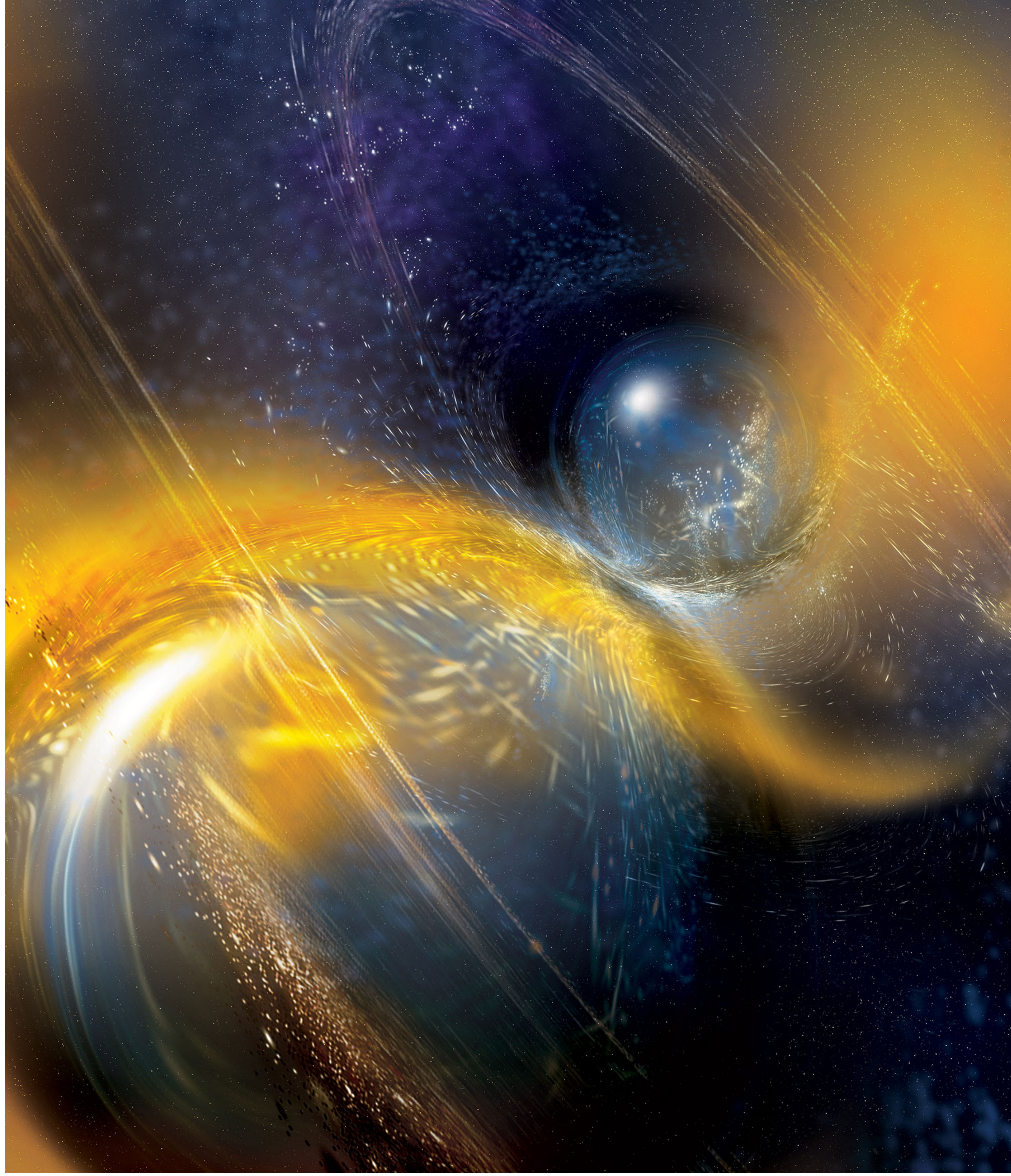
CONSTRAINING THE DENSE NUCLEAR-MATTER EQUATION OF STATE WITH THE DYNAMICAL TIDES OF NEUTRON STARS

Fabian Gittins

Astrophysics Seminar, MSSL-UCL
30 May 2024



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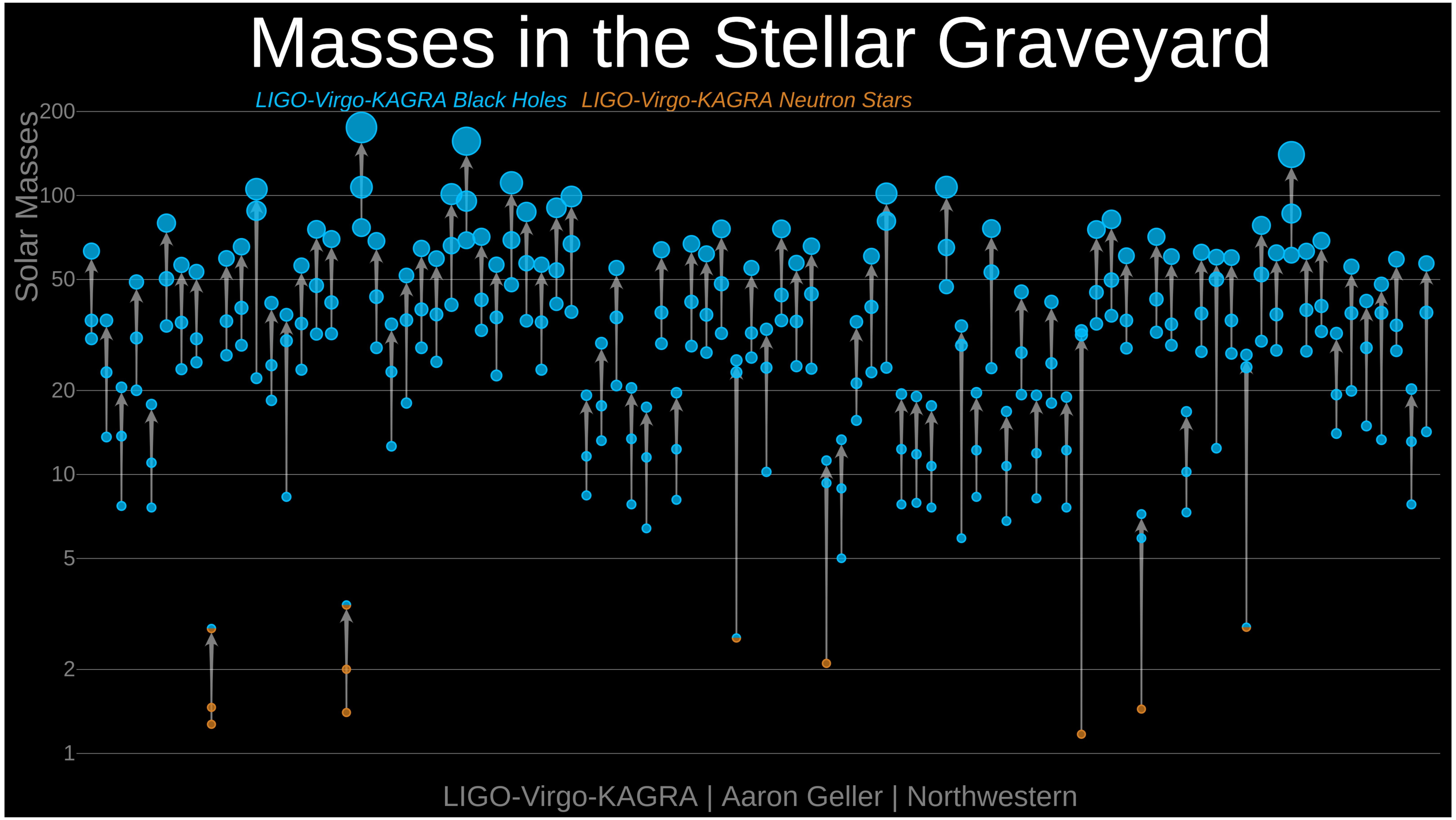
STAG RESEARCH CENTRE

physics of neutron stars

- Neutron stars are among the most complex objects in the Universe.
- A realistic description of a neutron star will inevitably require
  - general relativity
  - the equation of state
  - strong magnetic fields
  - superfluidity
  - a crust
  - thermal features
  - …

