Constraining the Neutron-Star Equation of State from Dynamical Tides

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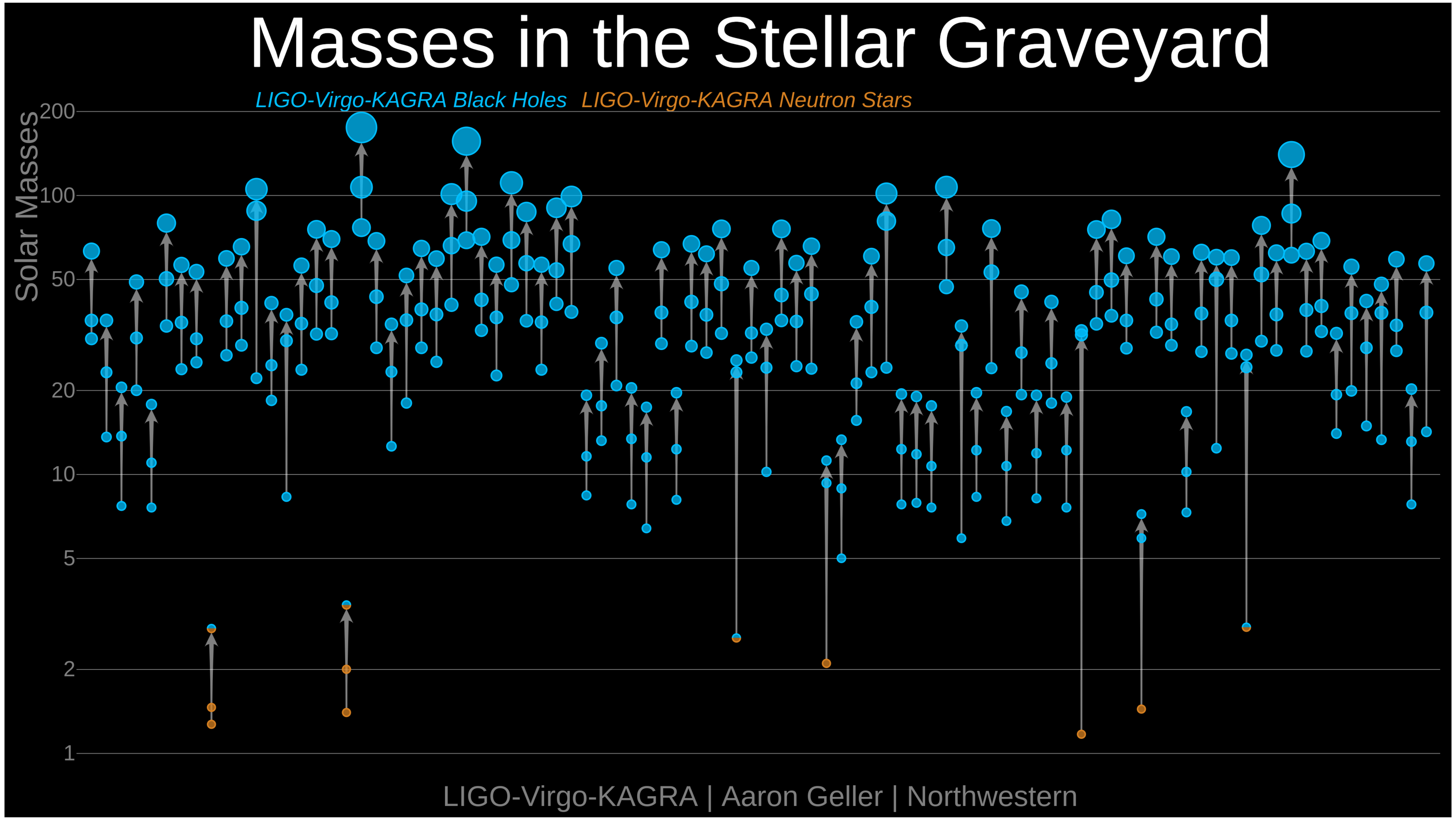
the physics of neutron stars

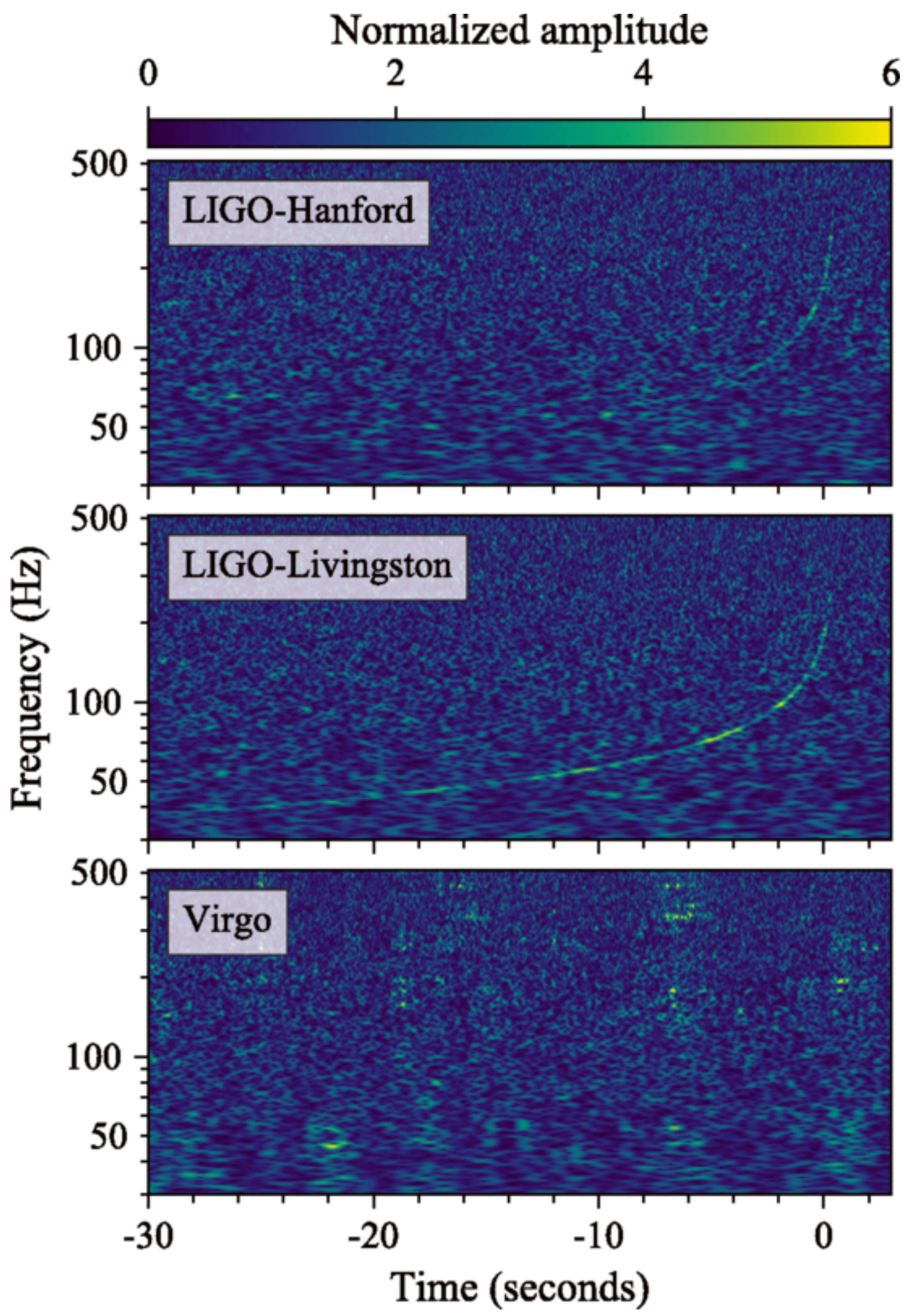
- Neutron stars are among the most complex objects in the Universe.
- A realistic description of a neutron star will inevitably require
  - general relativity
  - the equation of state
  - strong magnetic fields
  - superfluidity
  - a crust
  - thermal features
  - ...

