Constraining dense nuclear matter with gravitational waves


[credit: MIT]

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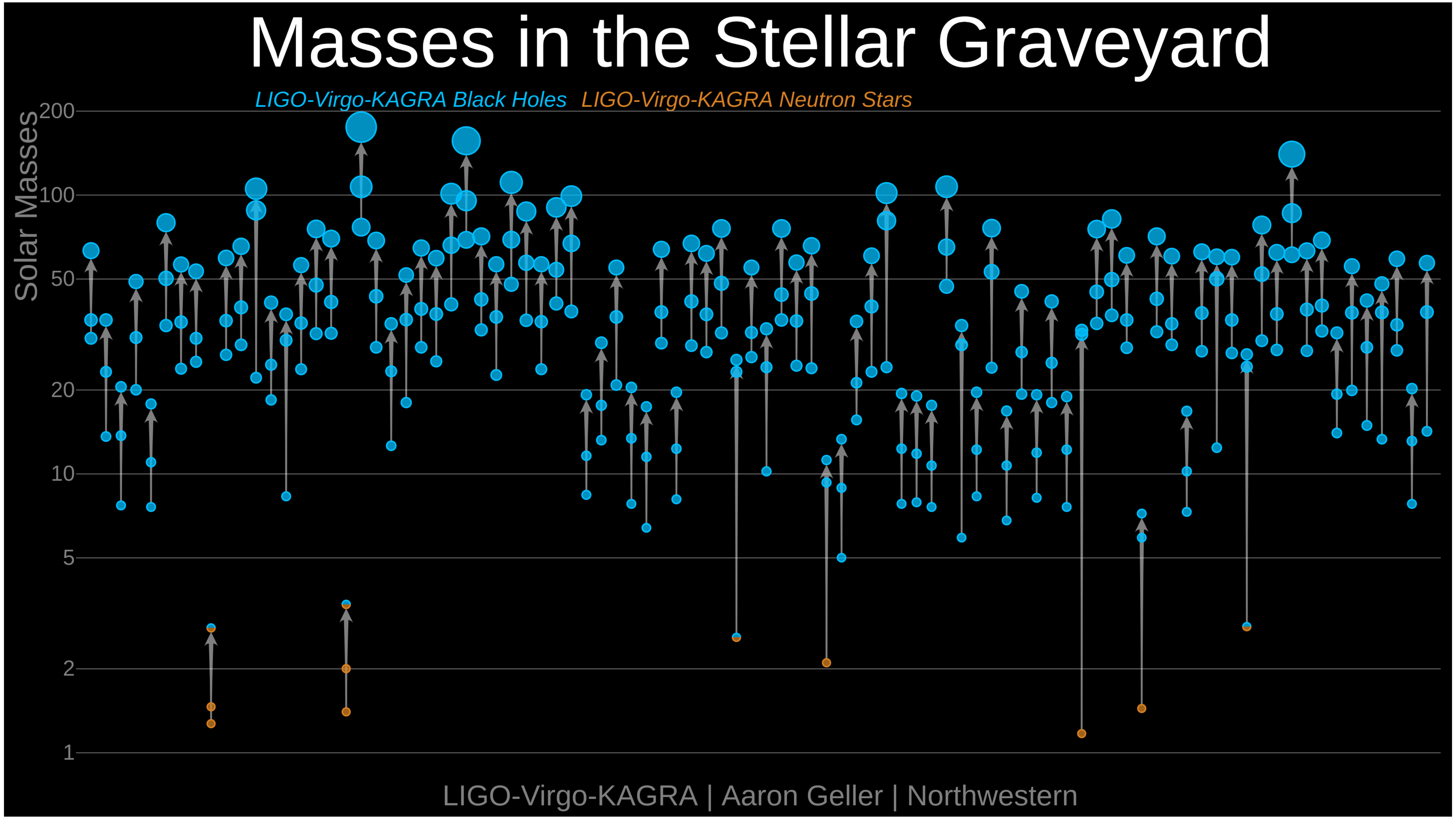
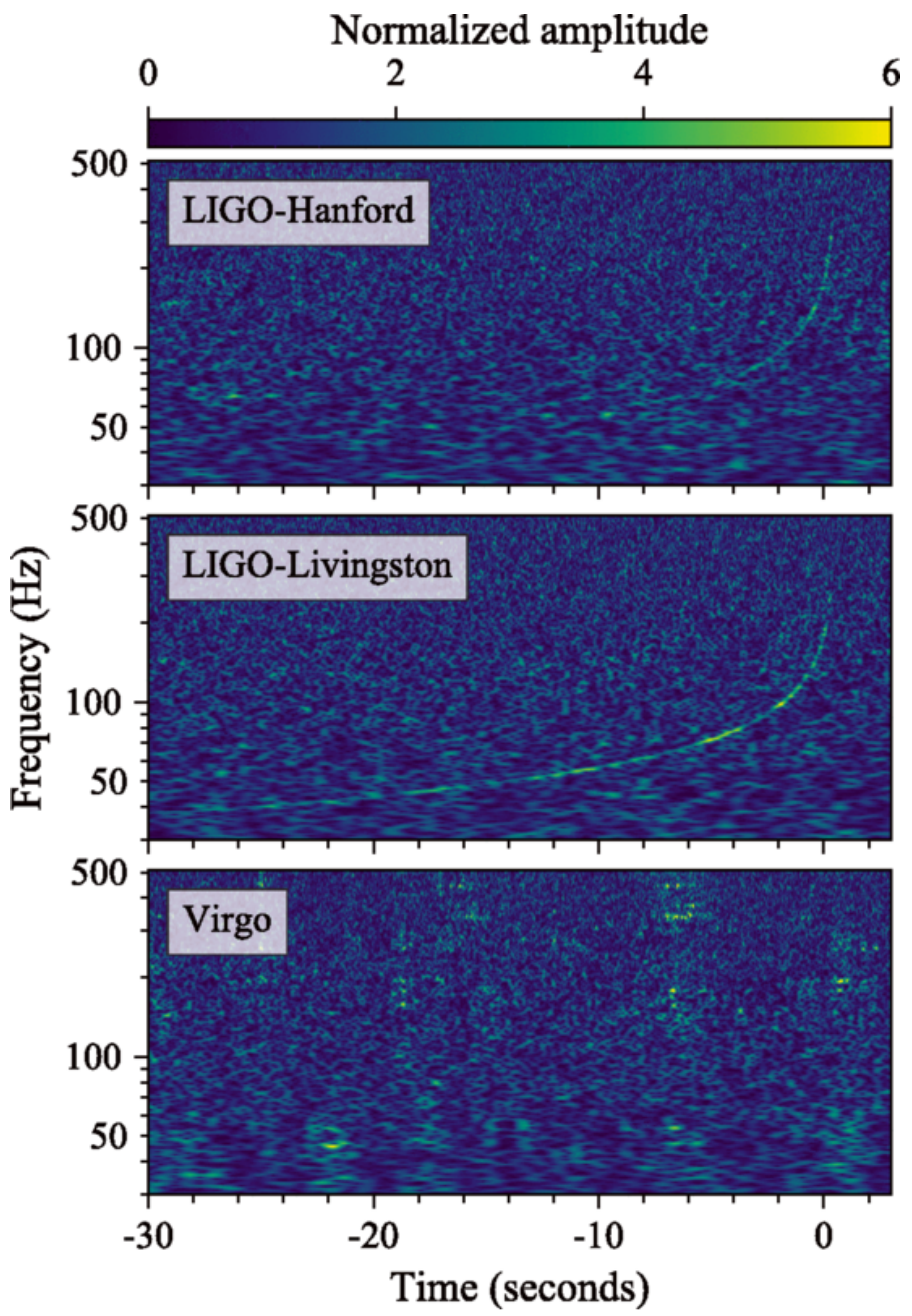
gravitational waves: observations

