Making (neutron-star) mountains out of molehills

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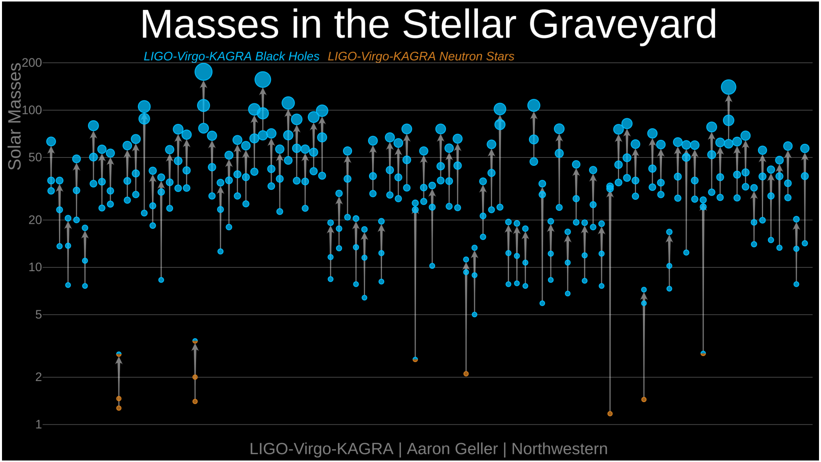
The story so far

- Since 2015, gravitational-wave detectors have witnessed 90 compact-binary coalescences – 2 neutron-star binaries and 3 neutron star-black hole binaries.



Neutron stars as gravitational-wave sources

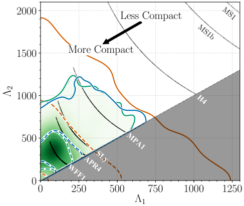
Configurations that radiate gravitational waves

(i) neutron stars in compact binaries

(ii) (rotating) neutron stars that host non-axisymmetric deformations known as *mountains*

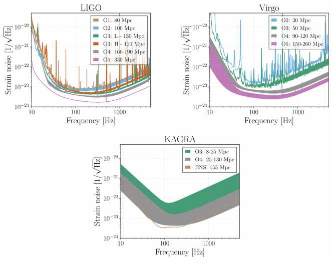
(iii) modes of oscillation (and associated instabilities)

- The first gravitational-wave detection of a neutron star came from the remarkable *multimessenger event* GW170817.
- Gravitational radiation presents an opportunity to constrain the elusive nuclear-matter *equation of state*.



(Abbott *et al.*, 2018)

Continuous-wave searches



(Abbott *et al.*, 2020)

- Rotating neutron stars that host mountains will *continuously* radiate (weak) gravitational waves.
- The sensitivity of searches through the data increases with observing time, but this comes at computational cost.

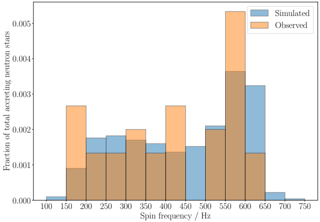
- There are three forms of searches:
  - (i) targeted – source parameters are known to sufficient accuracy;
  - (ii) directed – sky localisation is known, but spin is unknown;
  - (iii) all-sky – searches for unknown stars.

$$h_0 \approx \frac{G}{c^4}\frac{\epsilon I_{zz}(2\pi f)^2}{d} \approx 10^{-25}\left(\frac{\epsilon}{10^{-5}}\right)\left(\frac{f}{100\,\mathrm{Hz}}\right)^2\left(\frac{1\,\mathrm{kpc}}{d}\right)$$

Rapidly rotating pulsars

- Searches in the gravitational-wave data have, so far, only provided upper limits on the size of the deformations.

- No neutron star has been observed that rotates (even remotely) close to the *centrifugal break-up frequency* ($\sim$ 1 kHz for most equations of state).
- There are two competing explanations for this behaviour:
  (i) an interaction between the accreting gas and the magnetic field lines;
  (ii) gravitational-wave emission.