- Since 2015, gravitational-wave detectors have witnessed 90 compact-binary coalescences — 2 neutron-star binaries and 3 neutron star-black hole binaries.



gravitational waves: GW170817

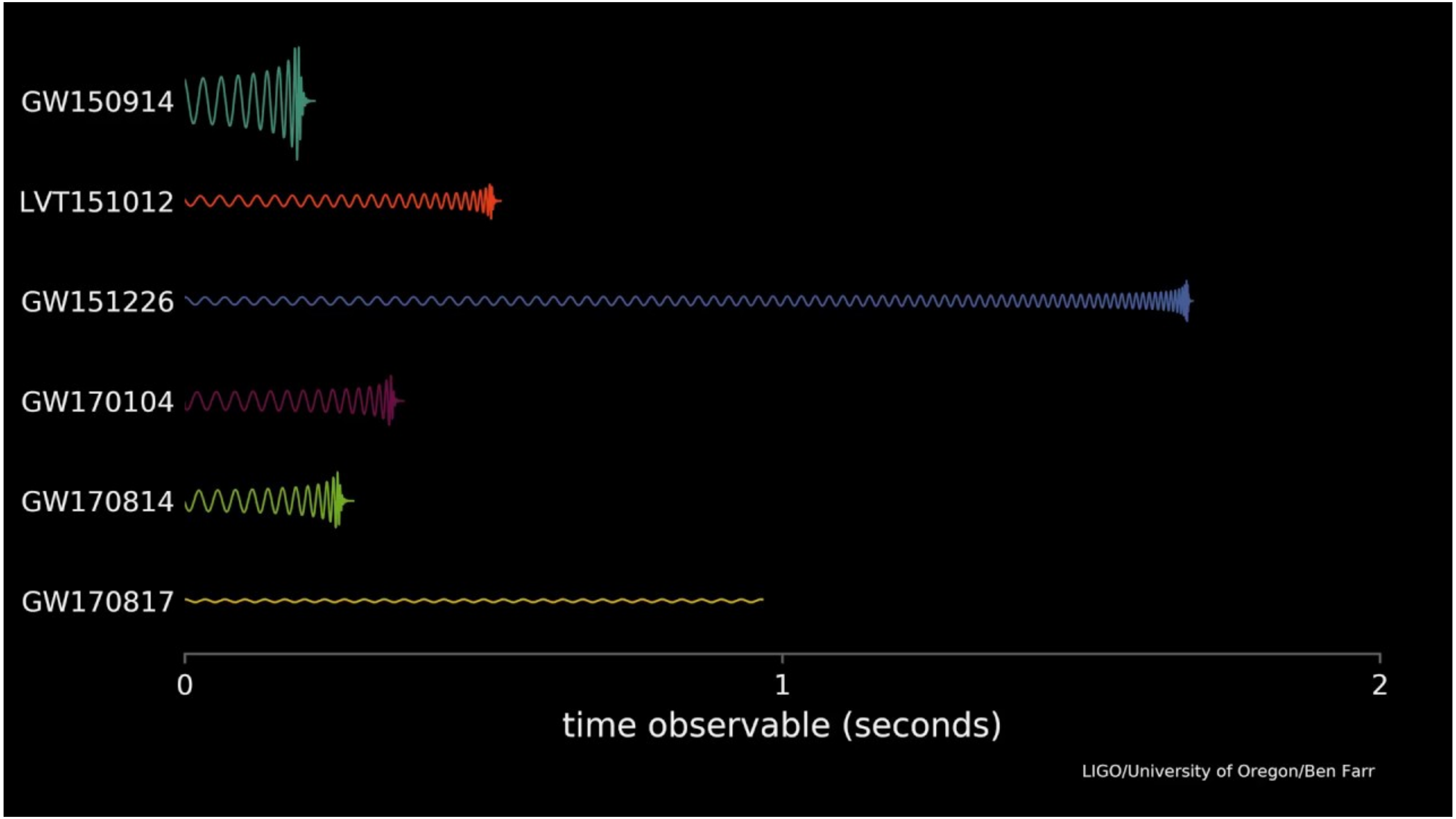
- On 17 Aug. 2017, gravitational-wave instruments detected the first neutron-star merger.



	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–$1.60\ M_\odot$	1.36–$2.26\ M_\odot$
Secondary mass m_2	1.17–$1.36\ M_\odot$	0.86–$1.36\ M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

[Abbott+ 2017, Phys. Rev. Lett. **119**, 161101]

gravitational waves: GW170817



the main take-away...

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Science

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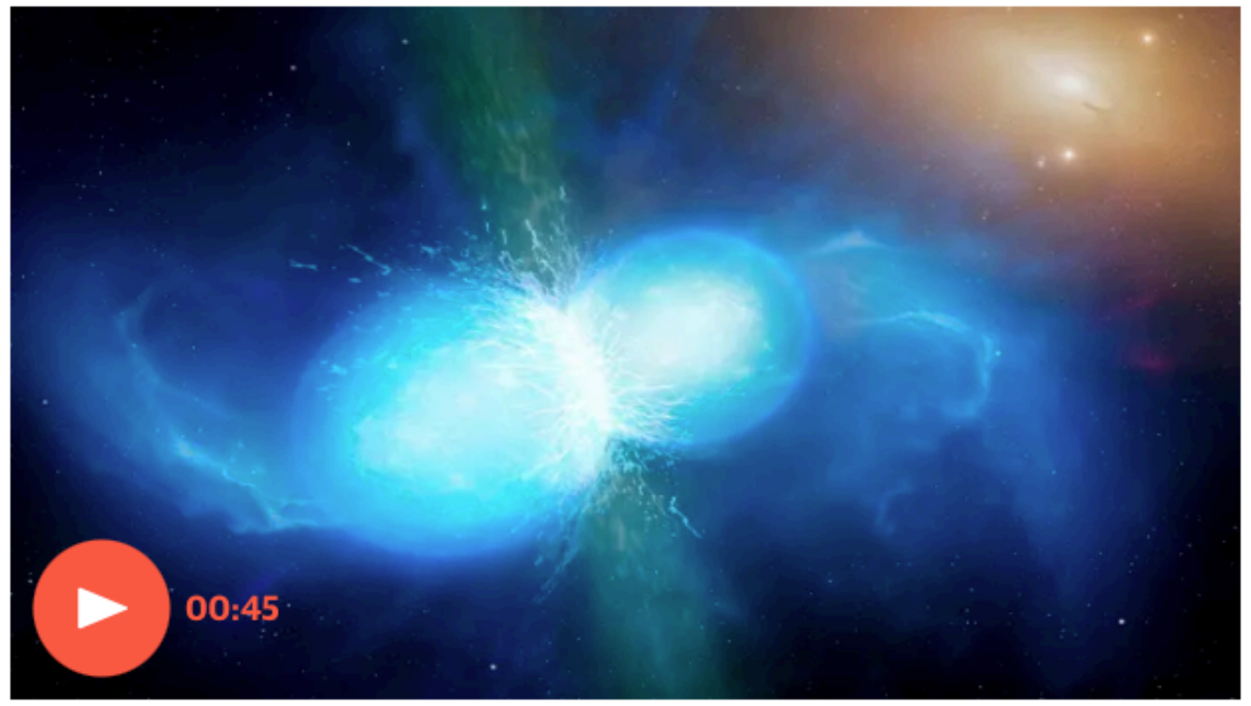
New frontier for science as astronomers witness neutron stars colliding

Extraordinary event has been 'seen' for the first time, in both gravitational waves and light - ending decades-old debate about where gold comes from



Hannah Devlin *Science correspondent*
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Mon 16 Oct 2017 15.00 BST

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Science & Environment

Einstein's waves detected in star smash

16 October 2017 · Comments

Vast cosmic 'kilonova' explosions that fling silver, gold, platinum and uranium across the universe may be far more common than thought

- Known as a kilonova, explosions are a luminous flash of radioactive light
- Immense explosions are caused by neutron stars colliding into each other
- Produces large quantities of elements like silver, gold, platinum and uranium
- The huge explosion rocked the fabric of the universe, distorting spacetime

By MARK PRIGG FOR DAILYMAIL.COM
UPDATED: 00:00, 17 October 2018

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Secret of gold finally found: precious metals are forged in cataclysmic collision of neutron stars

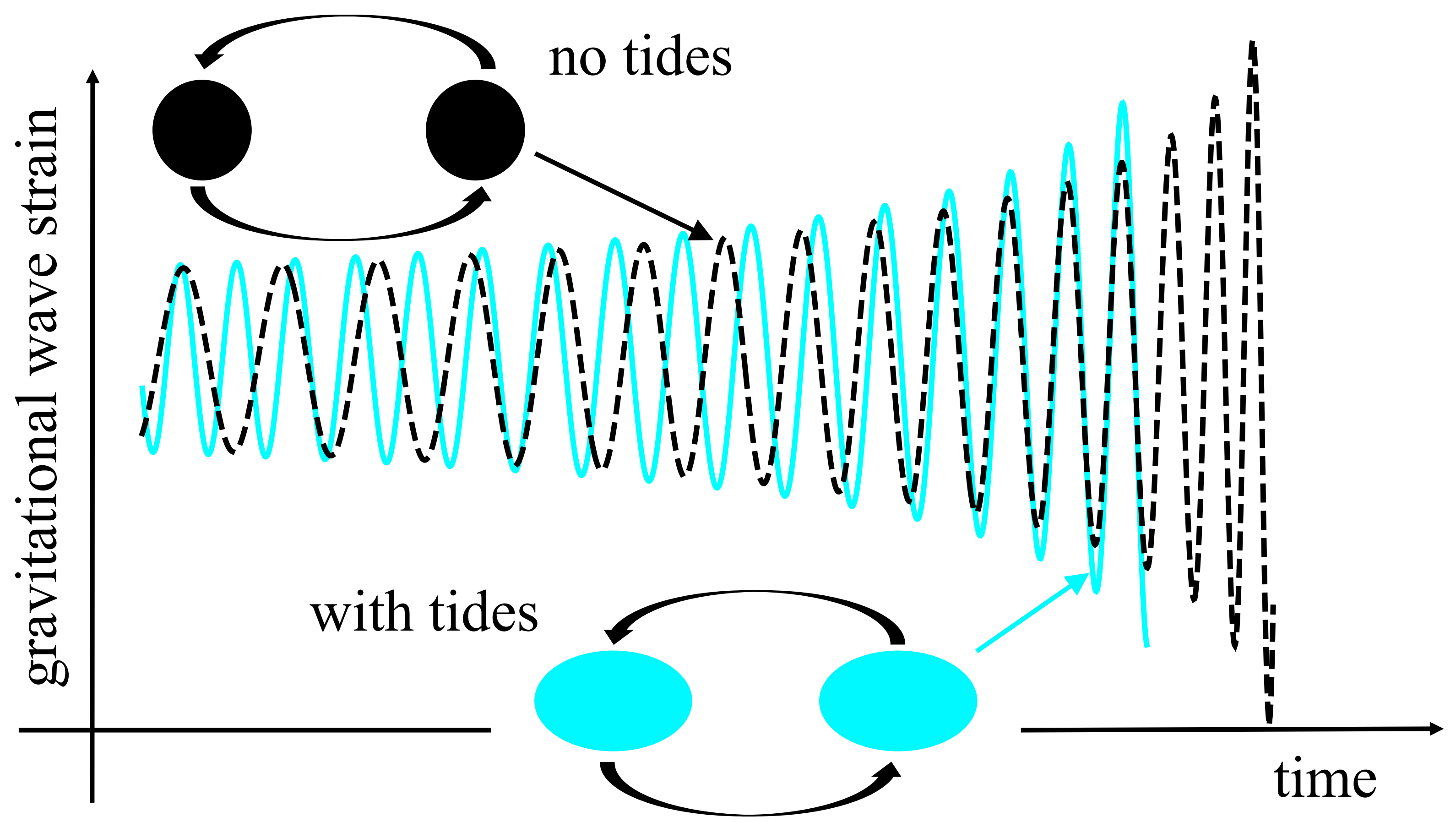
By Sarah Knapton, SCIENCE EDITOR
16 October 2017 • 3:00pm