- Since 2015, gravitational-wave instruments have witnessed over 90 compact-binary coalescences.



GW170817

- On 17 Aug. 2017, gravitational-wave instruments detected the first neutron-star merger.



	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

[Abbott+ 2017, *Phys. Rev. Lett.* **119**, 161101]

science potential of neutron-star binaries

1. Cosmology

LETTER

doi:10.1038/nature24471

A gravitational-wave standard siren measurement of the Hubble constant

The LIGO Scientific Collaboration and The Virgo Collaboration*, The 1M2H Collaboration*, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration*, The DLT40 Collaboration*, The Las Cumbres Observatory Collaboration*, The VINROUGE Collaboration* & The MASTER Collaboration*

2. Nuclear physics

PHYSICAL REVIEW LETTERS 121, 161101 (2018)

Editors' Suggestion

GW170817: Measurements of Neutron Star Radii and Equation of State

B. P. Abbott *et al.**
(The LIGO Scientific Collaboration and the Virgo Collaboration)

(Received 5 June 2018; revised manuscript received 25 July 2018; published 15 October 2018)

3. Astrophysics

The Astrophysical Journal Letters, 850:L40 (18pp), 2017 December 1
https://doi.org/10.3847/2041-8213/aa93fc

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On the Progenitor of Binary Neutron Star Merger GW170817

LIGO Scientific Collaboration and Virgo Collaboration
(See the end matter for the full list of authors.)
Received 2017 October 12; revised 2017 October 16; accepted 2017 October 16; published 2017 December 1

4. Testing general relativity

PHYSICAL REVIEW LETTERS 123, 011102 (2019)

Editors' Suggestion

Tests of General Relativity with GW170817

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 20 November 2018; revised manuscript received 21 March 2019; published 1 July 2019; corrected 20 August 2019)

5. Multi-messenger astronomy

The Astrophysical Journal Letters, 848:L12 (59pp), 2017 October 20
https://doi.org/10.3847/2041-8213/aa91c9

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Multi-messenger Observations of a Binary Neutron Star Merger

LIGO Scientific Collaboration and Virgo Collaboration, Fermi GBM, INTEGRAL, IceCube Collaboration, AstroSat Cadmium Zinc Telluride Imager Team, IPN Collaboration, The Insight-Hxmt Collaboration, ANTARES Collaboration, The Swift Collaboration, AGILE Team, The 1M2H Team, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, GRAWITA: GRAvitational Wave Inaf TeAm, The Fermi Large Area Telescope Collaboration, ATCA: Australia Telescope Compact Array, ASKAP: Australian SKA Pathfinder, Las Cumbres Observatory Group, OzGrav, DWF (Deeper, Wider, Faster Program), AST3, and CAASTRO Collaborations, The VINROUGE Collaboration, MASTER Collaboration, J-GEM, GROWTH, JAGWAR, Caltech-NRAO, TTU-NRAO, and NuSTAR Collaborations, Pan-STARRS, The MAXI Team, TZAC Consortium, KU Collaboration, Nordic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS: Transient Robotic Observatory of the South Collaboration, The BOOTES Collaboration, MWA: Murchison Widefield Array, The CALET Collaboration, IKI-GW Follow-up Collaboration, H.E.S.S. Collaboration, LOFAR Collaboration, LWA: Long Wavelength Array, HAWC Collaboration, The Pierre Auger Collaboration, ALMA Collaboration, Euro VLBI Team, Pi of the Sky Collaboration, The Chandra Team at McGill University, DFN: Desert Fireball Network, ATLAS, High Time Resolution Universe Survey, RIMAS and RATIR, and SKA South Africa/MeerKAT
(See the end matter for the full list of authors.)

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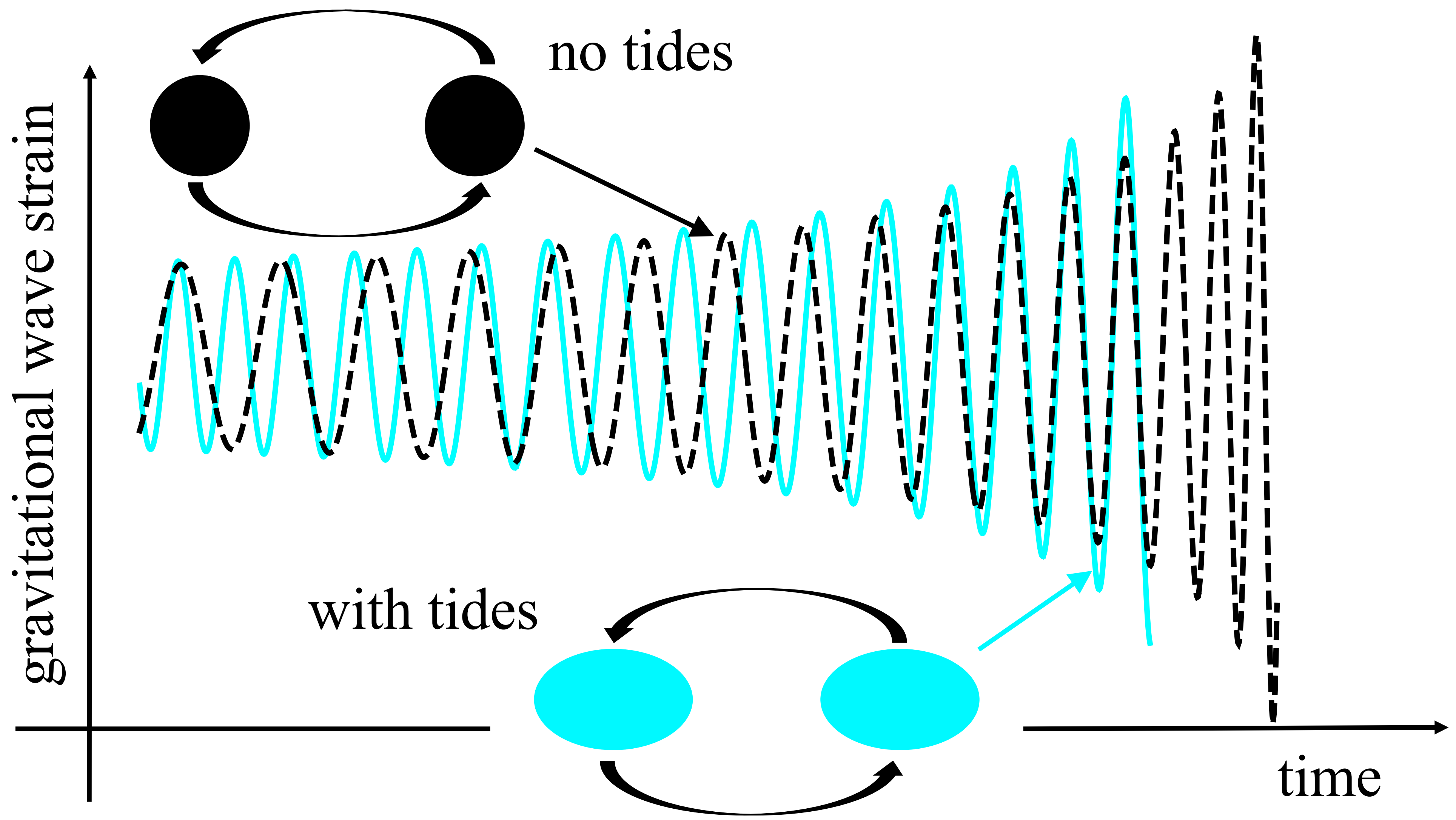
neutron-star binaries