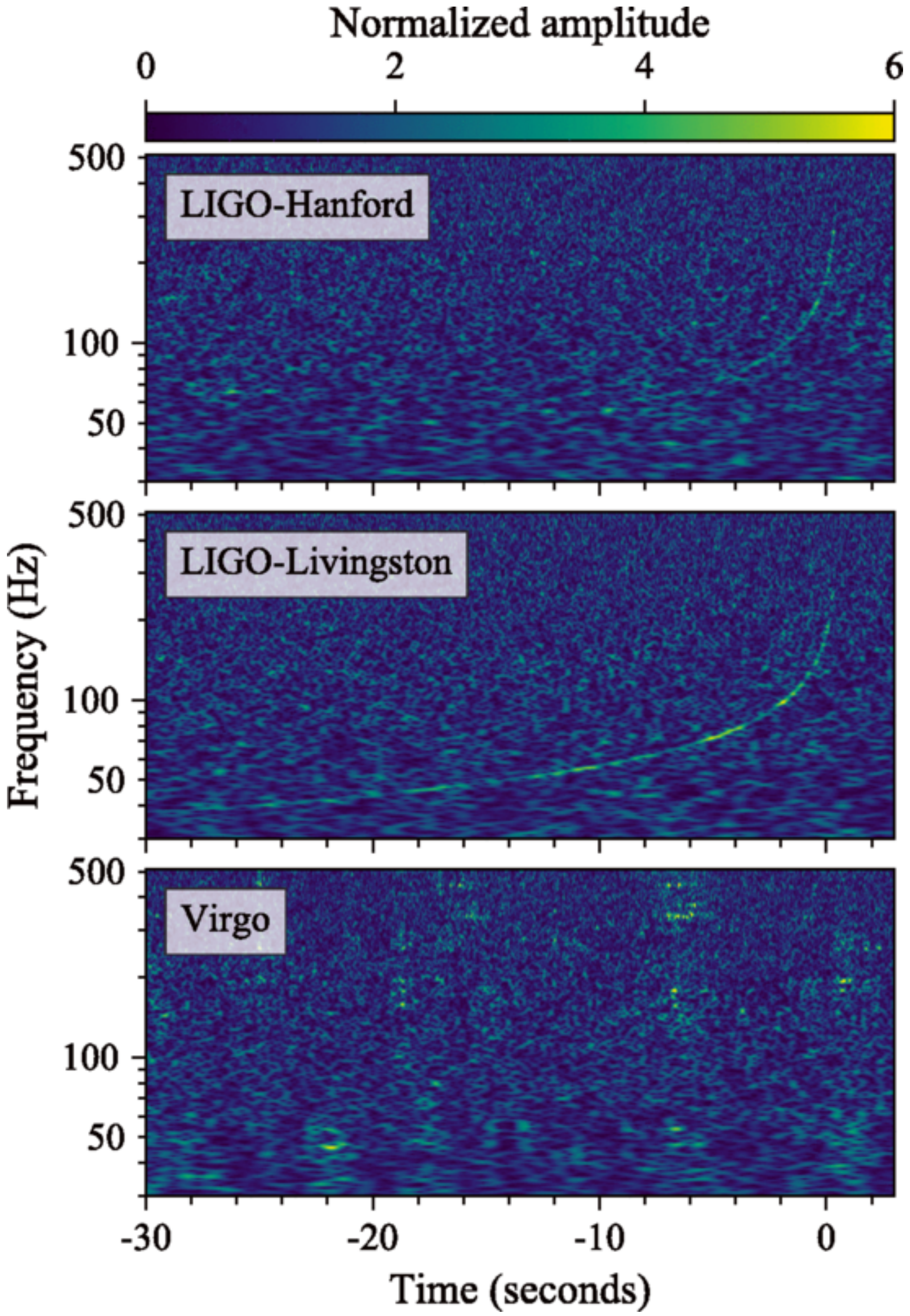
gravitational waves: observations

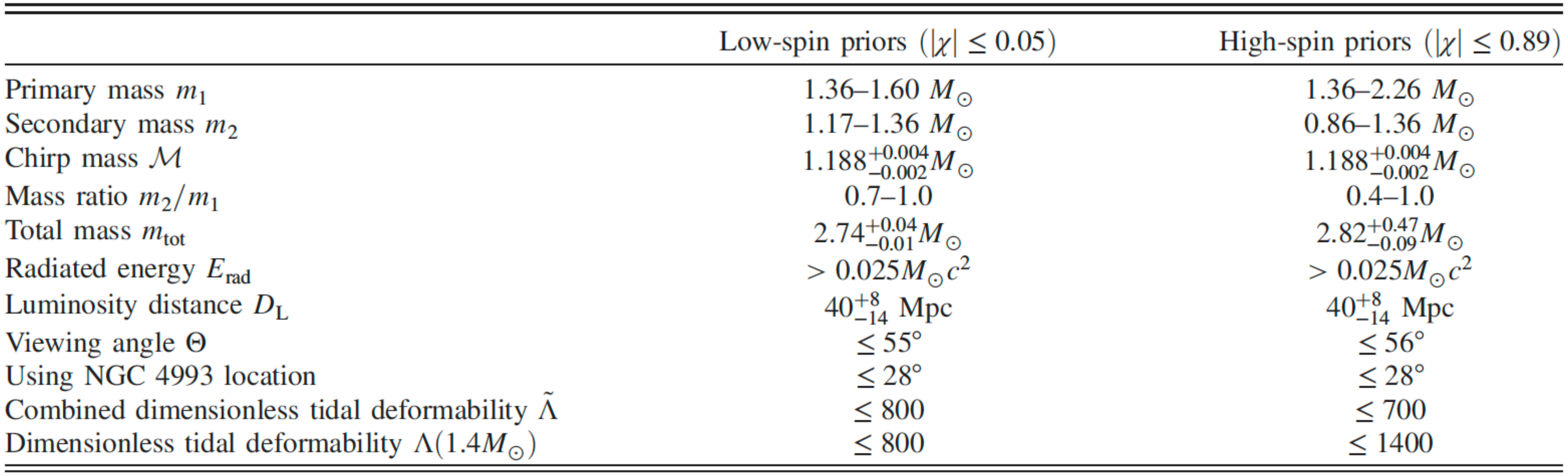
- Since 2015, gravitational-wave detectors have witnessed over 100 compact-binary coalescences, including 2 neutron-star binaries and 3 neutron star-black hole binaries.



gravitational waves: GW170817

- On 17 Aug. 2017, gravitational-wave instruments detected the first neutron-star merger.

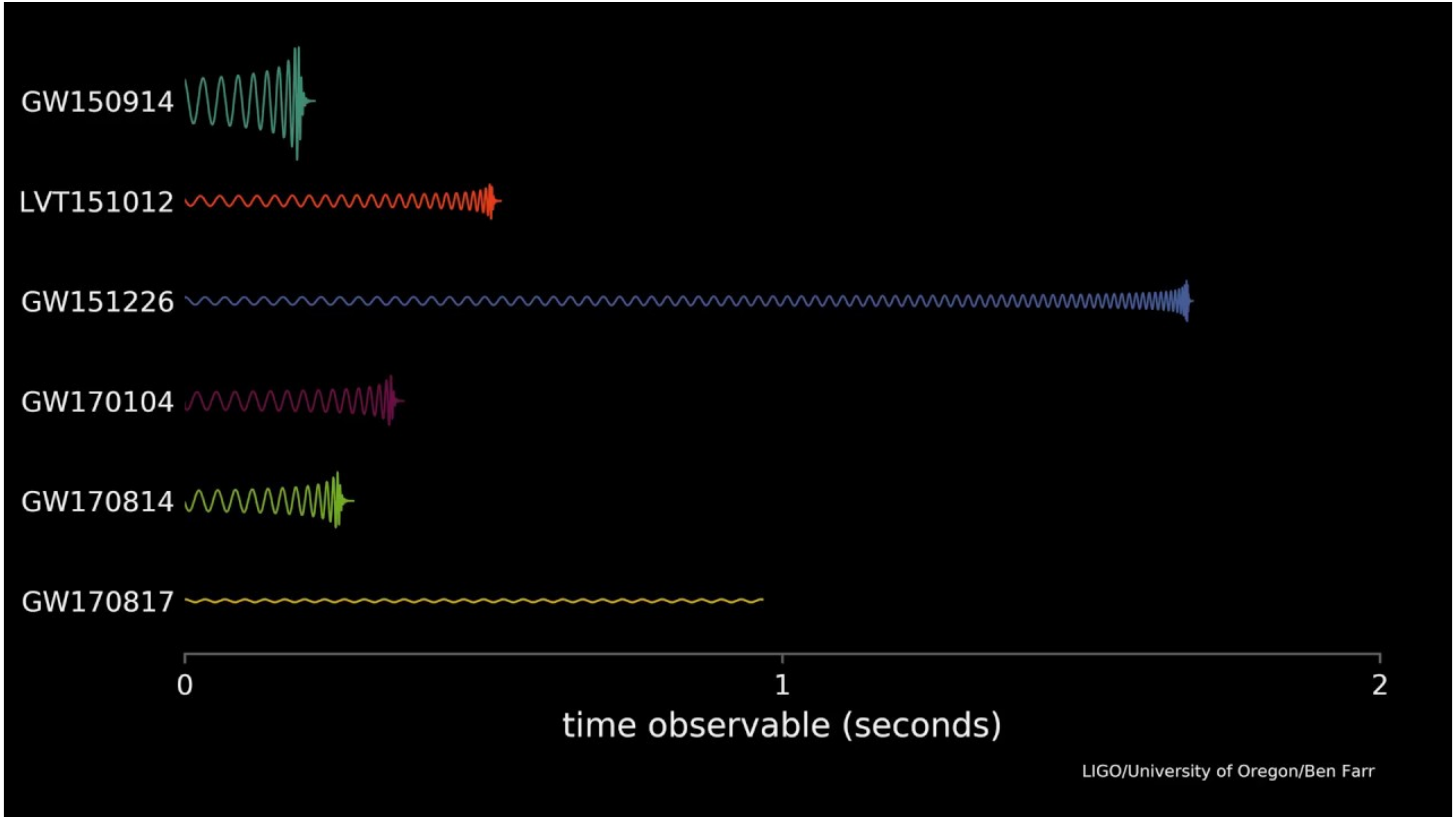




	Low-spin priors ($\lvert\chi\rvert \leq 0.05$)	High-spin priors ($\lvert\chi\rvert \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass m_{tot}	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy E_{rad}	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance D_{L}	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

[Abbott+, Phys. Rev. Lett. **119**, 161101 (2017)]

gravitational waves: GW170817



LIGO/University of Oregon/Ben Farr

science potential of neutron-star binaries

1. Cosmology

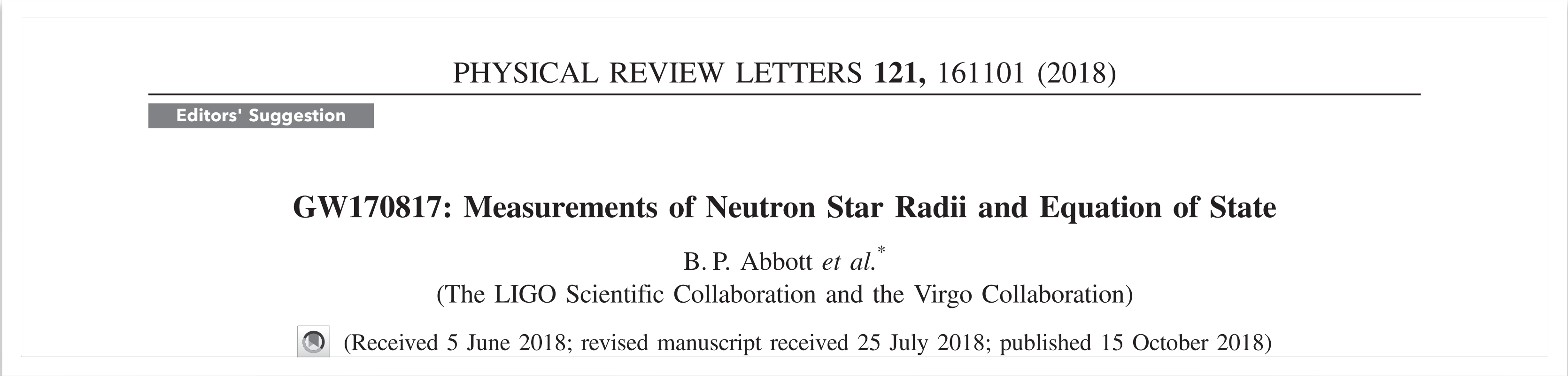
LETTER

doi:10.1038/nature24471

A gravitational-wave standard siren measurement of the Hubble constant

The LIGO Scientific Collaboration and The Virgo Collaboration*, The 1M2H Collaboration*, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration*, The DLT40 Collaboration*, The Las Cumbres Observatory Collaboration*, The VINROUGE Collaboration* & The MASTER Collaboration*

2. Nuclear physics



PHYSICAL REVIEW LETTERS **121,** 161101 (2018)

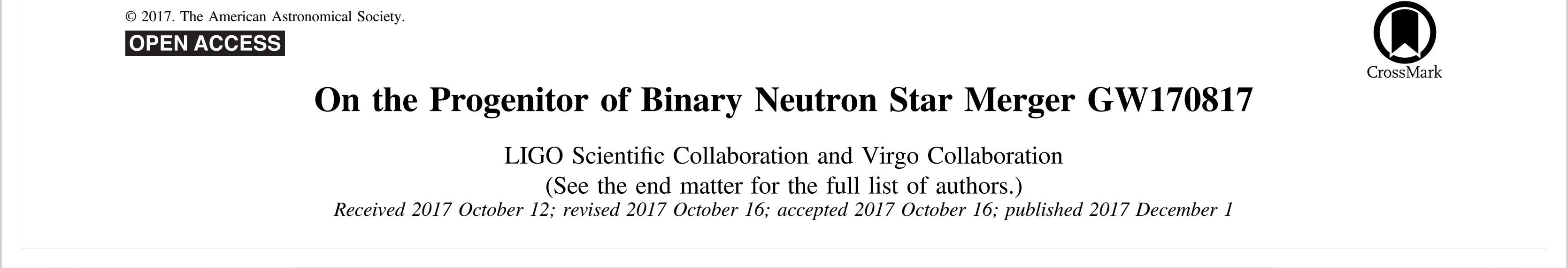
Editors' Suggestion

GW170817: Measurements of Neutron Star Radii and Equation of State

B. P. Abbott *et al.**
(The LIGO Scientific Collaboration and the Virgo Collaboration)

(Received 5 June 2018; revised manuscript received 25 July 2018; published 15 October 2018)