neutron-star binaries

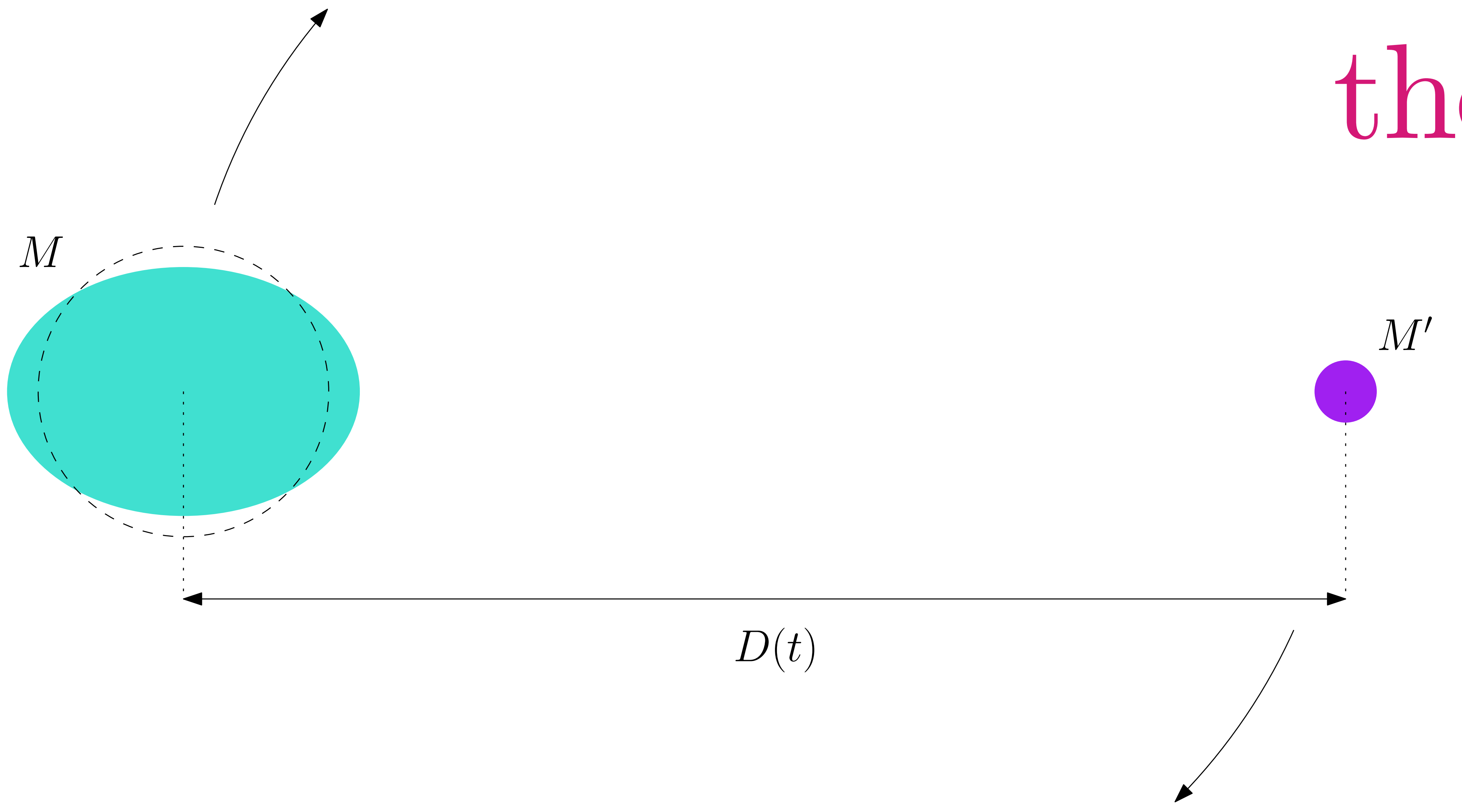
- The signal emitted from inspiralling neutron stars differs from that of black holes due to the material response to the tidal field.

- These features enter the waveform phase Ψ at 5PN through the induced quadrupole moment.

- The deformability of the stellar material is characterised by the *tidal Love numbers* k_{lm}, which depend on the interior composition and the equation of state.



the binary problem



- Assumptions:

  1. The bodies are well separated, $\epsilon = (M'/M)(R/D)^3 \ll 1$. The problem can be tackled perturbatively. (In the final few orbits, this breaks down completely and numerical relativity must be used.)

  2. The external field due to the companion is slowly varying, $\lambda = m\dot{\Psi}/\omega_\alpha \ll 1$. In this regime, the tidal field is *static*.

  3. The deformed neutron star is non-rotating.

the static tide: Newtonian gravity

- The Love numbers k_l are defined at the surface of the star $r = R$ by

$$\delta\Phi(R,\theta,\phi) = \sum_{l,m} \delta\Phi_l(R)\, Y_l^m(\theta,\phi) = \sum_{l,m} 2k_l\, \chi_l(R)\, Y_l^m(\theta,\phi)\,.$$

- Therefore, they can be read off from the exterior,

$$U_l \equiv \delta\Phi_l + \chi_l = \left[2k_l\left(\frac{R}{r}\right)^{2l+1} + 1\right]\left(\frac{r}{R}\right)^{l} \chi_l(R),$$

  where the field satisfies

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU_l}{dr}\right) - \frac{l(l+1)}{r^2}U_l = -\frac{4\pi G\rho}{dp/d\rho}U_l\,.$$

- This result generalises to relativity, where the field U is promoted to the (linearised) metric of the spacetime h_{ab}.

- In general relativity, the response of the star is obtained from the exterior behaviour of the metric, for example,

$$-\frac{h_{tt}}{2} = \frac{1}{2}\left[2k_2\left(\frac{R}{r}\right)^5 B_1 + A_1\right]\mathscr{E}_{jk}x^j x^k + \ldots,$$