- The signal emitted from inspiralling neutron stars differs to that of black holes due to the material response to the tidal field.
- These features enter the waveform phase Ψ at 5PN.
- The deformability of the stellar material is characterised by the *tidal Love numbers* k_l, which depend on the state of the nuclear matter.



- We start by assuming that the external field is *static*,

$$m\dot{\Psi} \ll \omega_\alpha.$$

- The tidal Love numbers k_l are defined at the surface of the neutron star $r = R$ by

$$\delta\Phi(R,\theta,\phi) = \sum_{l,m} \delta\Phi_l(R)\, Y_l^m(\theta,\phi) = \sum_{l,m} 2k_l\, \chi_l(R)\, Y_l^m(\theta,\phi).$$

- Therefore, they can be inferred from the behaviour in the exterior,

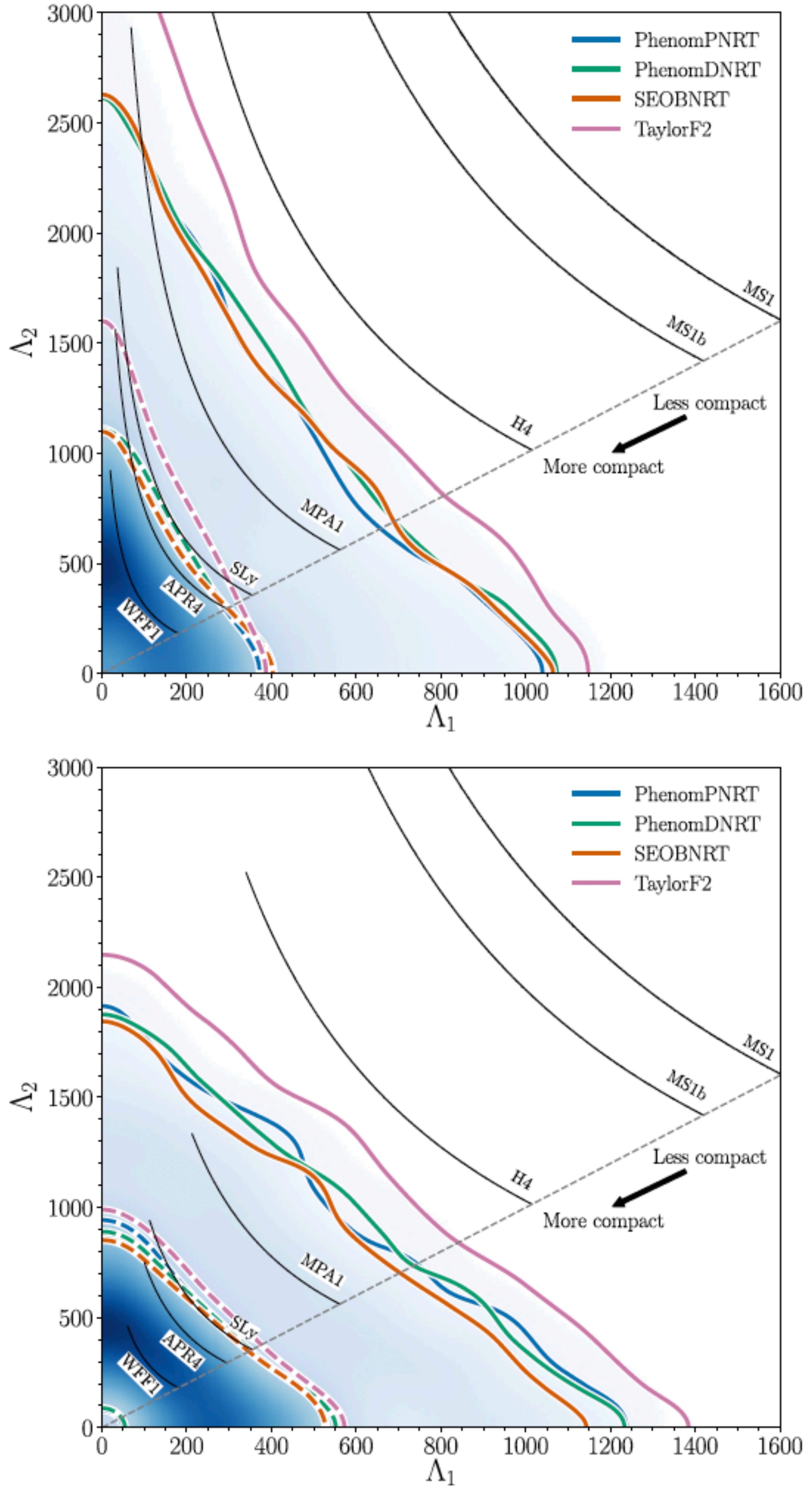
$$U_l \equiv \delta\Phi_l + \chi_l = \left[2k_l \left(\frac{R}{r}\right)^{2l+1} + 1\right] \left(\frac{r}{R}\right)^l \chi_l(R).$$