3. Astrophysics



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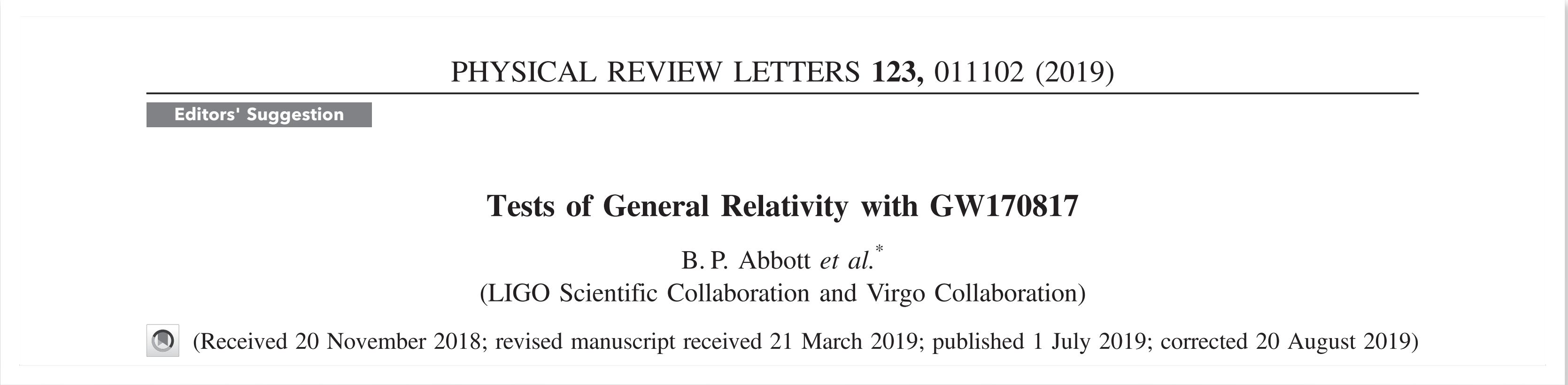
On the Progenitor of Binary Neutron Star Merger GW170817

LIGO Scientific Collaboration and Virgo Collaboration
(See the end matter for the full list of authors.)

Received 2017 October 12; revised 2017 October 16; accepted 2017 October 16; published 2017 December 1

4. Testing general relativity



PHYSICAL REVIEW LETTERS **123,** 011102 (2019)

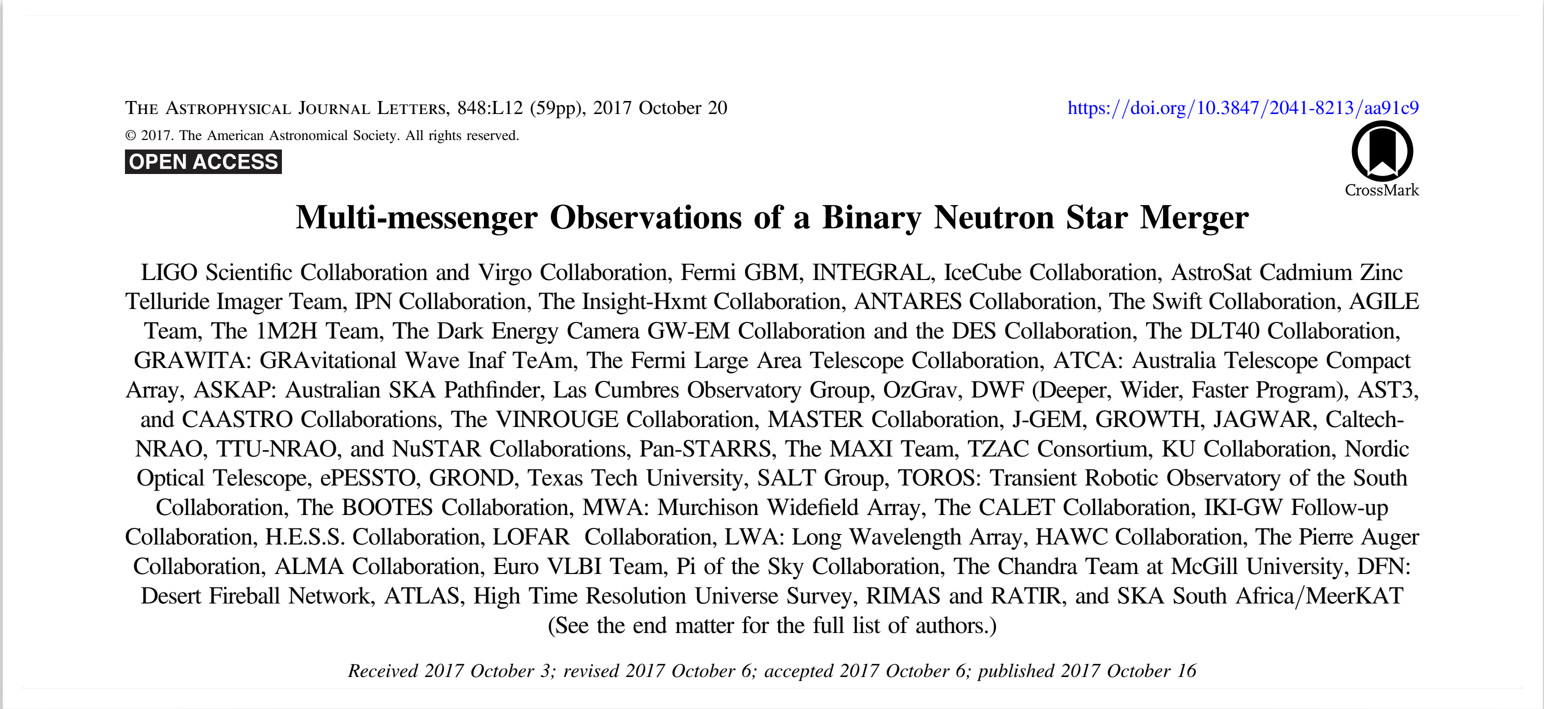
Editors' Suggestion

Tests of General Relativity with GW170817

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 20 November 2018; revised manuscript received 21 March 2019; published 1 July 2019; corrected 20 August 2019)

5. Multi-messenger astronomy



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Multi-messenger Observations of a Binary Neutron Star Merger

LIGO Scientific Collaboration and Virgo Collaboration, Fermi GBM, INTEGRAL, IceCube Collaboration, AstroSat Cadmium Zinc Telluride Imager Team, IPN Collaboration, The Insight-Hxmt Collaboration, ANTARES Collaboration, The Swift Collaboration, AGILE Team, The 1M2H Team, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, GRAWITA: GRAvitational Wave Inaf TeAm, The Fermi Large Area Telescope Collaboration, ATCA: Australia Telescope Compact Array, ASKAP: Australian SKA Pathfinder, Las Cumbres Observatory Group, OzGrav, DWF (Deeper, Wider, Faster Program), AST3, and CAASTRO Collaborations, The VINROUGE Collaboration, MASTER Collaboration, J-GEM, GROWTH, JAGWAR, Caltech-NRAO, TTU-NRAO, and NuSTAR Collaborations, Pan-STARRS, The MAXI Team, TZAC Consortium, KU Collaboration, Nordic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS: Transient Robotic Observatory of the South Collaboration, The BOOTES Collaboration, MWA: Murchison Widefield Array, The CALET Collaboration, IKI-GW Follow-up Collaboration, H.E.S.S. Collaboration, LOFAR Collaboration, LWA: Long Wavelength Array, HAWC Collaboration, The Pierre Auger Collaboration, ALMA Collaboration, Euro VLBI Team, Pi of the Sky Collaboration, The Chandra Team at McGill University, DFN: Desert Fireball Network, ATLAS, High Time Resolution Universe Survey, RIMAS and RATIR, and SKA South Africa/MeerKAT
(See the end matter for the full list of authors.)

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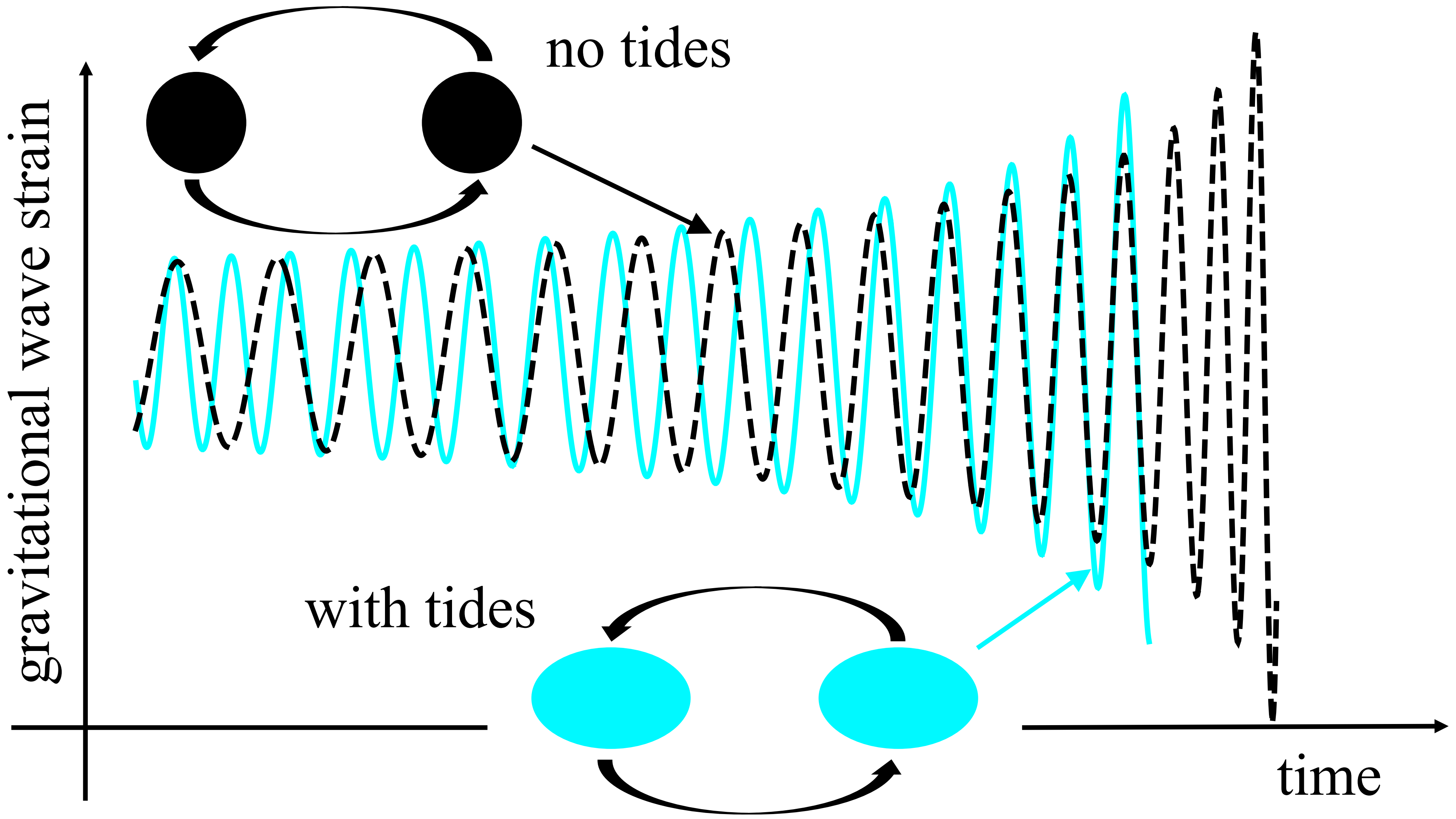
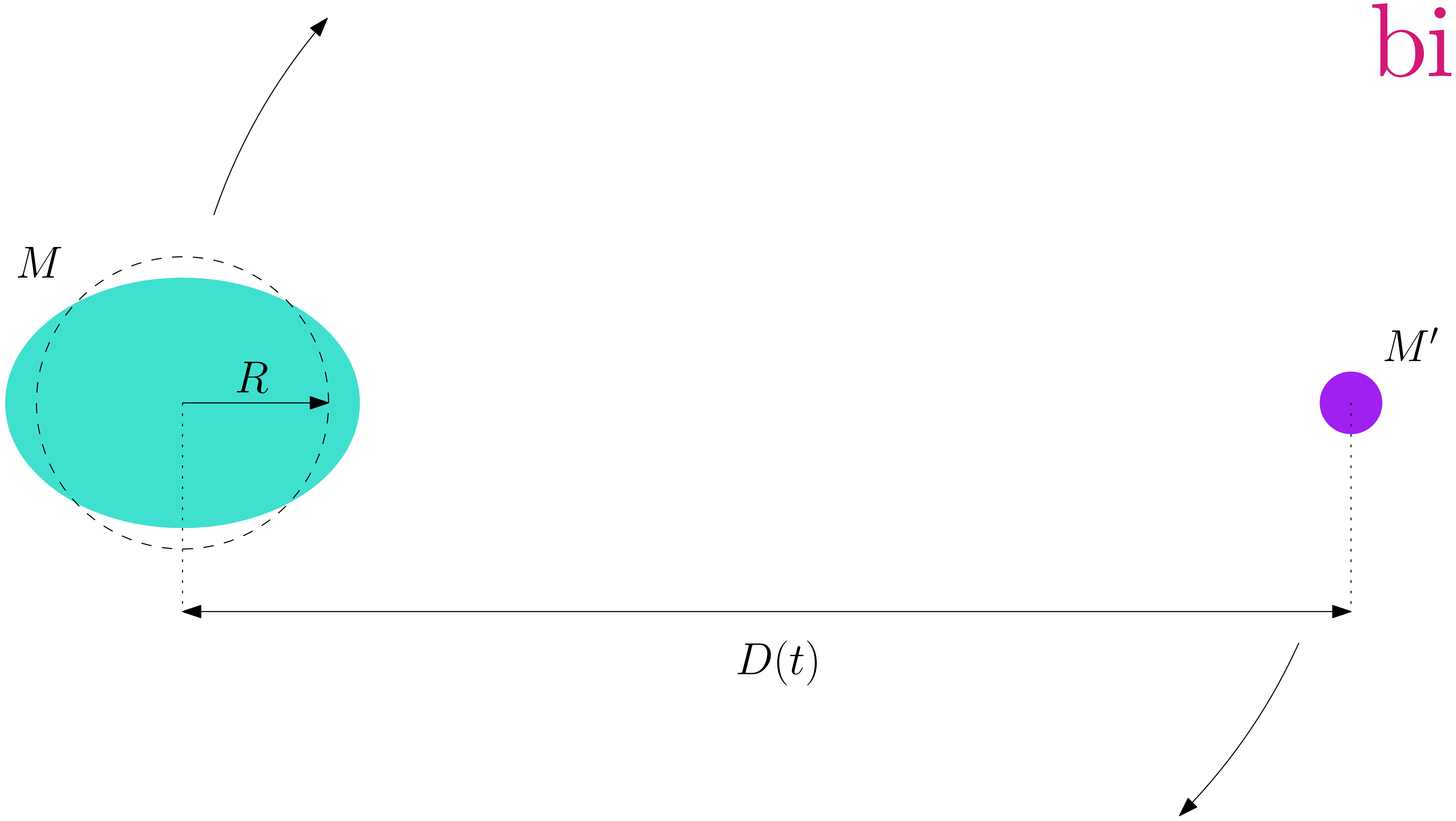
neutron-star binaries