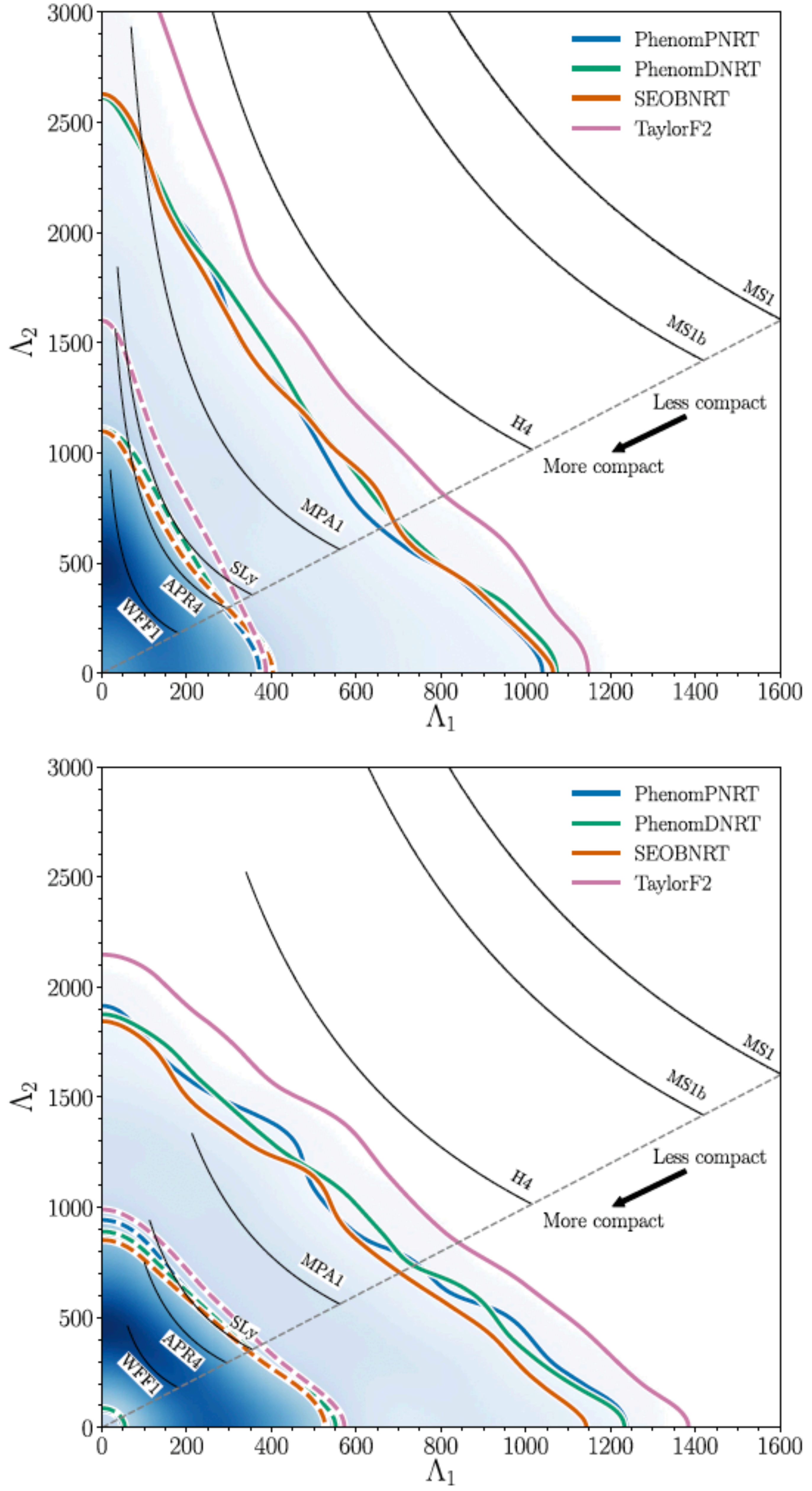
  where the functions A_1 and B_2 are determined from the Einstein field equations.

- New Love numbers appear: the gravitomagnetic Love numbers and (when the star's spin is considered) the rotational Love numbers.

the static tide: state of play

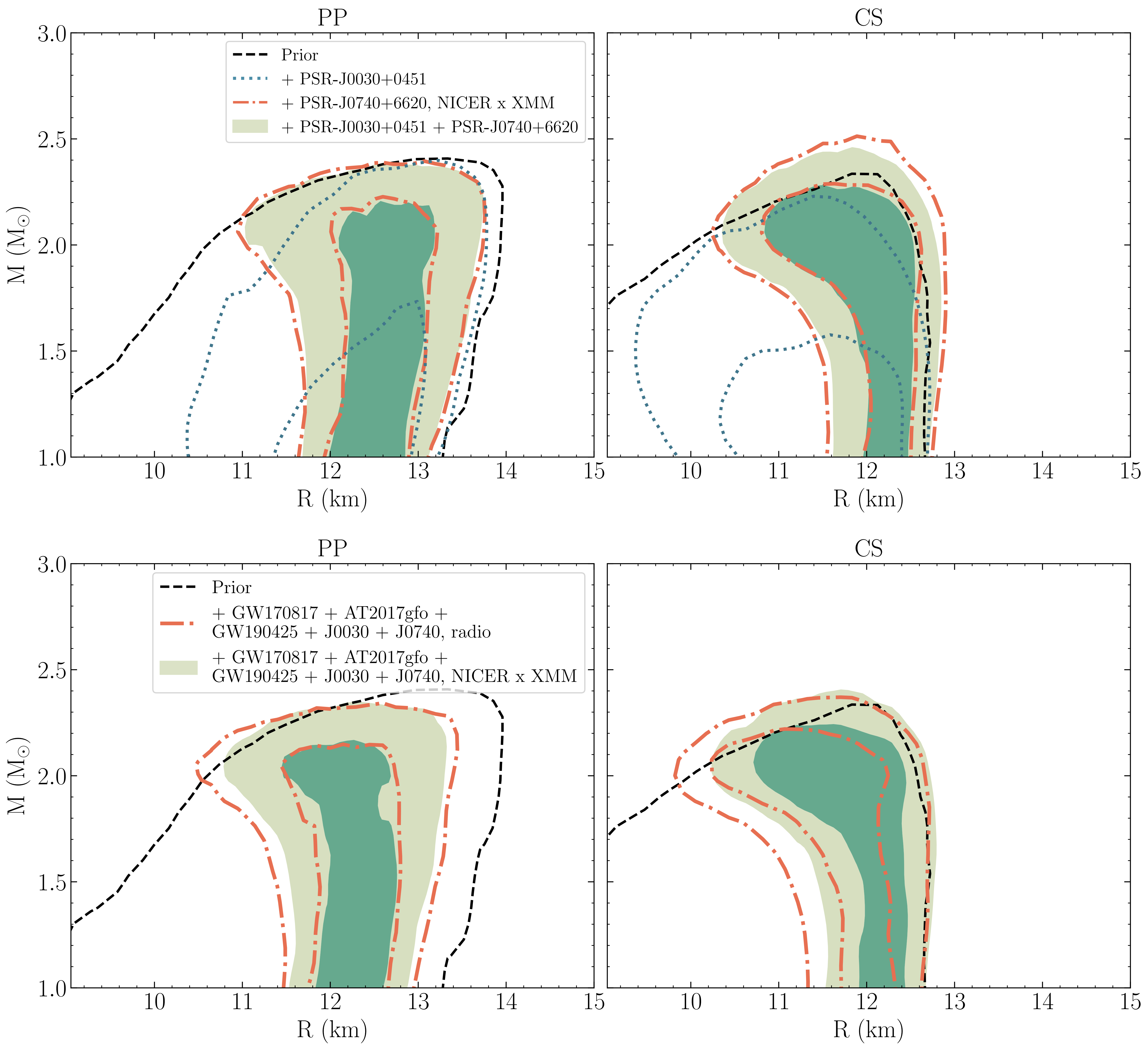
		Newtonian gravity	general relativity	notes
static tide	non-rotating stars	✓	✓ [Hinderer 2008; Binnington+Poisson 2009; Damour+Nagar 2009]	Relativistic neutron-star models with elastic crusts [Gittins+ 2020] and superfluidity [Yeung+ 2021].
	rotating stars		✓ [Landry+Poisson 2015; Landry 2015; Pani+ 2015a,b]	Calculations are at the level of slowly rotating fluid bodies.

equation-of-state constraints



[Abbott+ 2019, Phys. Rev. X **9**, 011001]

$$\Lambda_A = \frac{2}{3} k_{2A} \left(\frac{c^2 R_A}{G M_A} \right)^5$$



[Raaijmakers+ 2021, Astrophys. J. **918**, L29]

the dynamical tide

- At this point, we want to relax the assumption of a static tidal field.
- As the compact objects inspiral, the tidal frequency increases such that it eventually becomes comparable to the neutron star's natural modes of oscillation, $\lambda = m\dot{\Psi}/\omega_\alpha = O(1)$.
- Additional assumption: We ignore dissipation completely and work in Newtonian gravity.

the mode-sum: formalism

- Neutron stars host a spectrum of oscillation modes. Formally, the normal modes satisfy an eigenvalue problem,

$$\mathbf{C} \cdot \boldsymbol{\xi}_\alpha = \omega_\alpha^2 \, \boldsymbol{\xi}_\alpha \, .$$

- The normal modes form a complete basis [Chandrasekhar 1964, Astrophys. J. **139**, 664], such that a generic vector can be decomposed as

$$\boldsymbol{\xi}(t, \mathbf{x}) = \sum_\alpha q_\alpha(t) \, \boldsymbol{\xi}_\alpha(\mathbf{x}) \, .$$

- The equation of motion $\partial_t^2 \boldsymbol{\xi} + \mathbf{C} \cdot \boldsymbol{\xi} = -\nabla \chi$ becomes that of a driven harmonic oscillator,

$$\frac{d^2 q_\alpha}{dt^2} + \omega_\alpha^2 \, q_\alpha = Q_\alpha \propto e^{-im\Psi} \, .$$

- At resonance $m\dot{\Psi} = \omega_\alpha$, the mode will become excited and extract energy from the orbit. This will change the phase by

$$\frac{\Delta\Psi_\alpha}{2\pi} \approx -\frac{t_\mathrm{D}}{t_\mathrm{orb}}\frac{\Delta E_\alpha}{|E_\mathrm{orb}|} \propto \left(\frac{Q_\alpha}{\omega_\alpha}\right)^2 .$$