- This result generalises to relativity, where the potential U is promoted to the (linearised) metric of the spacetime h_{ab}.

state of play

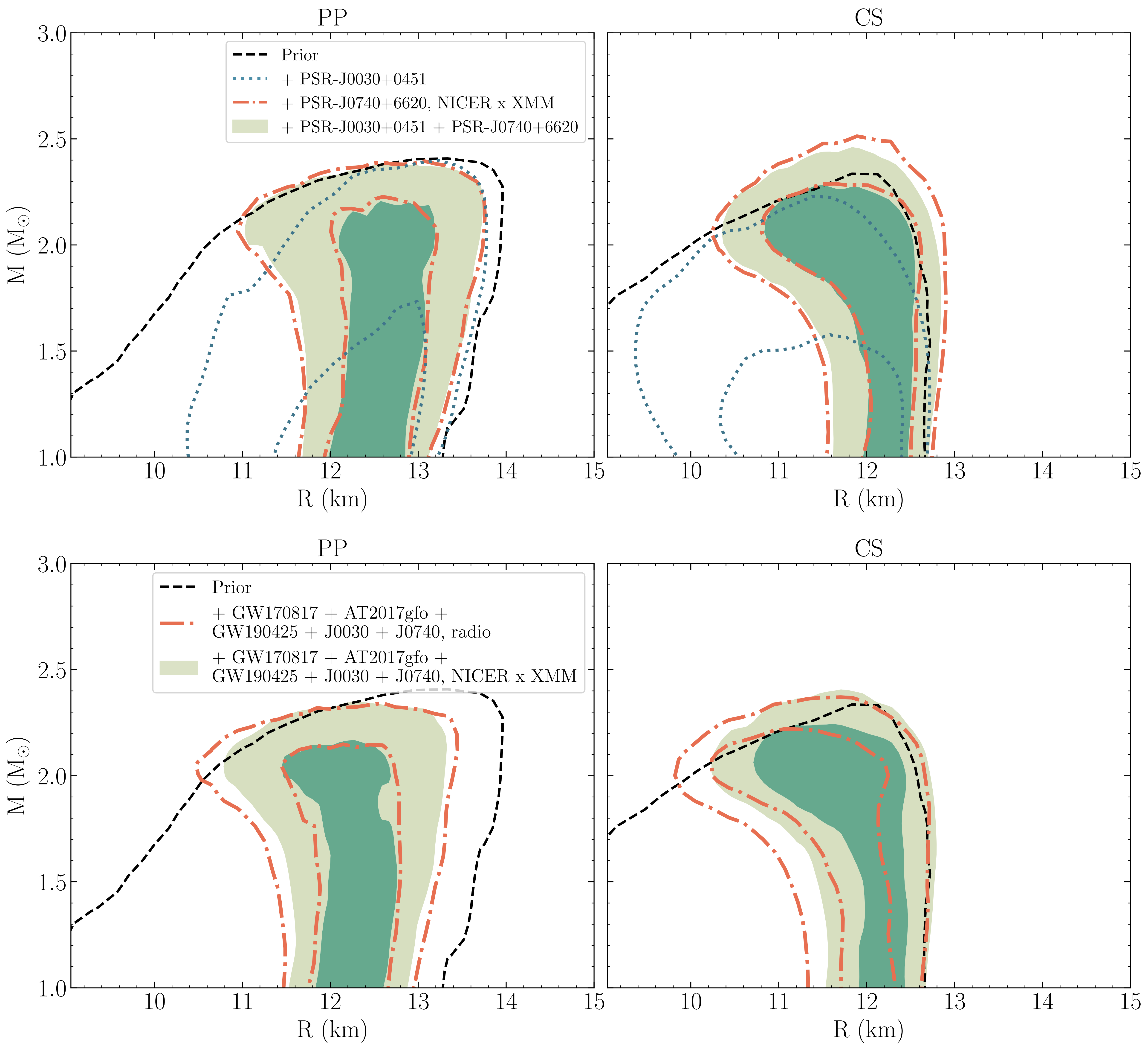
		Newtonian gravity	general relativity	notes
static tide	non-rotating stars	✓	✓ [Hinderer 2008; Binnington+Poisson 2009; Damour+Nagar 2009]	Relativistic neutron-star models with elastic crusts [Gittins+ 2020] and superfluidity [Yeung+ 2021].
	rotating stars		✓ [Landry+Poisson 2015; Landry 2015; Pani+ 2015a,b]	Calculations are at the level of slowly rotating fluid bodies.

equation-of-state constraints



[Abbott+ 2019, *Phys. Rev. X* **9**, 011001]

$$\Lambda_A = \frac{2}{3} k_{2A} \left(\frac{c^2 R_A}{G M_A} \right)^5$$



[Raaijmakers+ 2021, *Astrophys. J.* **918**, L29]

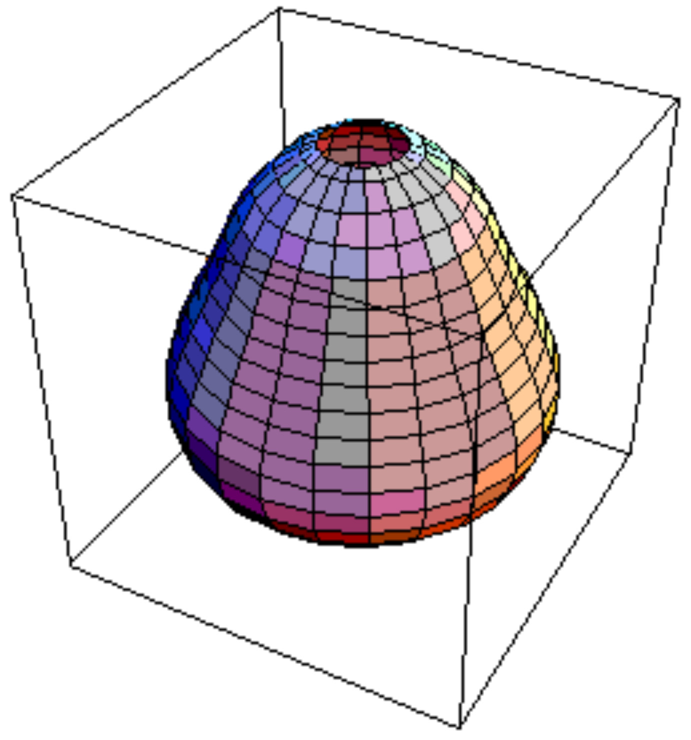
the dynamical tide

- However, the static tide approximation will inevitably break down.
- As the compact objects inspiral, the tidal frequency increases and eventually becomes comparable to the neutron star's natural modes of oscillation,