- The signal emitted from inspiralling neutron stars differs to that of black holes due to the material response to the tidal field.
- These features enter the waveform phase Ψ at 5PN.
- The deformability of the stellar material is characterised by the *tidal Love numbers* k_l, which depend on the state of the nuclear matter.



binary problem



- Assumptions:

  1. The bodies are well separated, $\epsilon = (M'/M)(R/D)^3 \ll 1$. The problem can be tackled perturbatively. (In the final few orbits, this breaks down completely and numerical relativity must be used.)

  2. The external field due to the companion is slowly varying, $\lambda = m\dot{\Psi}/\omega_\alpha \ll 1$. In this regime, the tidal field is *static*.

static tide: Newtonian gravity

- The Love numbers k_l are defined at the surface of the star $r = R$ by

$$\delta\Phi(R,\theta,\phi) = \sum_{l,m} \delta\Phi_l(R)\, Y_l^m(\theta,\phi) = \sum_{l,m} 2k_l\, \chi_l(R)\, Y_l^m(\theta,\phi)\,.$$

- Therefore, they can be read off from the exterior,

$$U_l(r) \equiv \delta\Phi_l(r) + \chi_l(r) = \left[2k_l \left(\frac{R}{r}\right)^{2l+1} + 1\right] \left(\frac{r}{R}\right)^{l} \chi_l(R),$$

where the field U_l satisfies Poisson's equation,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dU_l}{dr}\right) - \frac{l(l+1)}{r^2} U_l = -\frac{4\pi G\rho}{dp/d\rho} U_l\,.$$

- This result generalises to relativity, where U is promoted to the (linearised) metric of the spacetime h_{ab}.

static tide: relativity

- In general relativity, the response of the star is obtained from the exterior behaviour of the metric, for example,

$$-\frac{h_{tt}}{2} = \frac{1}{2}\left[2k_2\left(\frac{R}{r}\right)^5 B_1 + A_1\right]\mathcal{E}_{jk}x^j x^k + \ldots,$$

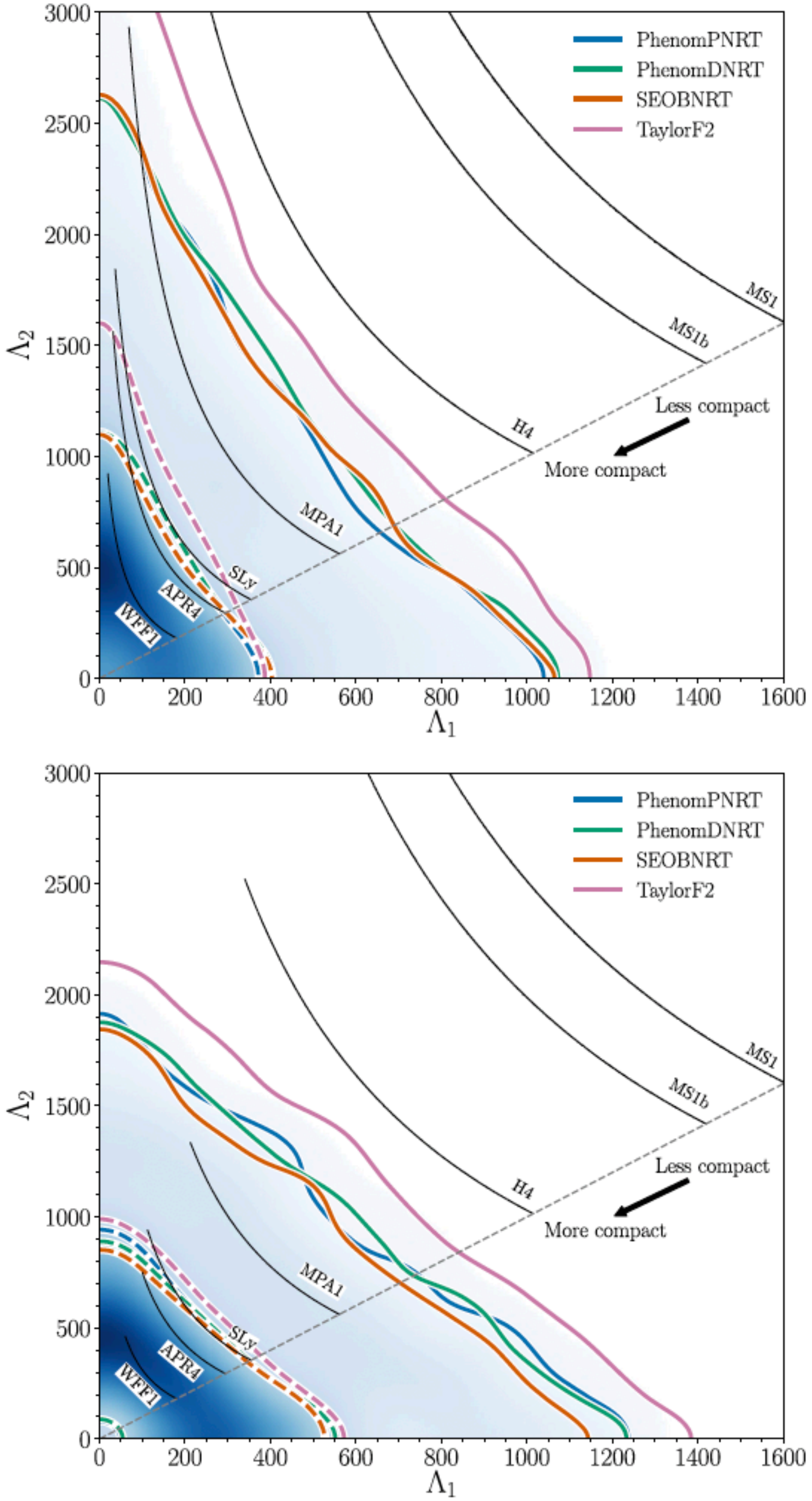
  where the hypergeometric functions A_1 and B_1 come from the Einstein field equations.

- New Love numbers appear: the *gravitomagnetic* Love numbers and (when the star's spin is considered) the *rotational* Love numbers.