$Q_{22} = 10^{36}\,\mathrm{g\,cm^2}, \epsilon \approx 10^{-9}$ (Gittins and Andersson, 2019)

Deformed stellar models

- Traditionally, one encounters the multipole moments $Q_{\ell m}$ by examining how the exterior gravitational potential of a non-spherical body deviates from sphericity. In the Newtonian limit, this is given by

$$\delta\Phi(r,\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \delta\Phi_{\ell m}(r)\, Y_{\ell m}(\theta,\phi), \quad \delta\Phi_{\ell m}(r) = -\frac{4\pi G}{2\ell+1}\frac{Q_{\ell m}}{r^{\ell+1}}$$

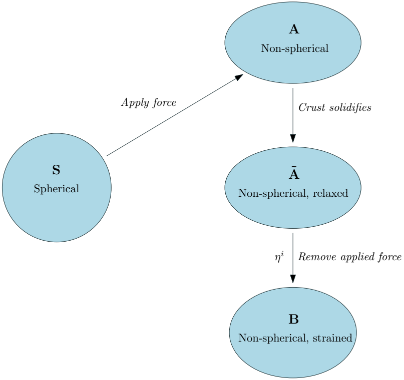
for $r \geq R$, where

$$Q_{\ell m} \equiv \int_0^R \delta\rho_{\ell m}(r) r^{\ell+2}\, dr.$$

- The dominant multipole in gravitational-wave emission is the *quadrupole moment* Q_{22}. Thus, we specialise to the $(\ell, m) = (2, 2)$ mode.
- Formally, one can use the law of momentum conservation – the Euler equation – to characterise a stellar model,

$$\begin{aligned} 0 &= -\nabla_i p - \rho\nabla_i\Phi + \nabla^j t_{ij} + f_i \\ &\equiv -H_i + \nabla^j t_{ij} + f_i. \end{aligned}$$

Building mountains: the usual approach



- One can describe star B with the perturbed Euler equation

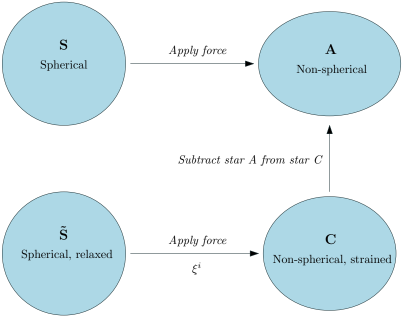
$$\delta H_i^{\mathrm{SB}} \equiv H_i^{\mathrm{B}} - H_i^{\mathrm{S}} = \nabla^j t_{ij}(\eta). \quad (1)$$

- Suppose one supplies a strain field $\sigma_{ij} = t_{ij}/(2\mu)$, then one can solve for the perturbations.
- Note that the fiducial force f_i is hidden in the calculation.

The question of the maximum mountain

- There have been theoretical attempts to estimate the *maximum* mountain that a neutron-star crust can support (Ushomirsky, Cutler, and Bildsten, 2000; Haskell, Jones, and Andersson, 2006; Johnson-McDaniel and Owen, 2013). Such an estimate provides a natural limit on the magnitude of the gravitational radiation from a rotating star.
- Previous calculations have generally followed the approach laid out by Ushomirsky *et al.* (2000): ensure the crust is maximally strained at *every point*. But such a technique does not respect the boundary conditions on the star.
- To this end, we introduce another scheme that makes explicit use of the deforming force.

A new mountain scheme



- The difference between stars C and A gives

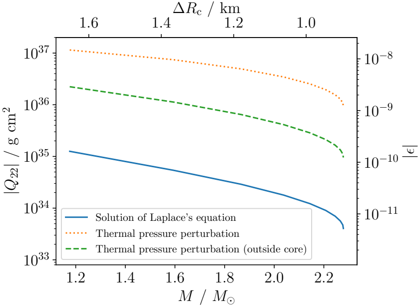
$$\delta H_i^{\text{AC}} = \nabla^j t_{ij}(\xi). \qquad (2)$$

- It turns out that the mountains calculated using (2) are equivalent to solving the previous perturbed Euler equation (1).

- Using the deforming force has two advantages: (i) we can calculate the relaxed configuration and (ii) we are able to explicitly satisfy the boundary conditions.

Examples of the deforming force

- We generated a set of fully relativistic neutron-star models (with a realistic equation of state) that were subjected to a few specific deforming forces.
- The amplitude of the force on each star was increased until the crust began to fracture. This produced the maximum mountain that each star could support for a given force.