- The impact of a resonance on the phase strongly depends on the *overlap* Q_α of the mode and the tidal potential,

$$Q_\alpha = -\int \delta\rho_\alpha^* \chi \, dV .$$

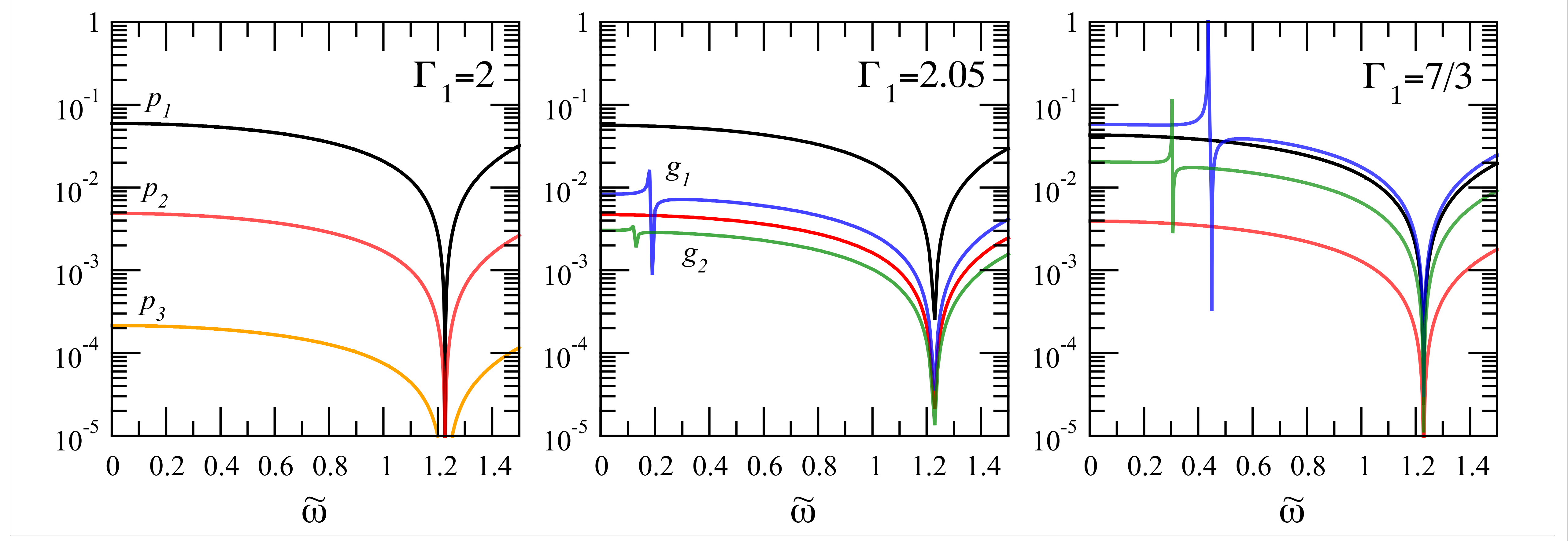
(some of the) neutron-star modes

- *f*-modes: Fundamental oscillations of the star; scale with the average density, $\omega_\alpha/(2\pi) \sim \sqrt{GM/R^3} \sim 1\,\mathrm{kHz}$.
- *p*-modes: Restored by the pressure of the fluid; high frequencies above the *f*-mode; possible instability with *g*-modes [Weinberg+ 2013].
- *g*-modes: Restored by buoyancy that arises from composition gradients; low frequencies below the *f*-mode, $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.
- inertial modes (including the *r*-mode): Restored by rotation; primarily excited by the gravitomagnetic tide (a relativistic effect) [Flanagan+Racine 2007]; $\omega_\alpha \sim \Omega$.
- *i*-modes: Oscillations that arise due to the core-crust interface; possible association with short gamma-ray bursts [Tsang+ 2012]; $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.

the mode-sum: application

- We expect the dynamical tide to be dominated by the *f*-mode, but it may be possible to see resonances during the inspiral.

$\Gamma_1 = 2$		$\Gamma_1 = 2.05$		$\Gamma_1 = 7/3$	
Mode	k_l	Mode	k_l	Mode	k_l
f	0.27528	f	0.27055	f	0.24685
$+p_1$	0.25887	$+p_1$	0.25526	$+g_1$	0.26115
$+p_2$	0.26021	$+p_2$	0.25653	$+p_1$	0.25052
$+p_3$	0.26015	$+g_1$	0.25878	$+g_2$	0.25556
		$+g_2$	0.25960	$+p_2$	0.25653
		$+g_3$	0.25993	$+g_3$	0.25856
		$+g_4$	0.26008	$+g_4$	0.25944
				$+g_5$	0.25983
	9×10^{-4}		7×10^{-4}		3×10^{-4}



Relative contributions to the tidal Love number k_2 compared to the *f*-mode.

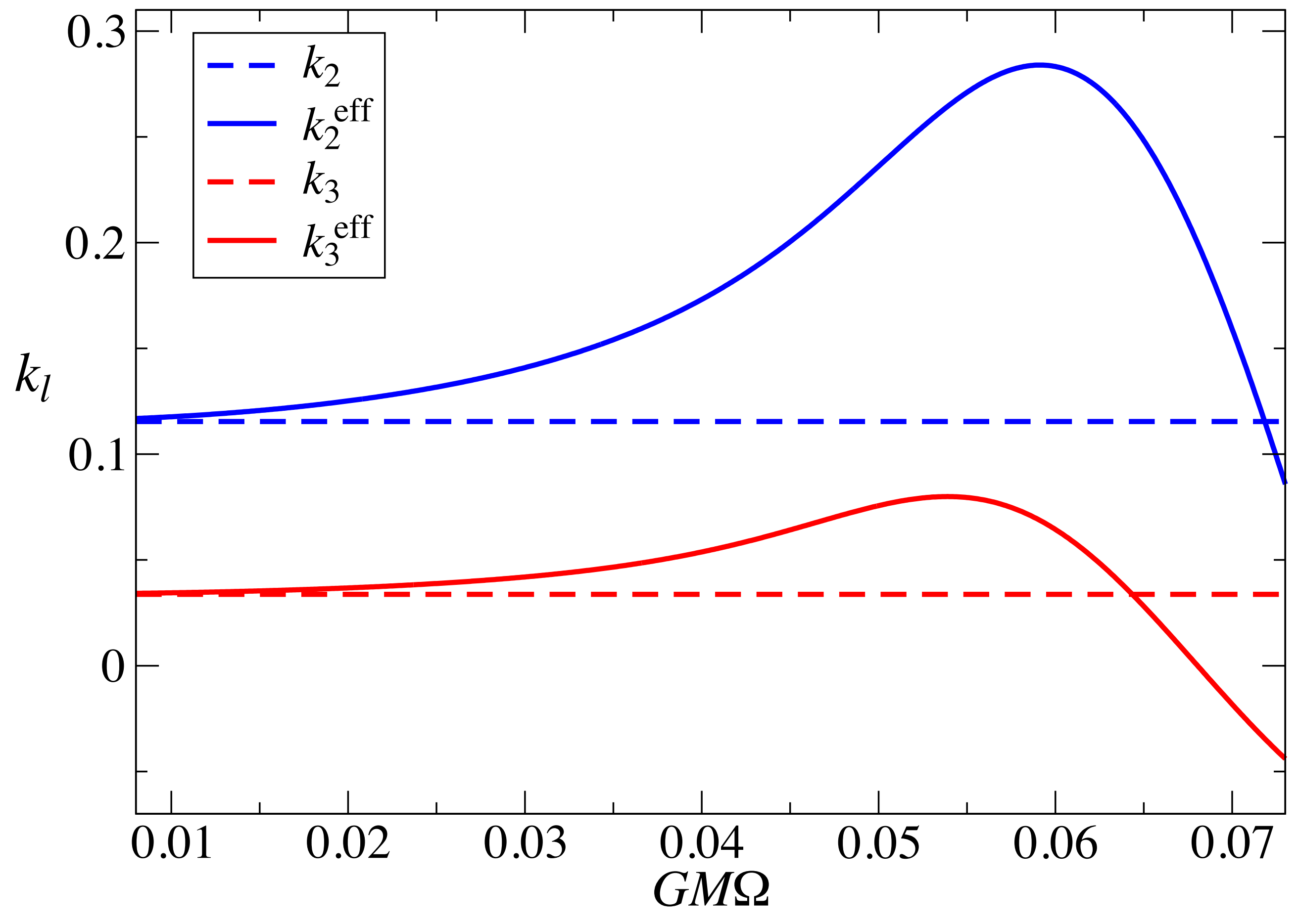
[Andersson+Pnigouras 2020, Phys. Rev. D **101**, 083001]