static tide: state of play

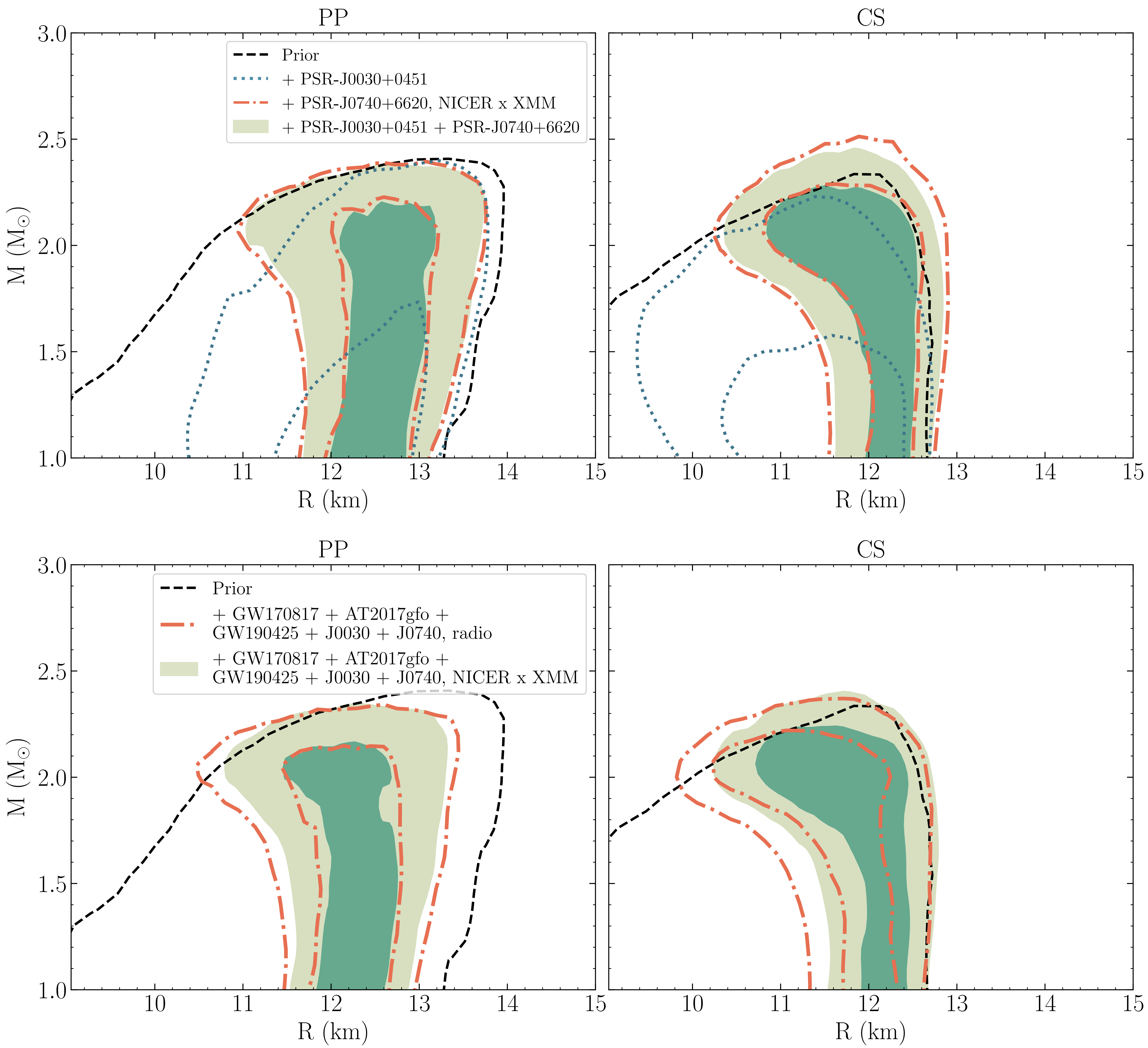
		Newtonian gravity	general relativity	notes
static tide	non-rotating stars	✓	✓ [Hinderer (2008); Binnington+Poisson (2009); Damour+Nagar 2009]	Relativistic neutron-star models with elastic crusts [Gittins+ (2020)] and superfluidity [Yeung+ (2021)].
	rotating stars		✓ [Landry+Poisson (2015); Landry (2015); Pani+ (2015a,b)]	Calculations are at the level of slowly rotating fluid bodies.

equation-of-state constraints



[Abbott+, Phys. Rev. X **9**, 011001 (2019)]

$$\Lambda_A = \frac{2}{3}k_{2A}\left(\frac{c^2R_A}{GM_A}\right)^5 = \frac{2}{3}\frac{k_{2A}}{C_A^5}$$



[Raaijmakers+, Astrophys. J. **918**, L29 (2021)]

dynamical tide

- However, the static tide approximation will inevitably break down.
- As the compact objects inspiral, the tidal frequency increases such that it eventually becomes comparable to the neutron star's natural modes,

$$\lambda = \frac{m\dot{\Psi}}{\omega_\alpha} = O(1)\,.$$

- This regime is known as the *dynamical tide*.
- Neglecting these effects could introduce severe biases in equation-of-state inference for third-generation instruments (Cosmic Explorer and the Einstein Telescope) [Pratten+, Phys. Rev. Lett. **129**, 081102 (2022)].

(some of the) modes

- *f*-modes: Fundamental oscillations of the star; scale with the average density, $\omega_\alpha/(2\pi) \sim \sqrt{GM/R^3} \sim 1\,\mathrm{kHz}$.
- *p*-modes: Restored by the pressure of the fluid; high frequencies above the *f*-mode; possible instability with *g*-modes [Weinberg+ (2013)].
- *g*-modes: Restored by buoyancy that arises from composition gradients; low frequencies below the *f*-mode, $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.
- inertial modes (including *r*-modes): Restored by rotation; primarily excited by the gravitomagnetic tide (a relativistic effect) [Flanagan+Racine (2007)]; $\omega_\alpha \sim \Omega$.
- *i*-modes: Oscillations that arise due to phase transitions; possible association with short gamma-ray bursts [Tsang+ (2012)]; $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.

mode-sum: formalism

- Neutron stars host a spectrum of oscillation modes. Formally, the normal modes satisfy an eigenvalue problem,

$$\mathbf{C}\cdot\boldsymbol{\xi}_\alpha = \omega_\alpha^2\,\boldsymbol{\xi}_\alpha\,.$$

- The normal modes form a complete basis [Chandrasekhar, Astrophys. J. **139**, 664 (1964)], such that a generic vector may be decomposed as

$$\boldsymbol{\xi}(t,\mathbf{x}) = \sum_\alpha q_\alpha(t)\,\boldsymbol{\xi}_\alpha(\mathbf{x})\,.$$

- The equation of motion $\partial_t^2\boldsymbol{\xi} + \mathbf{C}\cdot\boldsymbol{\xi} = -\nabla\chi$ becomes that of a driven harmonic oscillator,

$$\frac{d^2 q_\alpha}{dt^2} + \omega_\alpha^2\,q_\alpha = Q_\alpha \propto e^{-im\Psi}\,.$$

resonance

- At resonance $m\dot{\Psi} = \omega_\alpha$, the mode will become excited and extract energy from the orbit. This will change the phase by

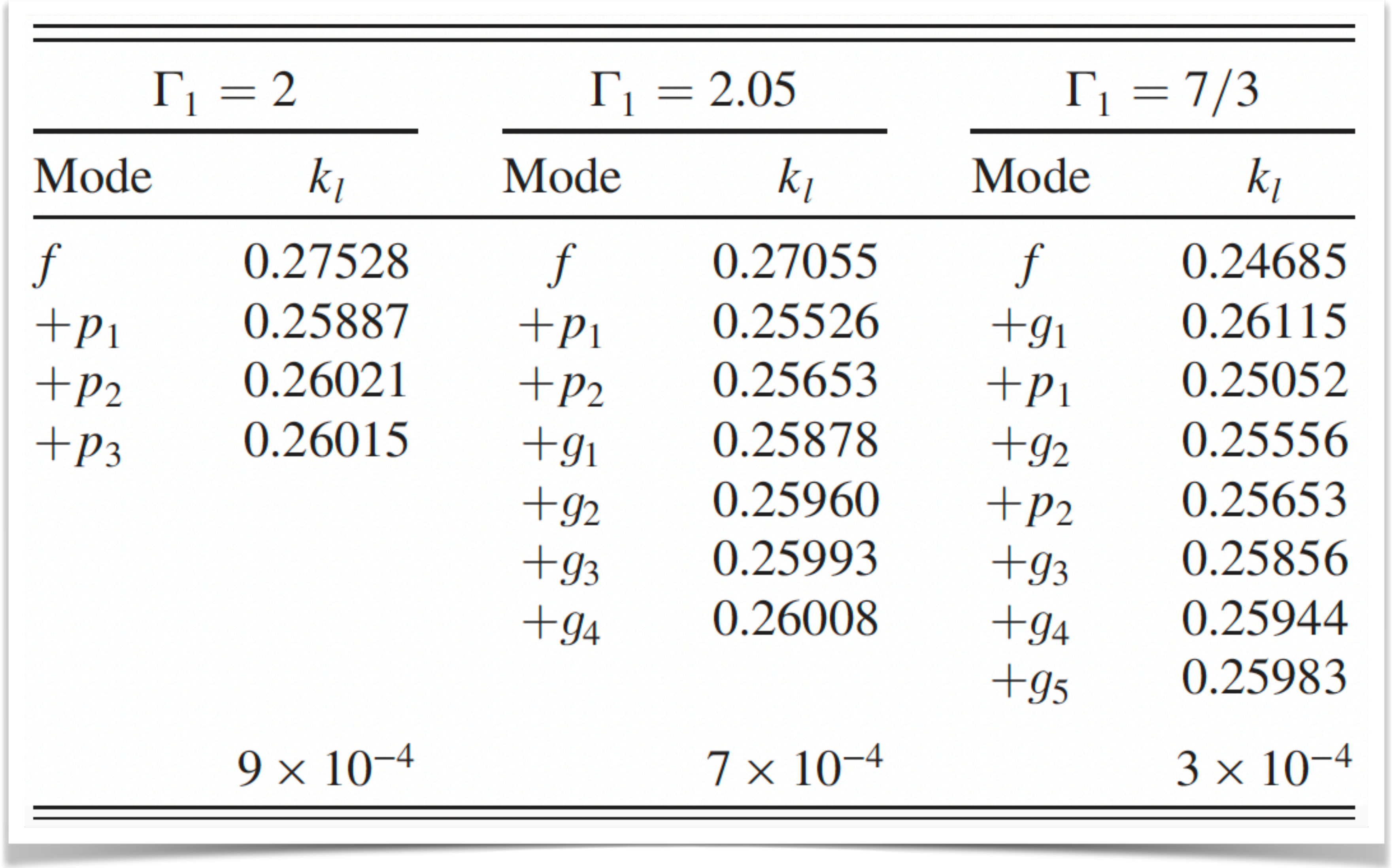
$$\frac{\Delta\Psi_\alpha}{2\pi} \approx -\frac{t_\mathrm{D}}{t_\mathrm{orb}}\frac{\Delta E_\alpha}{|E_\mathrm{orb}|} \propto \left(\frac{Q_\alpha}{\omega_\alpha}\right)^2 .$$

- The impact of a resonance on the phase strongly depends on the *overlap* Q_α of the mode with the tidal potential,

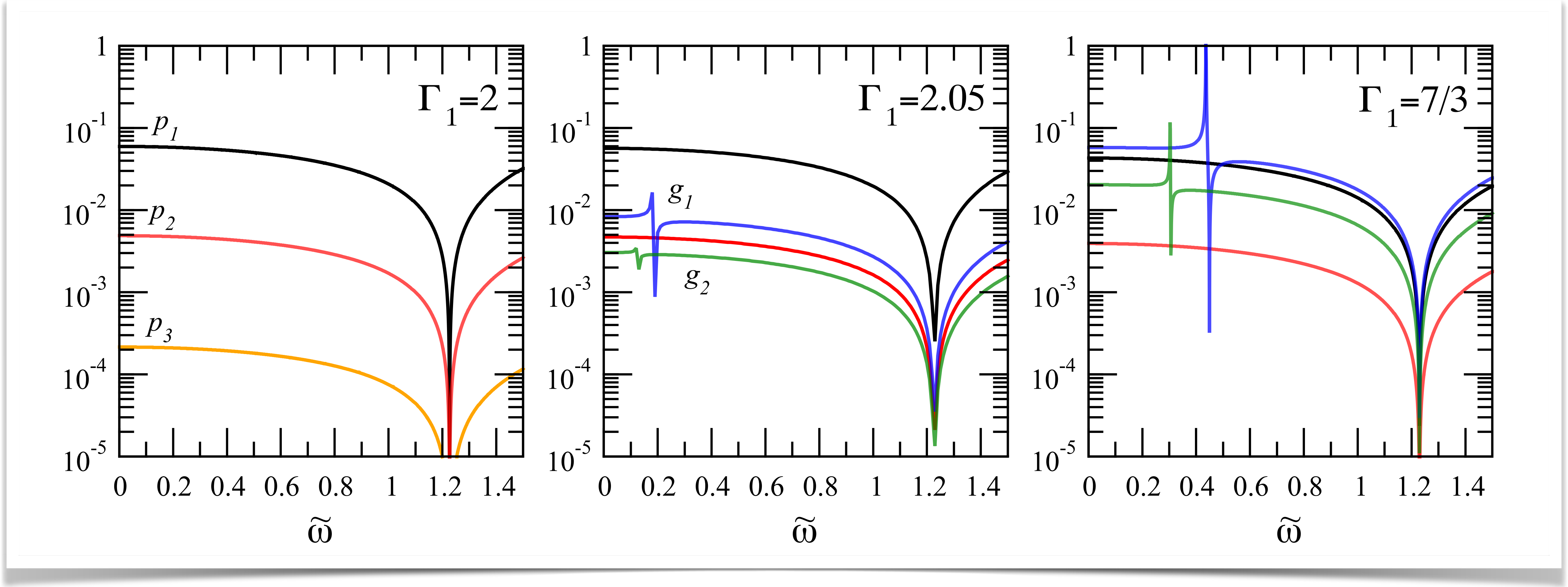
$$Q_\alpha = -\int \delta\rho_\alpha^* \chi \, dV .$$

mode-sum: application

- We expect the dynamical tide to be dominated by the *f*-mode, but it may be possible to see resonances during the inspiral.