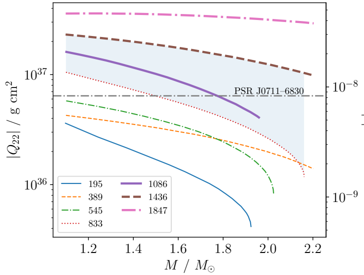
Cf., for a $1.4\,M_\odot$, 10 km Newtonian star, Ushomirsky *et al.* (2000) found

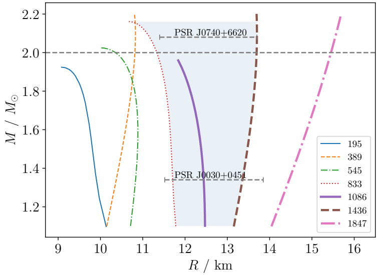
$$Q_{22}^{\max} \approx 1.2 \times 10^{39}\ \mathrm{g\,cm^2},$$
$$\epsilon^{\max} \approx 1.6 \times 10^{-6}.$$

- This illustrates the dependence of the mountain on the formation history of the star.

The equation of state

- We also considered the role of the equation of state in supporting the mountains, by implementing a subset of equations of state obtained from chiral effective field theory with a speed-of-sound parametrisation.





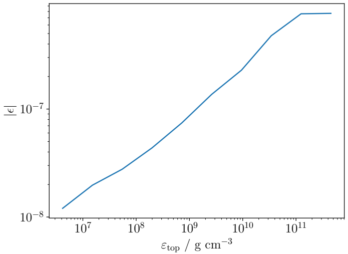
Summary & future work

- We introduce a new scheme to calculate mountains that explicitly satisfies the necessary boundary conditions. However, this scheme requires the introduction of a deforming force.
- Such a force will depend on the (possibly quite complex) formation history of the star. For this reason, we believe that evolutionary calculations will be necessary to make progress on this problem (Bildsten, 1998; Singh *et al.*, 2020; Osborne and Jones, 2020).
- The neutron-star equation of state plays an important role in supporting the mountains. In particular, the shear modulus of the crust (unsurprisingly) has a significant impact on how large the mountains can be.
- More accurate descriptions of the neutron-star crust may need to take into account plastic deformation.

A closing thought

- Suppose that when a point in the crust yields it becomes fluid. In most situations, the strain is largest at the top of the crust.
- This implies the following strategy:
  1. the crust is strained until the top breaks;
  2. when this point yields, it becomes fluid and the top of the crust is deeper in the star;
  3. since this new top will have less strain, the crust can be further strained until that point breaks.

  This procedure continues iteratively.



References

B. P. Abbott *et al.*, GW170817: Measurements of Neutron Star Radii and Equation of State, Phys. Rev. Lett. **121**, 161101 (2018), arXiv:1805.11581 [gr-qc].

B. P. Abbott *et al.*, Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA, Living Rev. Relativity **23**, 3 (2020).

F. Gittins and N. Andersson, Population synthesis of accreting neutron stars emitting gravitational waves, Mon. Not. R. Astron. Soc. **488**, 99 (2019), arXiv:1811.00550 [astro-ph.HE].

G. Ushomirsky, C. Cutler, and L. Bildsten, Deformations of accreting neutron star crusts and gravitational wave emission, Mon. Not. R. Astron. Soc. **319**, 902 (2000), astro-ph/0001136.

B. Haskell, D. I. Jones, and N. Andersson, Mountains on neutron stars: accreted versus non-accreted crusts, Mon. Not. R. Astron. Soc. **373**, 1423 (2006), arXiv:astro-ph/0609438 [astro-ph].

N. K. Johnson-McDaniel and B. J. Owen, Maximum elastic deformations of relativistic stars, Phys. Rev. D **88**, 044004 (2013), arXiv:1208.5227 [astro-ph.SR].

L. Bildsten, Gravitational Radiation and Rotation of Accreting Neutron Stars, Astrophys. J. **501**, L89 (1998), arXiv:astro-ph/9804325 [astro-ph].

N. Singh, B. Haskell, D. Mukherjee, and T. Bulik, Asymmetric accretion and thermal 'mountains' in magnetized neutron star crusts, Mon. Not. R. Astron. Soc. **493**, 3866 (2020), arXiv:1908.05038 [astro-ph.HE].

E. L. Osborne and D. I. Jones, Gravitational waves from magnetically induced thermal neutron star mountains, Mon. Not. R. Astron. Soc. **494**, 2839 (2020), arXiv:1910.04453 [astro-ph.HE].