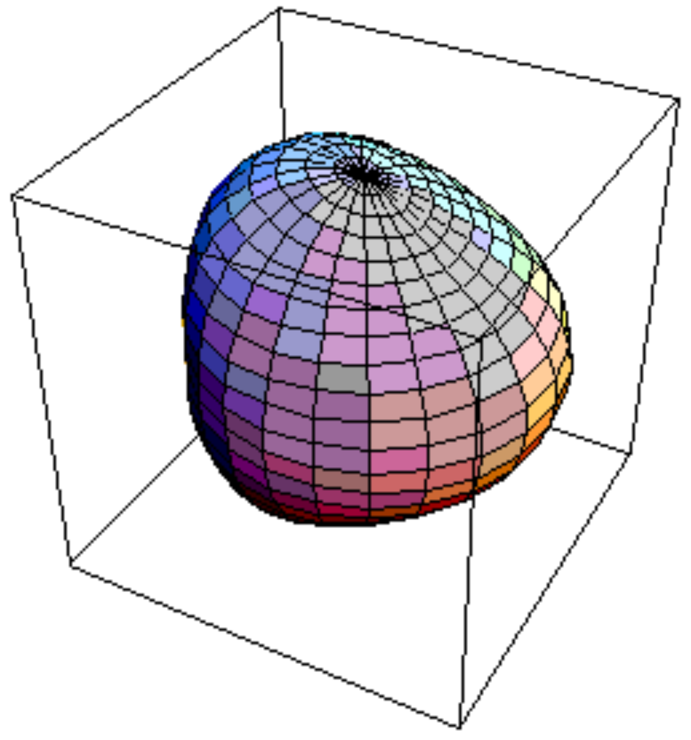
$$m\dot{\Psi} \sim \omega_\alpha .$$

- This regime is known as the *dynamical tide* and it has the exciting potential to probe the oscillation spectrum.

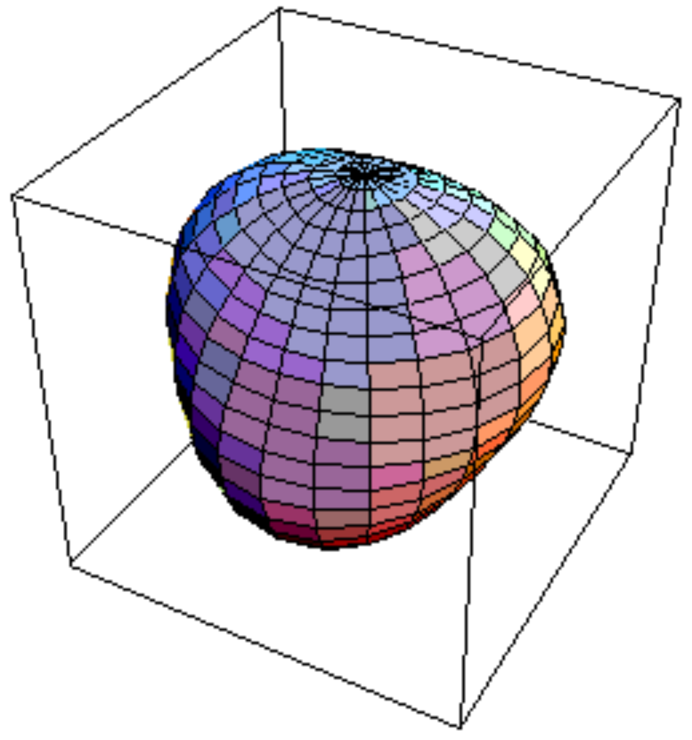
[credit: D. Guenther]