$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
Mode	k_l	Mode	k_l	Mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
	9×10^{-4}		7×10^{-4}		3×10^{-4}



Relative contributions to the tidal Love number k_2 compared to the *f*-mode.

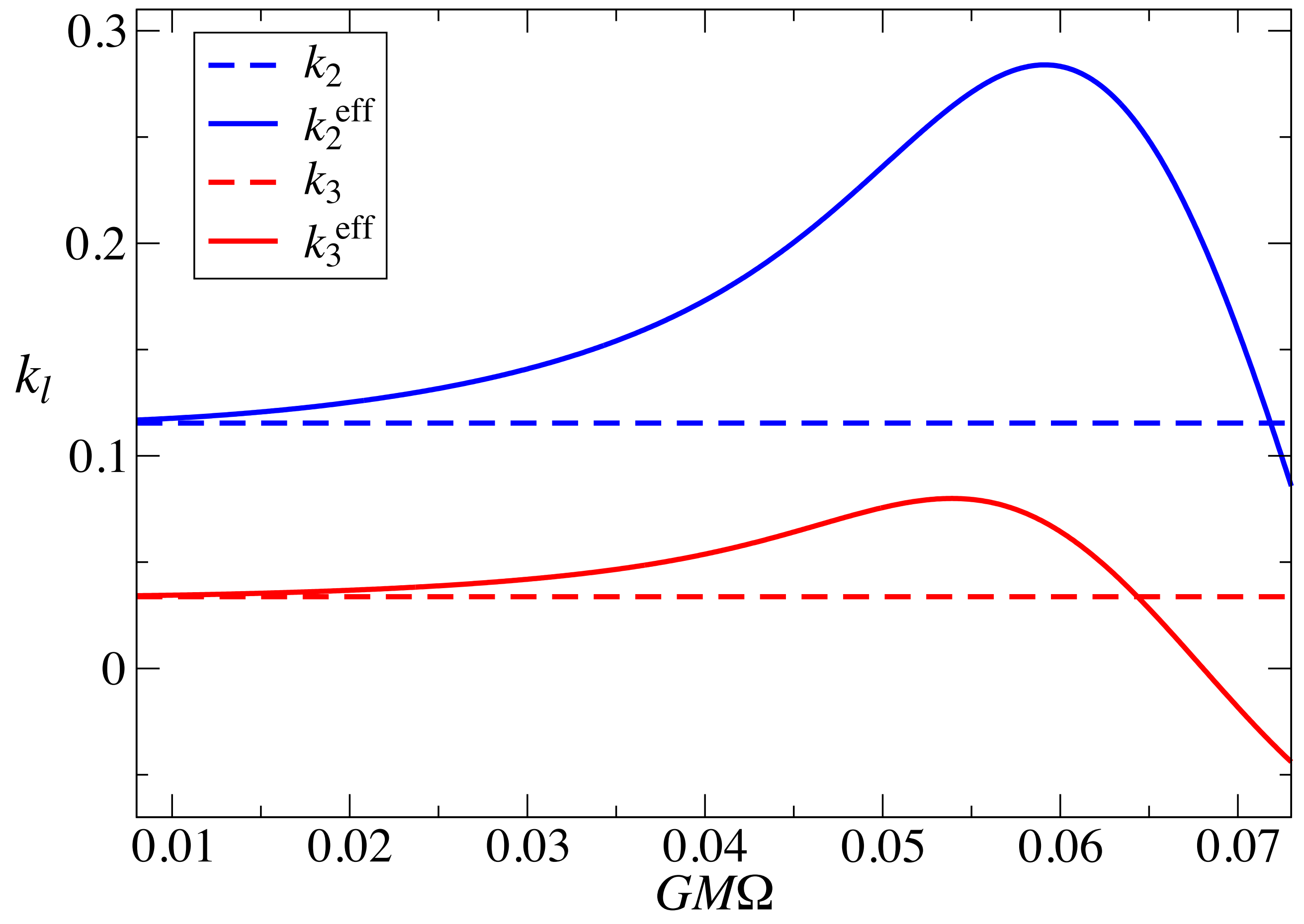
[Andersson+Pnigouras, Phys. Rev. D **101**, 083001 (2020)]

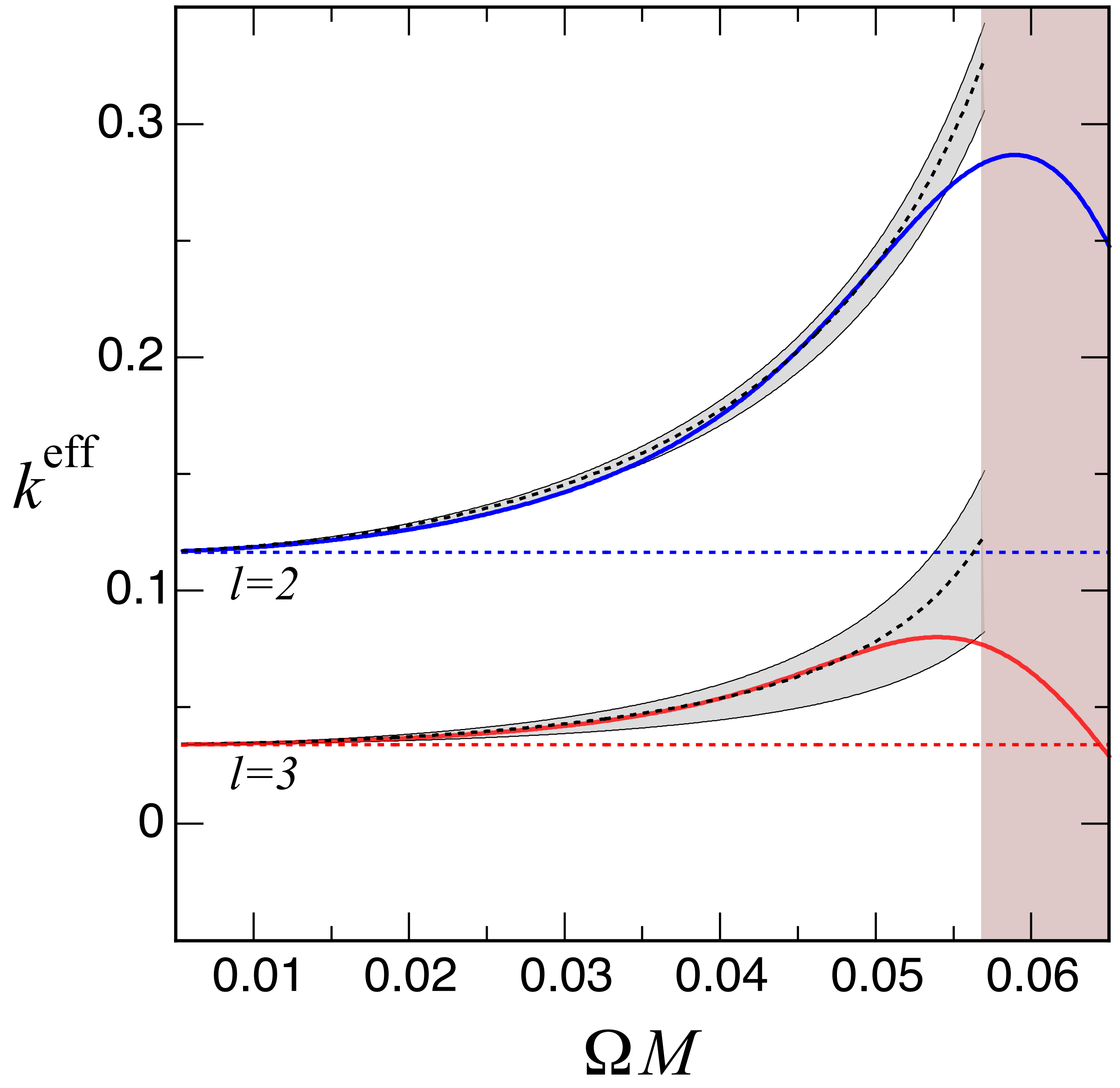
f-mode: approximation

- There has been some work in representing the dynamical tide using just the contribution from the *f*-mode.

  (i) Effective approach: generalising the Newtonian action for the orbital dynamics to relativity in the time domain [Steinhoff+, Phys. Rev. D **94**, 104028 (2016)] and frequency domain [Schmidt+Hinderer, Phys. Rev. D **100**, 021501 (2019)].

  (ii) Phenomenological approach [Andersson+Pnigouras, Mon. Not. R. Astron. Soc. **503**, 533 (2021)].