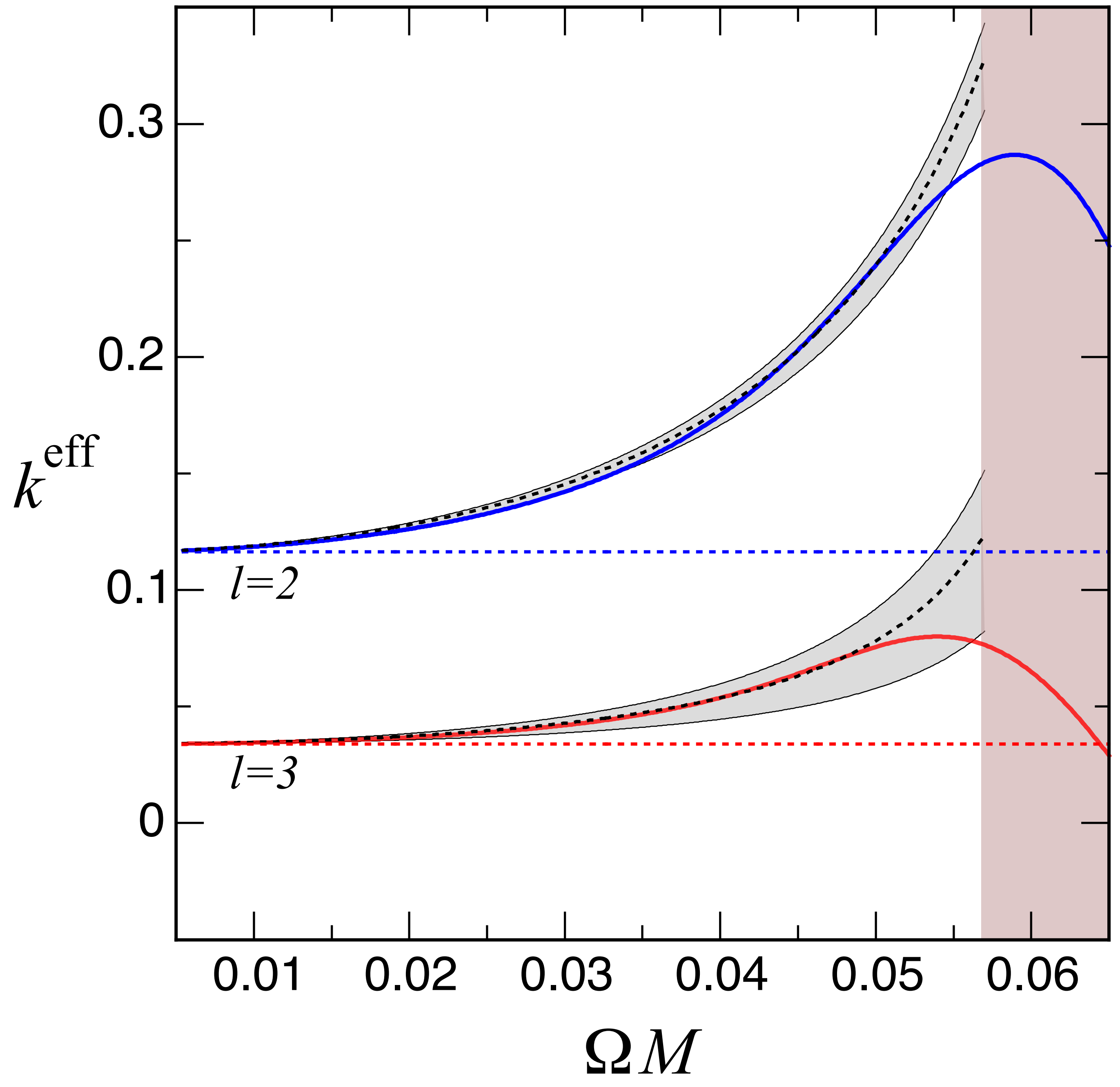
the f-mode: approximation

- There has been some work in representing the dynamical tide using just the contribution from the f-mode.

  (i) Effective approach: generalising the Newtonian action for the orbital dynamics to relativity in the time domain [Steinhoff+ 2016, Phys. Rev. D **94**, 104028] and frequency domain [Schmidt+Hinderer 2019, Phys. Rev. D **100**, 021501].

  (ii) Phenomenological approach [Andersson+Pnigouras 2021, Mon. Not. R. Astron. Soc. **503**, 533].