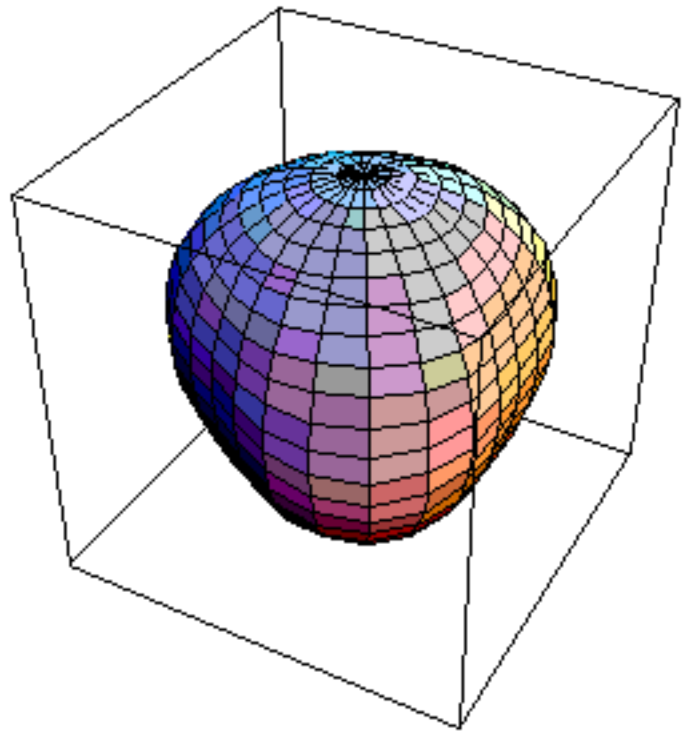
$(l, m) = (3,0)$



$(l, m) = (3,1)$



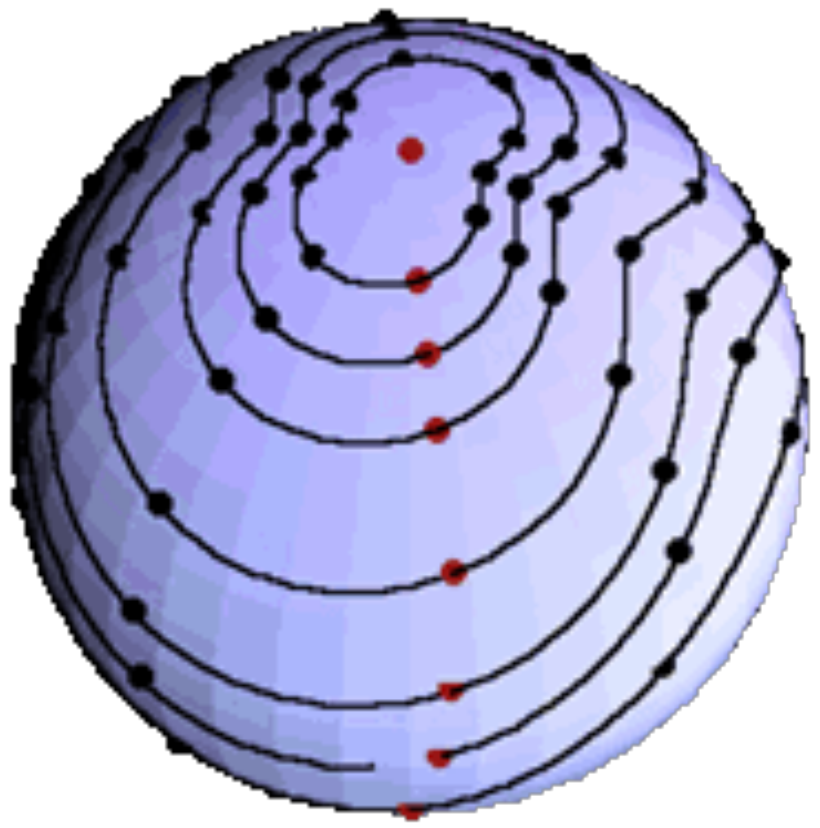
$(l, m) = (3,2)$



$(l, m) = (3,3)$

(some of) the modes

- *f*-modes: Fundamental oscillations of the star; scale with the average density, $\omega_\alpha/(2\pi) \sim \sqrt{GM/R^3} \sim 1\,\mathrm{kHz}$.
- *g*-modes: Restored by buoyancy that arises from composition gradients; $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.
- inertial modes (including the *r*-mode): Restored by rotation; primarily excited by the gravitomagnetic tide (a relativistic effect) [Flanagan+Racine 2007]; $\omega_\alpha \sim \Omega$.
- *i*-modes: Oscillations that arise due to the core-crust interface; possible association with short gamma-ray bursts [Tsang+ 2012]; $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.
- The natural oscillation modes depend on the nuclear-matter equation of state.



[credit: C. Hanna+B. Owen]

the mode-sum

- The normal modes form a complete basis [Chandrasekhar 1964, *Astrophys. J.* **139**, 664], such that the tidal response of the star can be decomposed as

$$\boldsymbol{\xi}(t, \mathbf{x}) = \sum_{\alpha} q_\alpha(t)\, \boldsymbol{\xi}_\alpha(\mathbf{x}) \,.$$

- Thus, the equation of motion becomes that of a driven harmonic oscillator,

$$\frac{d^2 q_\alpha}{dt^2} + \omega_\alpha^2 \, q_\alpha = Q_\alpha \propto e^{-im\Psi} \,.$$

- At resonance $m\dot{\Psi} = \omega_\alpha$, the mode will become excited and extract energy from the orbit. This will change the phase by

$$\frac{\Delta\Psi_\alpha}{2\pi} \approx -\frac{t_\mathrm{D}}{t_\mathrm{orb}} \frac{\Delta E_\alpha}{|E_\mathrm{orb}|} \propto \left(\frac{Q_\alpha}{\omega_\alpha}\right)^2 .$$