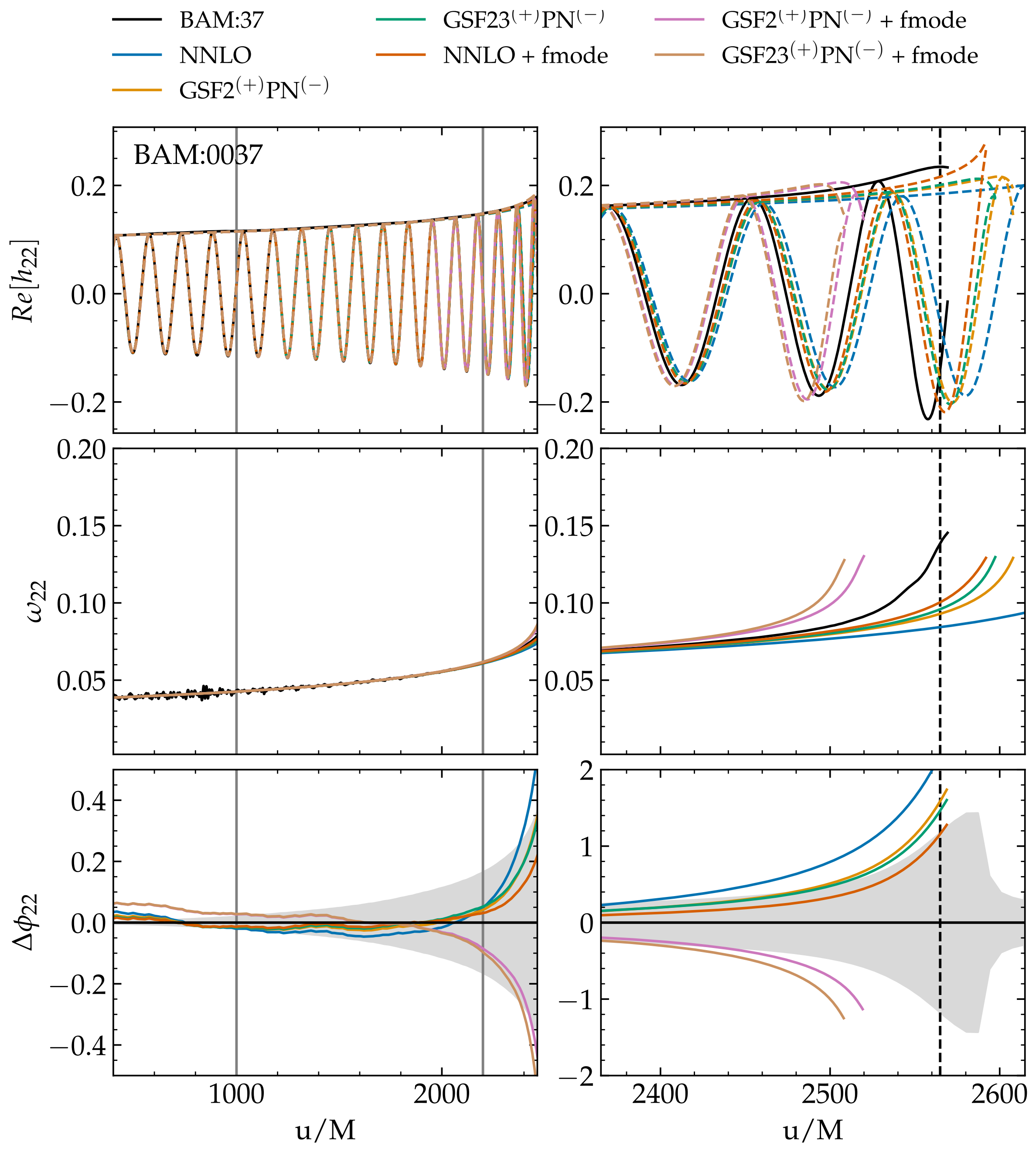


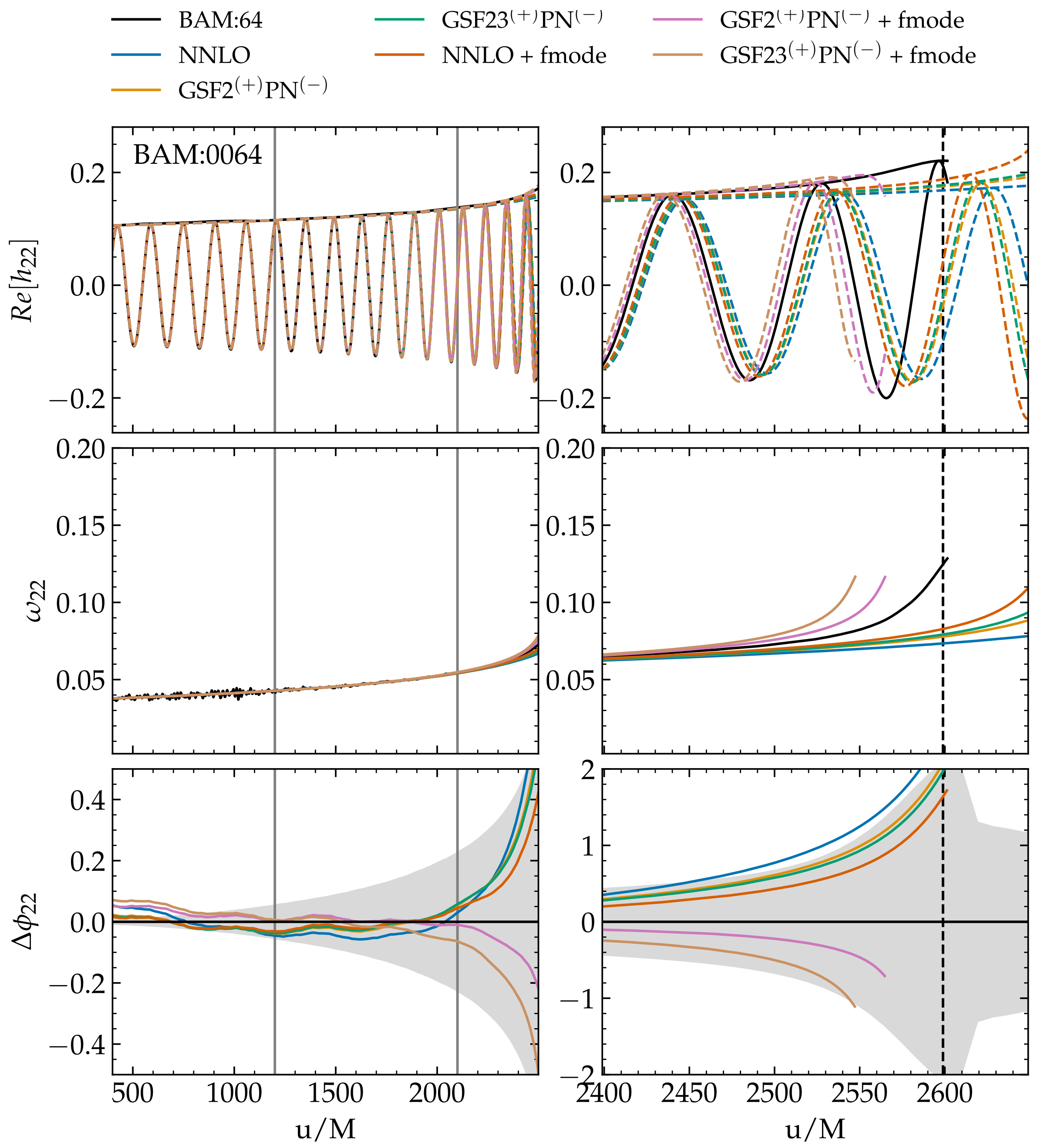
f-mode: results

- The effective approach has been used to constrain the $l = 2, 3$ f-mode frequencies from the larger component of GW170817 [Pratten+, Nat. Commun. **11**, 2553 (2020)],

$$\omega_{f,2}/(2\pi) \geq 1.39\,\mathrm{kHz}, \qquad \omega_{f,3}/(2\pi) \geq 1.86\,\mathrm{kHz}\,.$$

- However, while these approaches are improved compared to the static tide, they do not entirely match results from numerical simulations [Gamba+Bernuzzi, Phys. Rev. D **107**, 044014 (2023)].





state of play

		Newtonian gravity	general relativity	notes
static tide	non-rotating stars	✓	✓ [Hinderer (2008); Binnington+Poisson (2009); Damour+Nagar (2009)]	Relativistic neutron-star models with elastic crusts [Gittins+ (2020)] and superfluidity [Yeung+ (2021)].
	rotating stars		✓ [Landry+Poisson (2015); Landry (2015); Pani+ (2015a,b)]	Calculations are at the level of slowly rotating fluid bodies.
dynamical tide	non-rotating stars	✓ [Lai (1994); Andersson+Pnigouras (2020)]	[Steinhoff+ (2016); Schmidt+Hinderer (2016); Pitre+Poisson (2024); Hegade K. R.+ (2024)] • How to treat a dynamical tidal field? • The (quasi-normal) modes are incomplete. • Can we go beyond just the *f*-mode?	Newtonian neutron-star models with elastic crusts and superfluidity [Passamonti+ (2021)].
	rotating stars	✓ [Ho+Lai (1999); Pnigouras+ (2024)]		Planetary studies [Lai (2021); Dewberry+Lai (2021)].

g-modes: origins

- Not a new idea [Cowling, Mon. Not. R. Astron. Soc. **101**, 367 (1941)].
- Start with the first law of thermodynamics,

$$dE = T\,dS - p\,dV + \sum_{\mathrm{x}} \mu_{\mathrm{x}}\,dN_{\mathrm{x}}\,.$$

- Assuming cold ($T = 0$), electrically neutral ($N_{\mathrm{p}} = N_{\mathrm{e}}$), pure npe-matter,

$$d\varepsilon = \frac{\varepsilon + p}{n_{\mathrm{b}}}\,dn_{\mathrm{b}} + n_{\mathrm{b}}\mu_{\Delta}\,dY_{\mathrm{e}} \qquad \Longrightarrow \qquad \varepsilon = \varepsilon(n_{\mathrm{b}}, Y_{\mathrm{e}}),$$

  where $\mu_{\Delta} = \mu_{\mathrm{p}} + \mu_{\mathrm{e}} - \mu_{\mathrm{n}}$ encodes the deviation from chemical equilibrium and $Y_{\mathrm{e}} = N_{\mathrm{e}}/N_{\mathrm{b}}$.

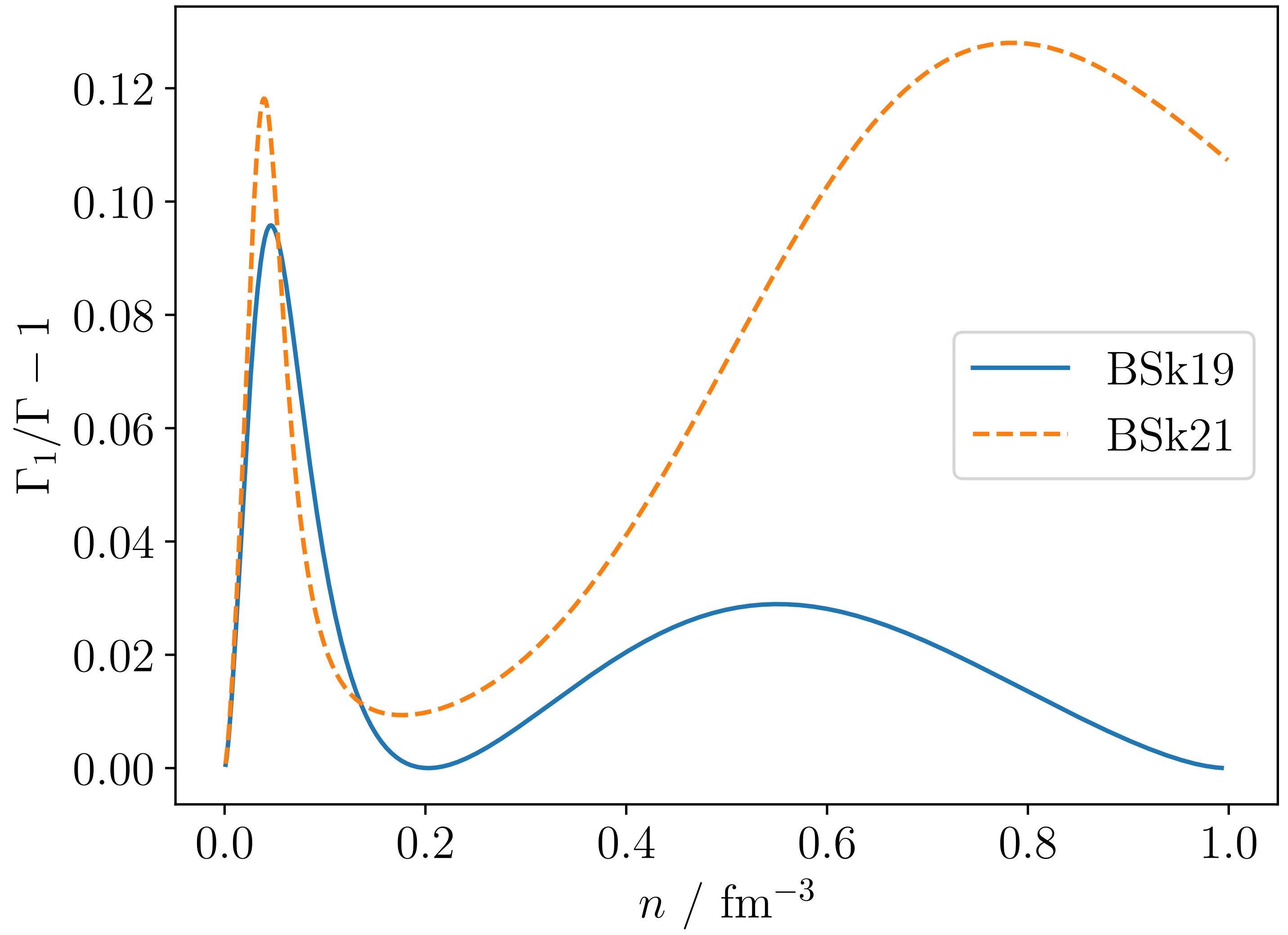
- When the fluid maintains equilibrium $\mu_{\Delta} = 0$ through an oscillation, the equation of state is *barotropic* $\varepsilon = \varepsilon(n_{\mathrm{b}})$ and there are no g-modes.

g-modes: realistic composition

- In a neutron star, the weak-interaction timescale is much longer than the characteristic oscillation period. Thus, the fluid does not maintain β equilibrium as it pulsates, giving rise to *g*-modes.