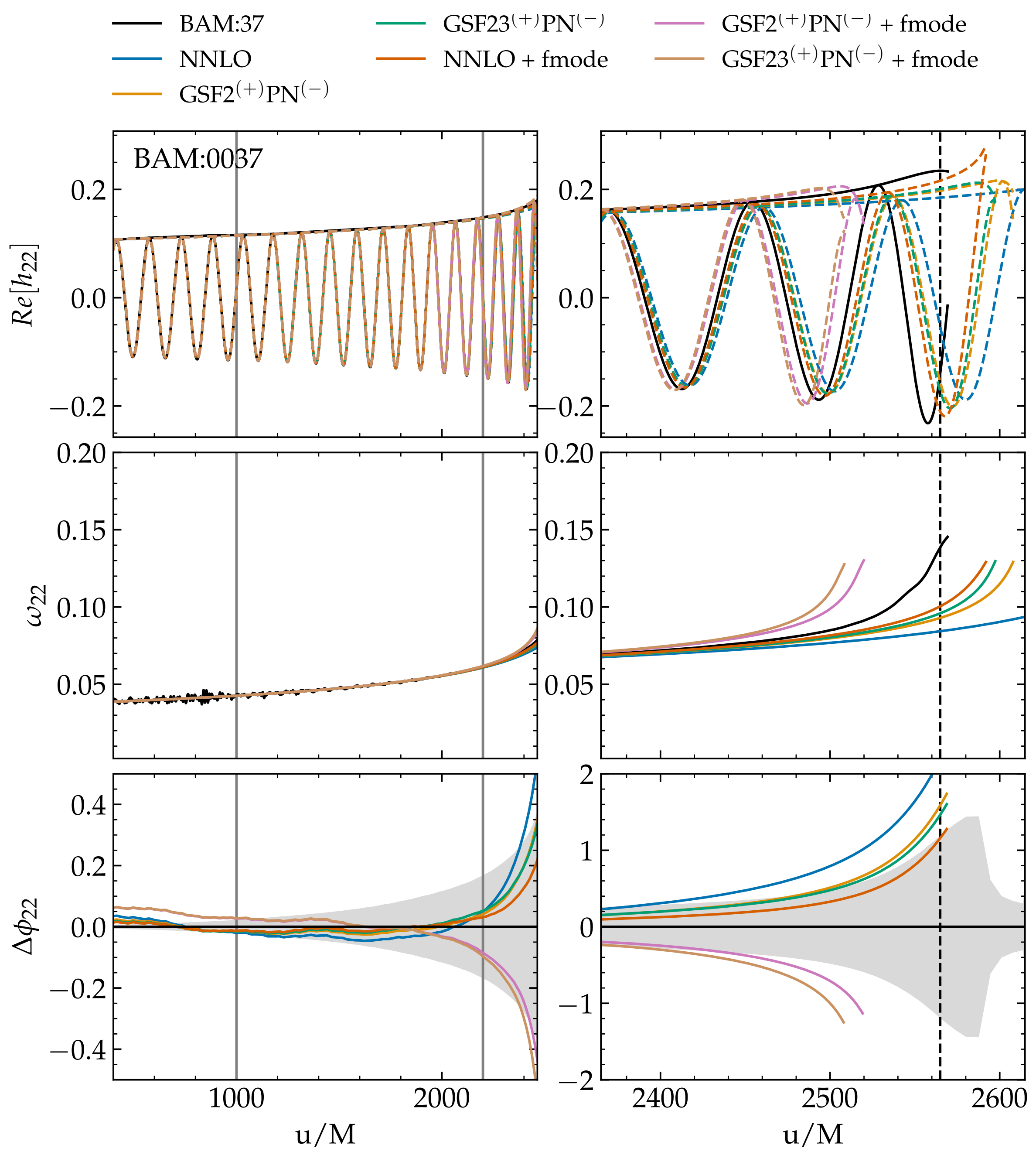


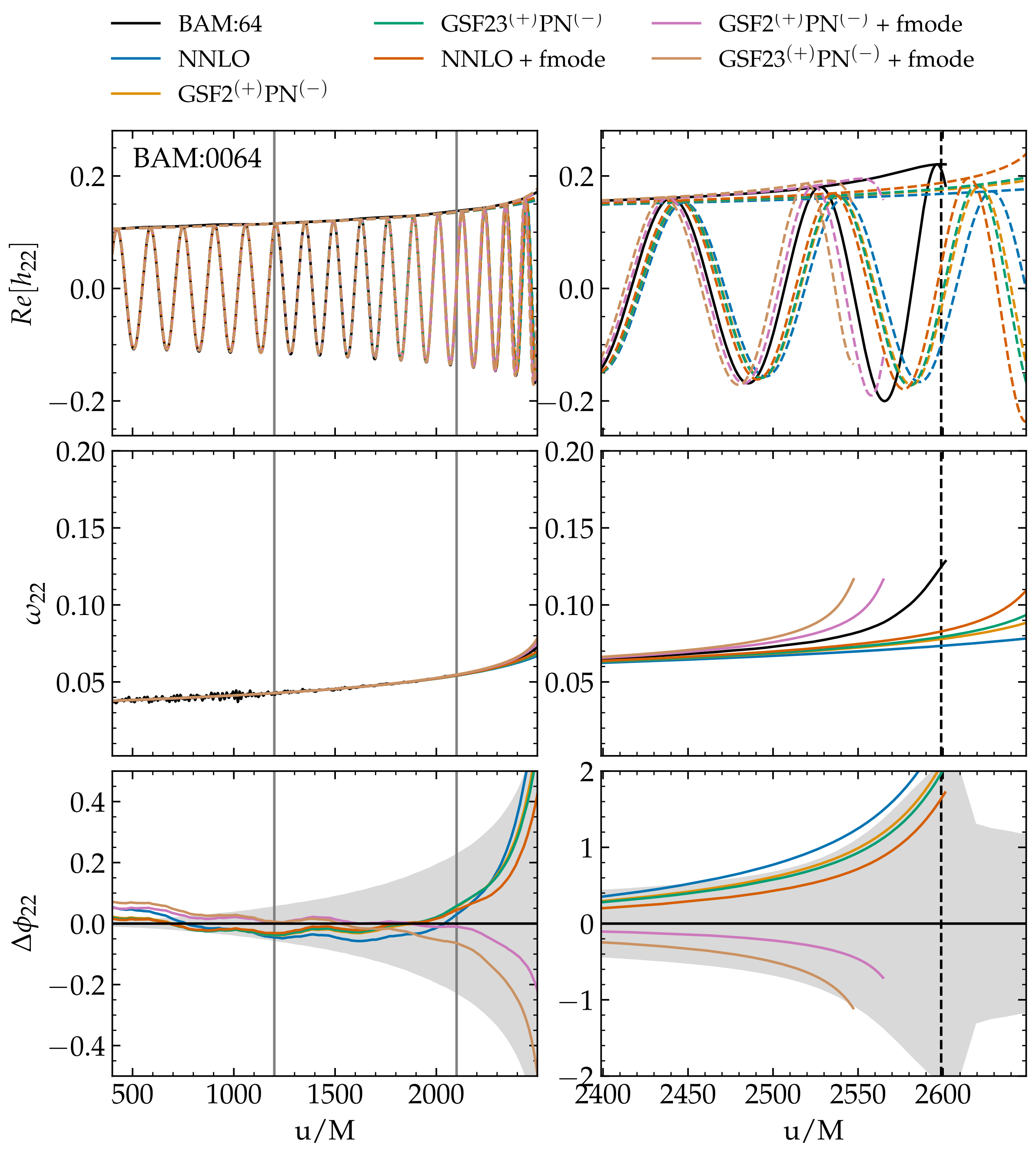
the f-mode: results

- The effective approach has been used to constrain the $l = 2, 3$ f-mode frequencies from the larger component of GW170817 [Pratten+ 2020, Nat. Commun. **11**, 2553],

$$\omega_{f,2}/(2\pi) \geq 1.39\,\mathrm{kHz}, \qquad \omega_{f,3}/(2\pi) \geq 1.86\,\mathrm{kHz}\,.$$

- However, while these approaches are improved compared to the static tide, they do not entirely match results from numerical simulations [Gamba+Bernuzzi 2023, Phys. Rev. D **107**, 044014].





state of play

		Newtonian gravity	general relativity	notes
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	rotating stars		✓ [Landry+Poisson 2015; Landry 2015; Pani+ 2015a,b]	Calculations are at the level of slowly rotating fluid bodies.
dynamical tide	non-rotating stars	✓ [Lai 1994; Andersson+Pnigouras 2020]	• How to treat a dynamical tidal field? • The modes are incomplete. • Can we go beyond just the *f*-mode?	Newtonian neutron-star models with elastic crusts and superfluidity [Passamonti+ 2021].
	rotating stars	✓ [Ho+Lai 1999; Pnigouras+ in prep.]		Planetary studies [Lai 2021; Dewberry+Lai 2021].

the g-modes: origins

- Not a new idea [Cowling 1941, Mon. Not. R. Astron. Soc. **101**, 367].
- Start with the first law of thermodynamics,

$$d\varepsilon = T\,ds + \sum_{\mathrm{x}} \mu_{\mathrm{x}}\,dn_{\mathrm{x}}\,.$$

- Assuming cold, electrically neutral, pure npe-matter,

$$d\varepsilon = (\mu_{\mathrm{n}} - \beta x_{\mathrm{p}})\,dn - \beta n\,dx_{\mathrm{p}} \qquad \Longrightarrow \qquad \varepsilon = \varepsilon(n, x_{\mathrm{p}}),$$

  where $\beta = \mu_{\mathrm{n}} - (\mu_{\mathrm{p}} + \mu_{\mathrm{e}})$ encodes the deviation from chemical equilibrium and $x_{\mathrm{p}} = n_{\mathrm{p}}/n$.

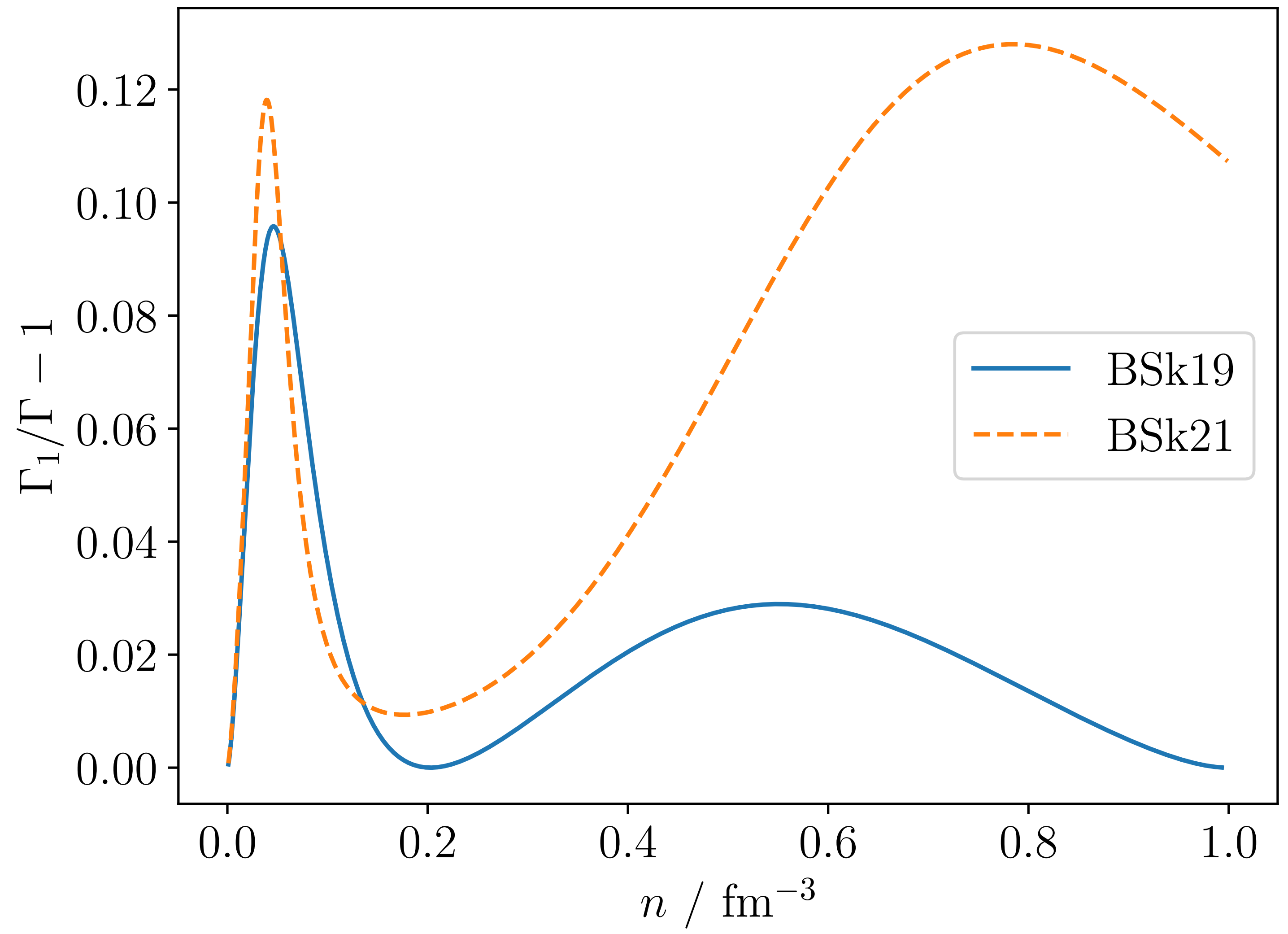
- When the fluid is in equilibrium $\beta = 0$, the equation of state is barotropic $\varepsilon = \varepsilon(n)$ and there are no g-modes.

the g-modes: realistic composition

- In principle, the g-modes will contain information about the non-barotropic nature of the equation of state.