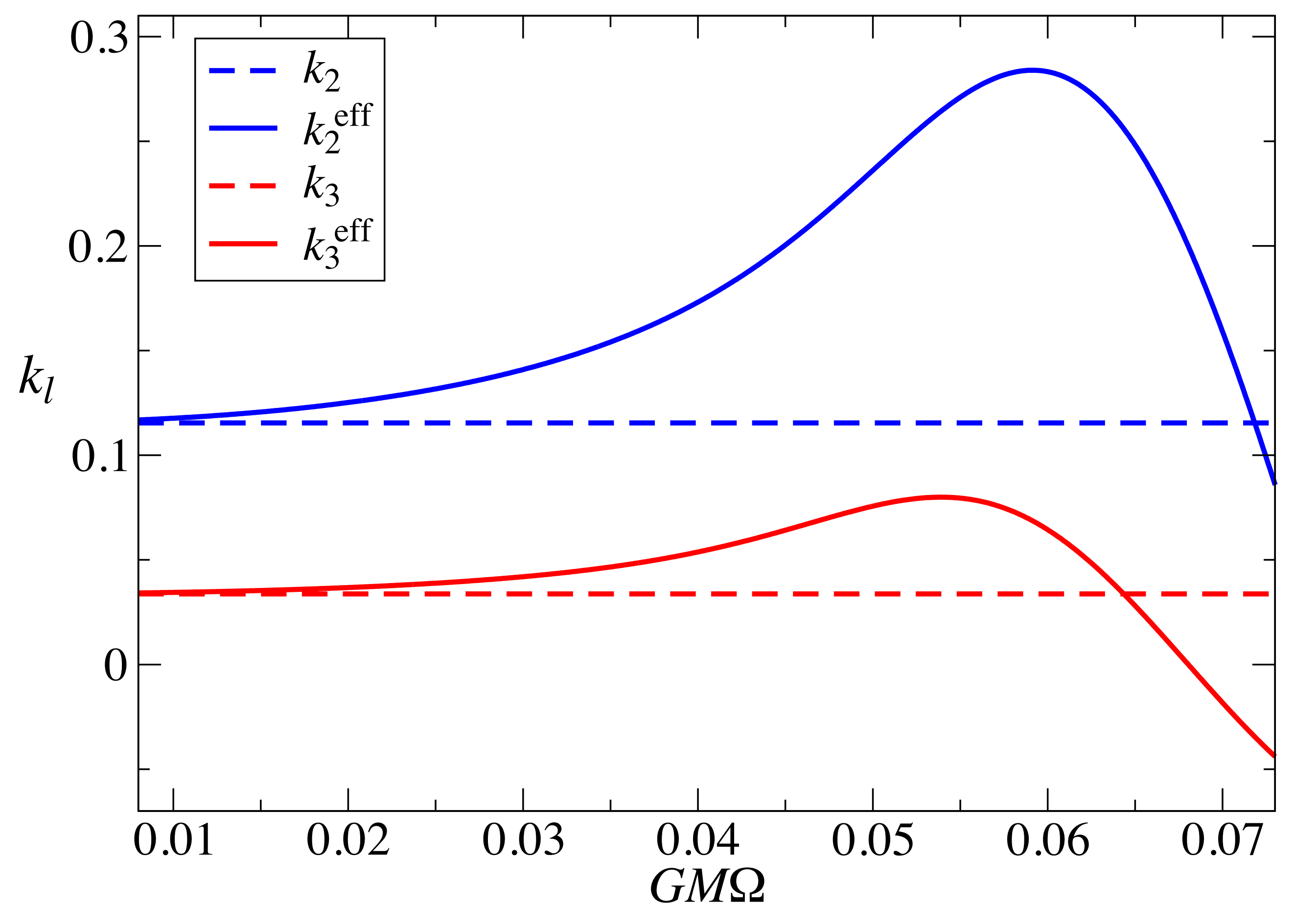
the f-mode approximation

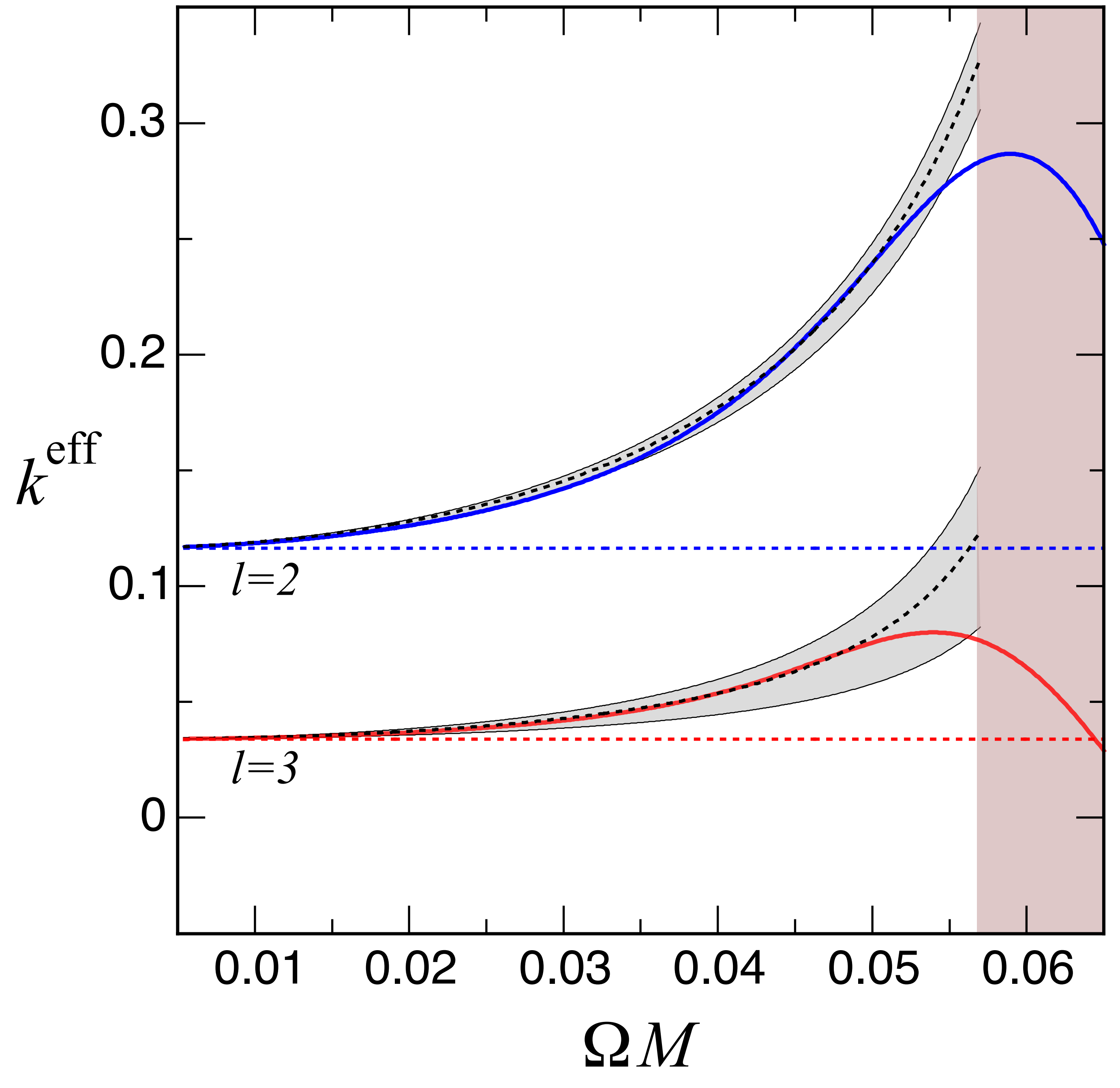
- There has been some work in representing the dynamical tide using just the f-mode.

  (i) Effective approach: generalising the Newtonian action for the orbital dynamics to relativity [Steinhoff+ 2016, *Phys. Rev. D* **94**, 104028; Schmidt+Hinderer 2019, *Phys. Rev. D* **100**, 021501].

  (ii) Phenomenological approach [Andersson+Pnigouras 2021, *Mon. Not. R. Astron. Soc.* **503**, 533].

- However, it seems they do not match results from numerical simulations [Gamba+Bernuzzi 2023, *Phys. Rev. D* **107**, 044014].