- Hence, *g*-modes contain information about the chemical composition.

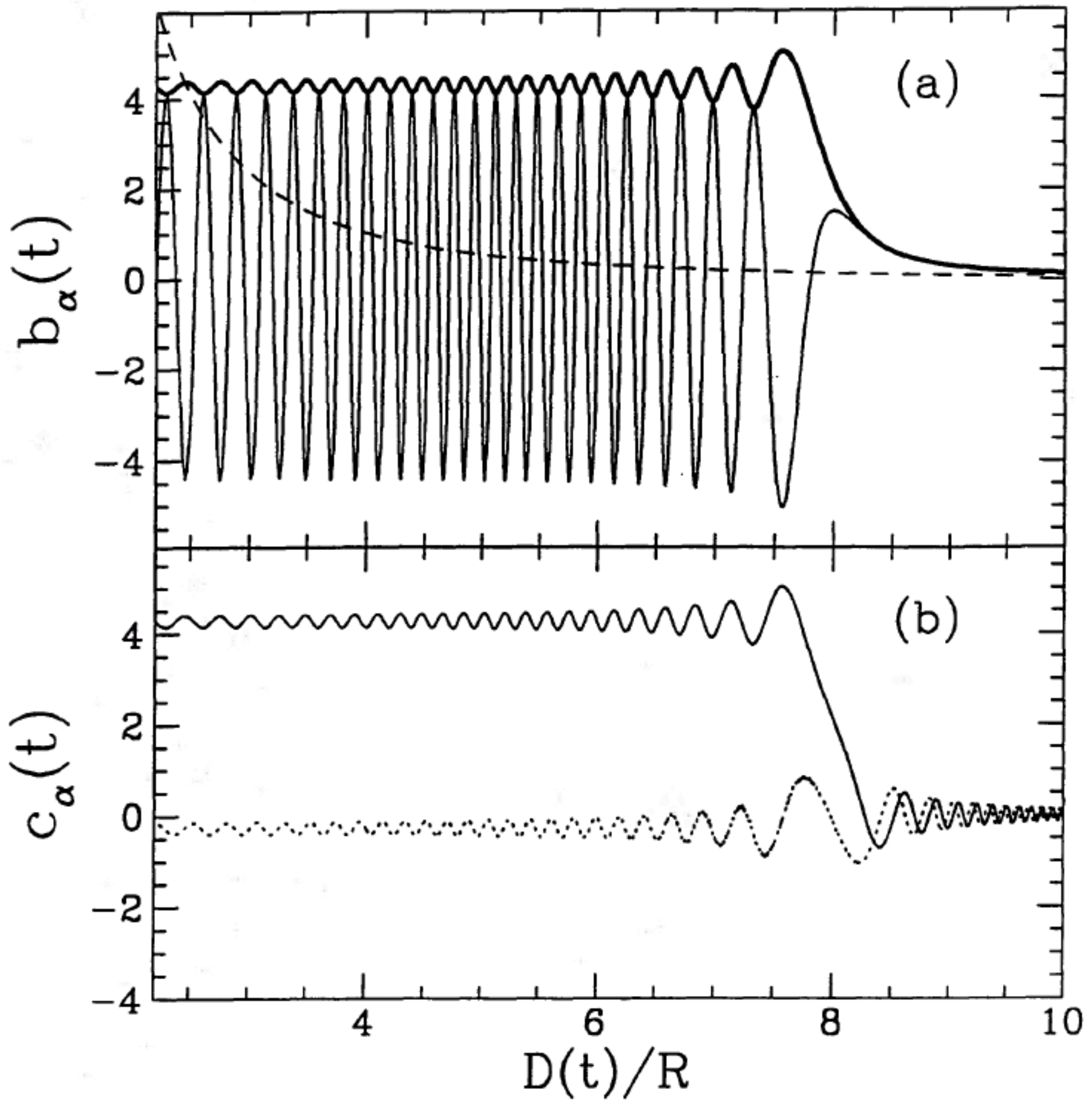
- The *g*-modes are sensitive to the deviations from chemical equilibrium. This is characterised by the (local) Brunt-Väisälä frequency N,

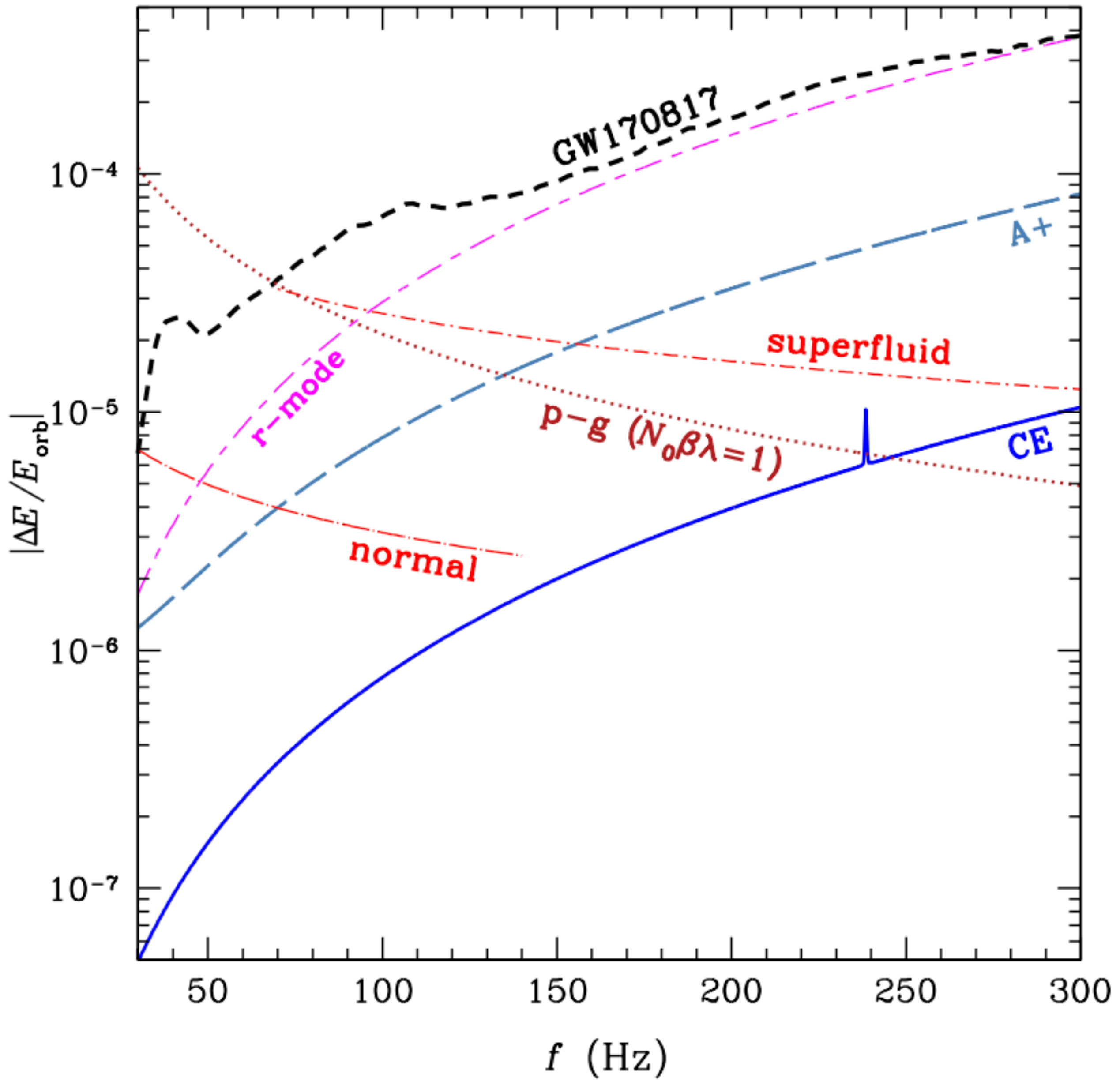


$$N^2 = \frac{\rho g^2}{p}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma_1}\right).$$

[Gittins+Andersson, Mon. Not. R. Astron. Soc. **521**, 3043 (2023)]

g-modes: prospects





- The phase shifts are expected to be very small [Lai, Mon. Not. R. Astron. Soc. **270**, 611 (1994)],

$$\frac{\Delta\Psi_g}{2\pi} \approx -4.3\times10^{-4}\left[\frac{100\,\mathrm{Hz}}{\omega_g/(2\pi)}\right]^2\left(\frac{Q_g}{0.0003}\right)^2 .$$

- But some recent work in light of third-generation detectors are more optimistic [Ho+Andersson, Phys. Rev. D **108**, 061104 (2023)].

- Even without direct measurements of the *g*-modes, the sensitivity improvements will place constraints on the nuclear matter.

beyond Newton

- In general relativity, all motion is dissipative due to gravitational radiation,

$$\mathbf{f}_{\mathrm{GW}} = -\frac{2G}{5c^5}\rho\frac{d^5\mathbf{Q}}{dt^5}\cdot\mathbf{x} \qquad \rightarrow \qquad \rho\frac{d\mathbf{v}}{dt} = -\nabla p - \rho\nabla\Phi + \mathbf{f}_{\mathrm{GW}},$$

$$\Longrightarrow \frac{dE}{dt} = \int \mathbf{v}\cdot\mathbf{f}_{\mathrm{GW}}\,dV \neq 0.$$

- This is formally a 2.5PN feature and inevitably spoils the completeness of the modes.

- In the hope of doing (at the very least) better than Newtonian models, we are exploring whether progress can be made in PN theory [Andersson+ (2023); Andersson+Gittins (in prep.)].

- Ultimately, we will need calculations in full general relativity to describe neutron stars.

summary

- Gravitational waves provide the exciting opportunity to probe the behaviour of ultra-dense nuclear matter through observations of neutron stars.

what we know

- We understand the static tide well and are able to obtain constraints on the equation of state.
- Third-generation observatories will have enhanced sensitivities to the dynamical tide…
- …and neglecting these effects will lead to systematics.

what we need to know

- Formulate the dynamical tide in full general relativity.
- Incorporate oscillation modes beyond the *f*-mode.
- What the dynamical tide will teach us about nuclear matter.