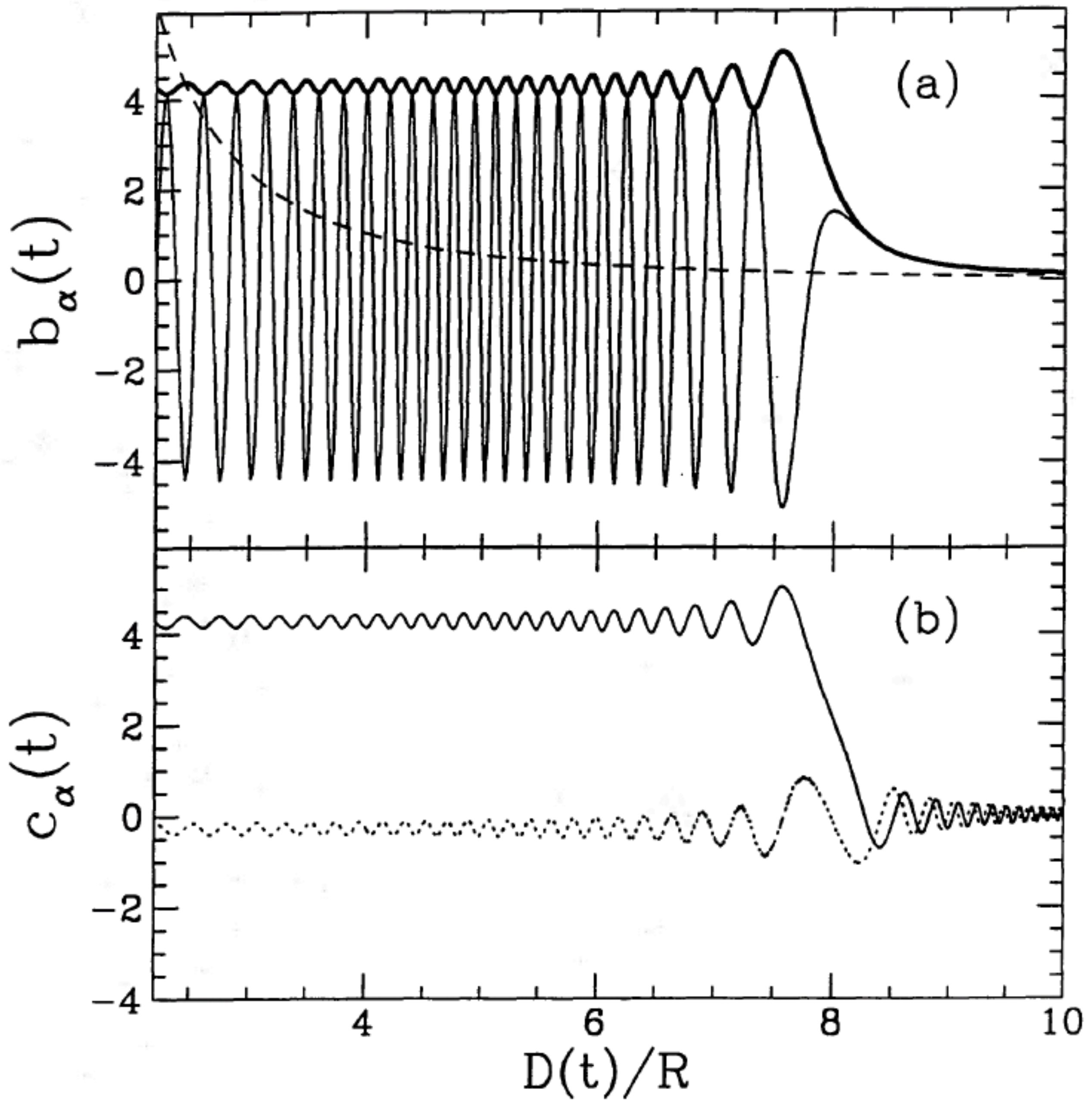
- The g-modes are sensitive to the deviations from chemical equilibrium. This is characterised by the (local) Brunt-Väisälä frequency N,

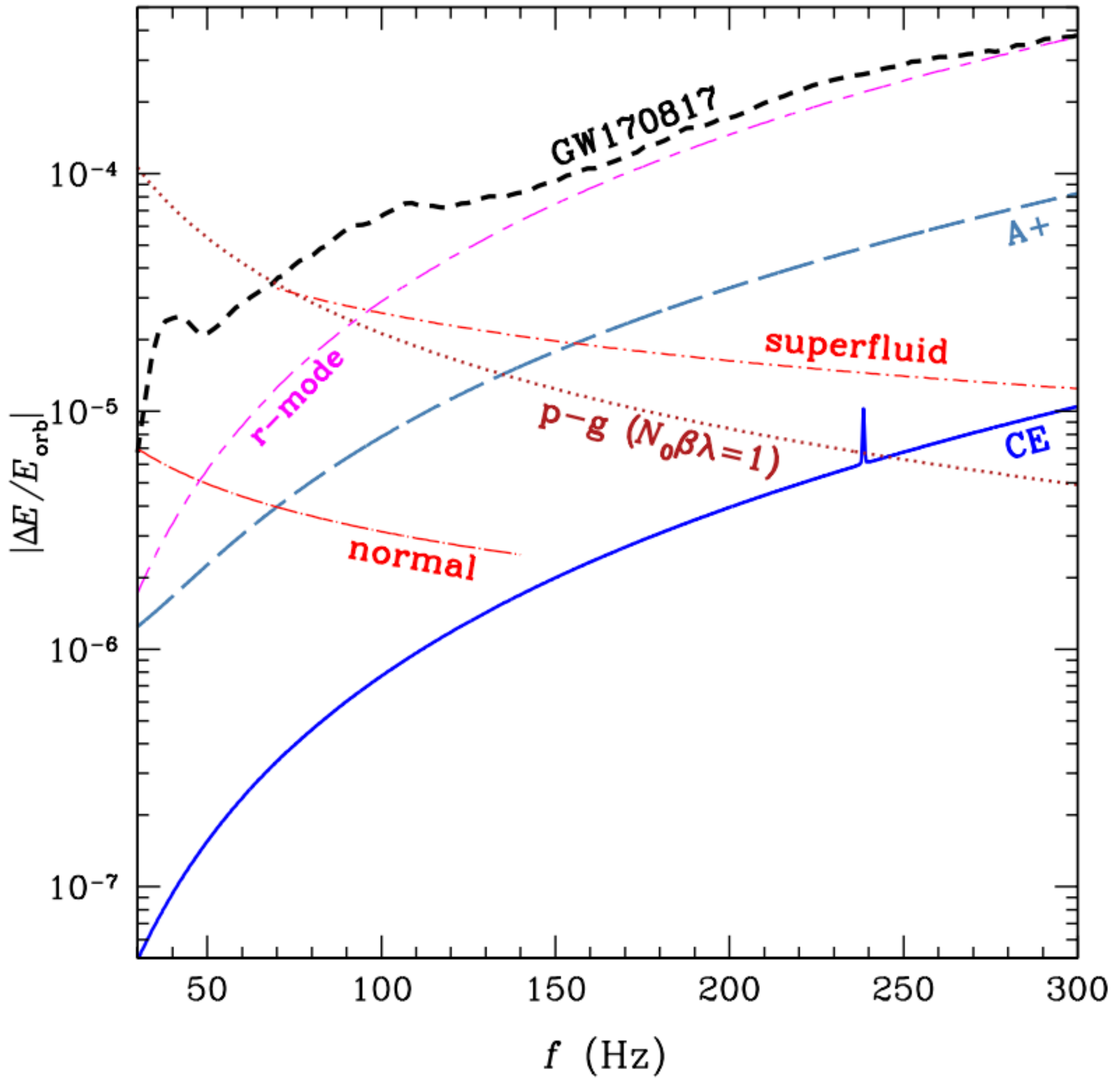
$$N^2 = \frac{\rho g^2}{p}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma_1}\right).$$



[Gittins+Andersson 2023, Mon. Not. R. Astron. Soc. **521**, 3043]

the g-modes: prospects





- The phase shifts are expected to be very small [Lai 1994, Mon. Not. R. Astron. Soc. **270**, 611],

$$\frac{\Delta\Psi_g}{2\pi} \approx -4.3 \times 10^{-4} \left[\frac{100\,\mathrm{Hz}}{\omega_g/(2\pi)}\right]^2 \left(\frac{Q_g}{0.0003}\right)^2 .$$

- But some recent work in light of third-generation detectors — Cosmic Explorer and the Einstein Telescope — are more optimistic [Ho+Andersson in prep.]

- In general relativity, all motion is dissipative due to gravitational radiation,

$$\mathbf{f}_{\mathrm{GW}} = -\frac{2G}{5c^5}\rho\frac{d^5\mathbf{Q}}{dt^5}\cdot\mathbf{x} \qquad \rightarrow \qquad \rho\frac{d\mathbf{v}}{dt} = -\nabla p - \rho\nabla\Phi + \mathbf{f}_{\mathrm{GW}},$$

$$\implies \frac{dE}{dt} = \int \mathbf{v}\cdot\mathbf{f}_{\mathrm{GW}}\,dV \neq 0.$$

- This is formally a 2.5PN feature and inevitably spoils the completeness of the modes.

- In the hope of doing (at the very least) better than Newtonian models, we are exploring whether progress can be made in PN theory [Andersson+Gittins in prep.].

- Ultimately, we will need calculations in full general relativity to describe neutron stars.

summary

- Gravitational waves carry information about the material properties of neutron stars.

- We understand the static tidal regime well and can develop realistic neutron-star models.

- The dynamical tide is less well-understood and much of our understanding still relies on Newtonian gravity. In particular, the presence of dissipation through gravitational radiation hampers our ability to make progress.

- Opportunities to detect resonances are quite tantalising and the resonances will hold information about the interior stellar physics. Third-generation detectors will be more sensitive and may give us an opportunity to see these effects.