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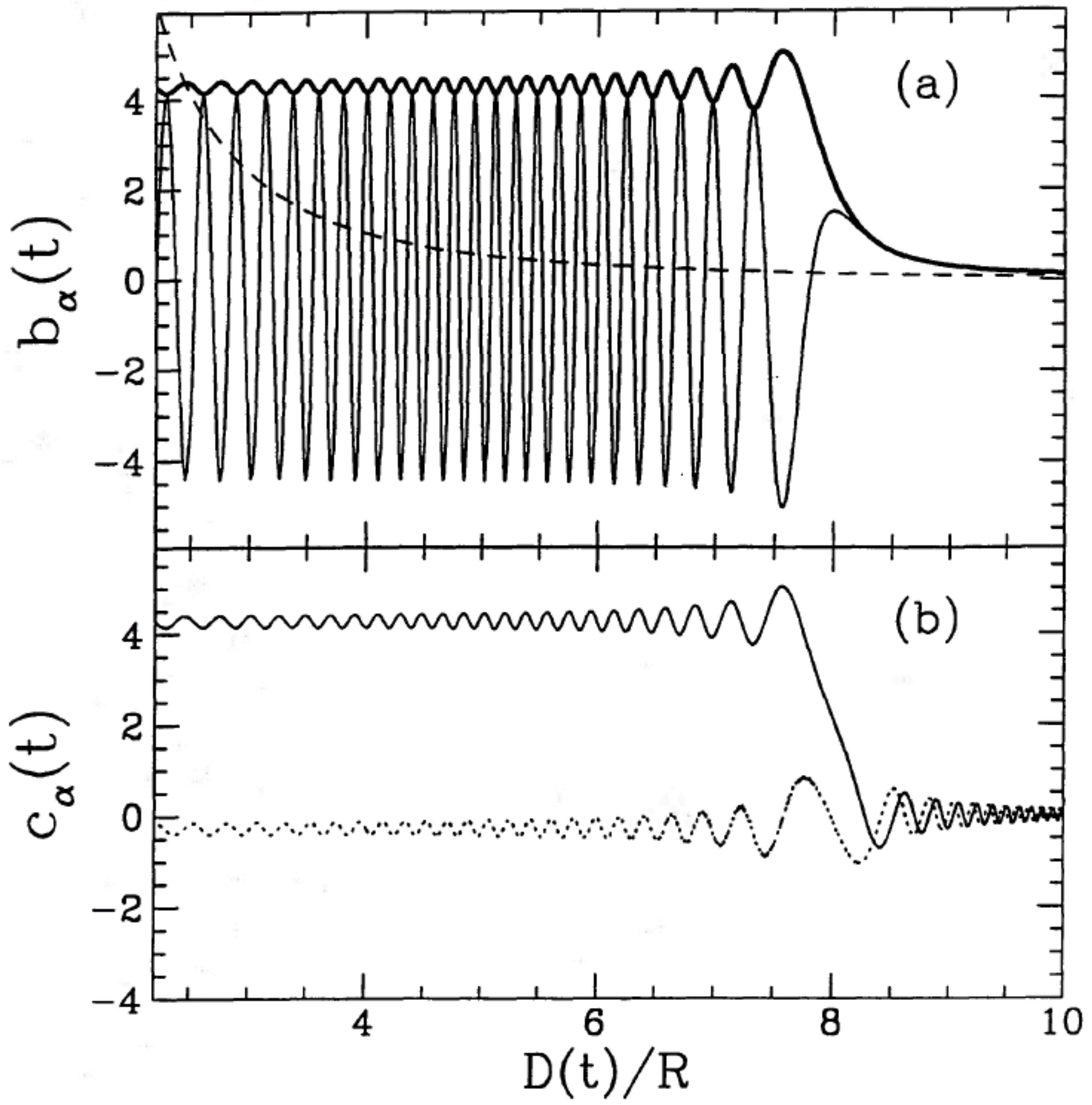
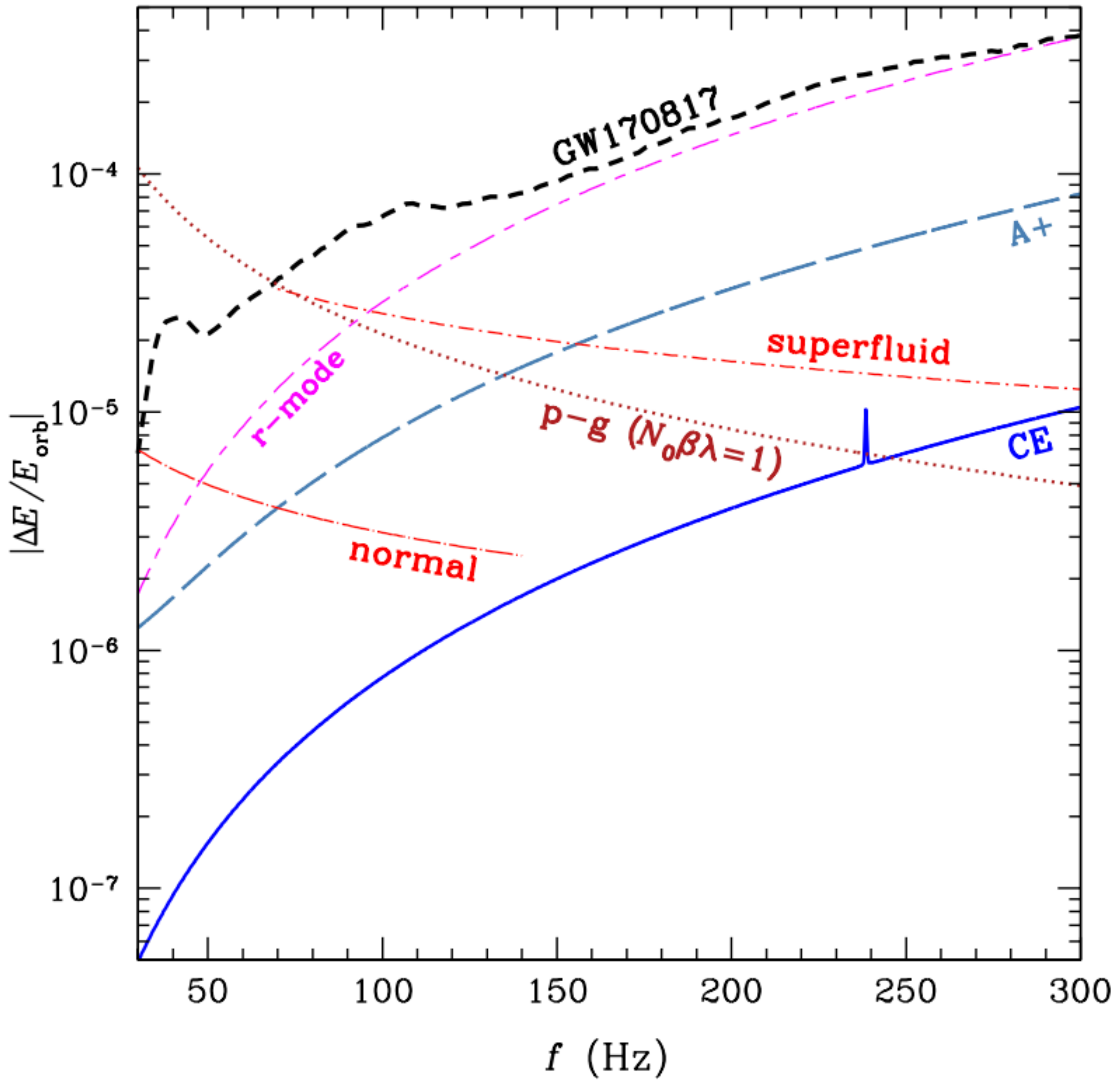
	rotating stars		✓ [Landry+Poisson 2015; Landry 2015; Pani+ 2015a,b]	Calculations are at the level of slowly rotating fluid bodies.
dynamical tide	non-rotating stars	✓ [Lai 1994; Andersson+Pnigouras 2020]	• How to treat a dynamical tidal field? • The modes are incomplete. • Can we go beyond just the *f*-mode?	Newtonian neutron-star models with elastic crusts and superfluidity [Passamonti+ 2021].
	rotating stars	✓ [Ho+Lai 1999; Pnigouras+ 2023.]		Planetary studies [Lai 2021; Dewberry+Lai 2021].

the g-modes





- The phase shifts are expected to be very small [Lai 1994, *Mon. Not. R. Astron. Soc.* **270**, 611],

$$\frac{\Delta\Psi_g}{2\pi} \approx -4.3\times 10^{-4}\left[\frac{100\,\mathrm{Hz}}{\omega_g/(2\pi)}\right]^2\left(\frac{Q_g}{0.0003}\right)^2 .$$

- But some recent work in light of third-generation observatories (Cosmic Explorer and The Einstein Telescope) are more optimistic [Ho+Andersson 2023, *Phys. Rev. D* **108**, 043003].

- Even without direct measurements of the g-modes, the sensitivity improvements will place constraints on properties of the nuclear matter.

summary

- Gravitational waves carry information about the dense nuclear matter inside neutron stars.
- We understand the static tide well. However, this approximation *will* break down during the inspiral.
- The dynamical tide is less well-understood and much of our understanding still relies on Newtonian gravity.
- Opportunities to detect resonances are quite tantalising and the resonances will hold information about the interior stellar physics. Third-generation detectors will be more sensitive and may give us an opportunity to see these effects.